MCA 2009 – EXAMINATION PAPER

1. The function \( f(x) = q + \lfloor \sin x \rfloor \) is
   1) Discontinuous everywhere
   2) Not differentiable at \( x = 0 \)
   3) Not differentiable at an infinite number of points
   4) All of the above

2. The function \( f(x) = 2 \log(x - 2) - x^2 + 4x + 1 \) increase in the interval
   1) (1, 2)
   2) (2, 3)
   3) (3, 2)
   4) (1, 5)

3. The equation \( e(x - 1) + x - 2 = 0 \) has
   1) One real root
   2) Two real roots
   3) Three real roots
   4) Four real roots

4. The area of the figure bounded by the curves \( y = [x - 1] \) and \( y = 3-[x] \) is
   1) 2
   2) 3
   3) 4
   4) 1

5. The order and degree of the differential equation \( (y'')^2 = 1 + (y''')^3 \) are respectively
   1) 2 and 3
   2) 2 and 2
   3) 3 and 3
   4) 3 and 2

Directions for question (6-10)

Read the passage and answer the questions given below the passage.
A family consisting of six members AA, BB, CC, DD, EE and FF is travelling together.

- BB is the son of CC but CC is not the mother of BB
- FF is the brother of BB
- EE is the brother of CC
- DD is the daughter of AA
- AA and CC are a married couple

6. How many male members are there in the family?
   1) 1
   2) 2
   3) 3
   4) 4

7. Who is the mother of BB?
   1) AA
   2) DD
   3) CC
   4) EE

8. How many children does AA have?
   1) 1
   2) 2
   3) 3
   4) 4

9. How many brother-brother pair can be made from the above family?
   1) 1
   2) 2
   3) 3
   4) 4

10. How is EE related to DD?
    1) Father
    2) Brother
    3) Uncle
    4) Sister-in-law

Directions for questions (11-15):

In each of the questions below consists two statements followed by four conclusions. You have to assume they are true, even if they seem to be different from commonly known facts. Then decide which of the alternative conclusions logically follows.

11. Statements:
    i) All boys are fathers
    ii) Some fathers are children
    Conclusions:
        1. All boys are children
        2. Some boys are children
        3. All fathers are boys
        4. None of the above

12. Statements:
    i) Some circles are lines.
    ii) Some circles are squares.
    Conclusions:
        1. All circles that are lines are squares.
        2. All circles are either lines or squares.
        3. Some circles are neither squares nor lines.
        4. None of the above

13. Statements:
    i) All teachers are fathers.
    ii) Some students are teachers.
    Conclusions:
        1. All students are fathers.
        2. Some teachers are students.
        3. Some students are fathers.
4. No student is a father

14. **Statements:**
i) Some people are scientists.
ii) Some people are rich

**Conclusions:**
1. Some rich are scientists
2. Some scientists may be rich
3. No scientist is rich
4. Some people are not scientist but rich

15. **Statements:**
i) All players are healthy.
ii) Kapil Dev is healthy.

**Conclusions:**
1. Kapil Dev is a player
2. Kapil Dev is not a player
3. All healthy people are player
4. Kapil Dev may be a player

16. The value of $x$ in the series 9, 35, 91, 189, $x$, 559, ..... is

1) 341  2) 241
3) 211  4) 391

**Directions for questions (17-20):**
Each question given have some computation steps to execute. Then a PRINT statement follows to print the value of variable following the PRINT statement. Identify the right alternative that can be said about the output.

17. $X = 10$  
$P = 10$

For I = 1 to 10 days
{
    $P = P*(X-1)$
    $X = X-1$
}

PRINT P
1) 10!  2) 9!  3) 11!  4) 8!

18. $X = 1$
$Y = 2$

For I = 1 to 5 do
{
    $X = X + Y$;
    $Y = Y + X$;
}

PRINT X, Y;
1) 34, 35  2) 55, 90
3) 144, 233  4) 233, 377

19. $X = 1$
$Y = 0.5$
$Z = Y$

For I = 1 to 100 do
{
    $X = X + Y$;
    $Z = Z * Y$;
}

PRINT X
The following can be said about the value of X:
1) 2  2) $1 < X < 2$
3) $X = 2$  4) $X < 2$

20. $X = 0$
$M = 1$

Which (M ≤ 100) do

N = 1
{
    While (N < M do
        $X = X + 4$;
        $N = N + 1$;
    }

M = M + 1;

PRINT X;
1) 99 * 100  2) 99 * 200
3) 99 * 300  4) 99 * 400

21. The solution of the differential equation ydx – xdy + e^{(1/x)} dx = 0 is

1) $y + xe^{(1/x)} = cx$  2) $y + xe^{(1/x)} = cx$
3) $y + xe^{(1/x)} = c$  4) $y + xe^{(1/x)} = cx + cy$

22. How many license plates consisting of three letters followed by three digits contain no letter or digit twice?

1) 11,232,000  2) 11,00,000
3) 26! 10!  4) 11,132,00
23. How many bijective (both one-one and onto) functions are there from an N element set to an N element set?
   1) \( N^2 \)  
   2) \( N^N \)  
   3) \( N! \)  
   4) \( 2^N \)  

24. Suppose that a die is biased so that 3 appears twice as often as each other number but that the other five outcomes are equal likely. What is the probability that an odd number appears when we roll this die?
   1) 1/2  
   2) 4/7  
   3) 3/7  
   4) 2/7  

25. If A and B are any two arbitrary events, then which one of the following is true?
   1) \( P(A \cap B) = P(A) + P(B) \)  
   2) \( P(A \cap B) = P(A) \times P(B) \)  
   3) \( P(A | B) = P(A \cap B) / P(B) \)  
   4) \( P(A \cap B) \leq P(A) + P(B) \)  

26. What is the expected value of the sum of numbers that appear when a pair of dice is rolled?
   1) 7  
   2) 5  
   3) 1  
   4) 4  

27. What is the variance of the random variable \( X \) such that \( X(i, j) = 2i \), where \( i \) is the number appearing on the first die and \( j \) is the number appearing on the second die, when two dice are rolled?
   1) \( 35/3 \)  
   2) \( 1/36 \)  
   3) \( 36 \)  
   4) \( 6 \)  

28. The series \( a_1 - a_2 + a_3 - a_4 + \ldots \) is convergence if
   1) \( a_n \)’s are all positive  
   2) \( a_n > a_{n+1} \) for every \( n \)  
   3) \( a_0 = 0 \)  
   4) All of the above  

29. Which one of the following is true?
   1) Every linear programming problem has an optional solution  

30. The event A is independent of itself if and only if
   1) \( P(A) = 1 \)  
   2) \( P(A) = 1/2 \)  
   3) \( P(A) = 1/3 \)  
   4) \( P(A) = 1/4 \)  

31. If the sum of two numbers is 200, then the largest value of their product is
   1) 9000  
   2) 10000  
   3) 100000  
   4) 1900  

32. The equation of the curve which passes through the point (1, -1) and has slope \( 3x^2 \) is
   1) \( y = 3x^3 - 2 \)  
   2) \( y = 3x^2 \)  
   3) \( y = 3x^3 \)  
   4) \( y = 3x \)  

33. Which one of the following is true?
   1) A semi-group with more than one idempotent element cannot be a group  
   2) If G is a group, then it may not be a monoid  
   3) G is group if and only if it is a semi-group  
   4) None of the above  

34. The points of maximum and minimum curvature on the curve \( y = \log x \) are real and \( -\pi \leq x \leq \pi \) are
   1) \( \pm \pi \)  
   2) 0, \( \pm \pi/4 \) respectively  
   3) \( \pm \pi/2, \pm \pi \) respectively  
   4) \( \pm \pi, \pm \pi/2 \) respectively  

35. The area of the circle whose center is at (0,0) is 25\( \pi \). The circle passes through all the points except
   1) (-5, 0)  
   2) (5, 0)  
   3) (5, 5)  
   4) (0, 5)  

36. A classroom has \( r \) rows of desks with \( d \) desks in each row. On a particular day when all pupils are present 3 seats are
left vacant (one student per desk). The number of pupils in the class is
1) dr-3       2) d + r + 3
3) dr + 3      4) r/d + 3

37. The length of a rectangle is increased by 50%. By what per cent the width has to be decreased to maintain the same area?
1) 33.33      2) 50
3) 66.67      4) 150

38. If the radius of a circle is 0.5 m, how many revolutions does the wheel make per kilometer?
1) 1000       2) 2000
3) 1000/π      4) 1000/π

39. The average of 5, 10, 15 and X is 20. What is X?
1) 20         2) 25
3) 45         4) 50

40. What is the largest prime factor of 255?
1) 15         2) 5
3) 51         4) 17

41. A and B are in the ratio 5:4. B and C are in the ratio 6:7 then A:B:C is
1) 30 : 24 : 28  2) 5 : 10 : 7
3) 5 : 4 : 7     4) 15 : 12 : 14

42. Some pumps discharge at 50 kl a minute that can irrigate 4 hectares in 8 hours. If the flow is 40 kl a minute, how long it will take to irrigate 6 hectares?
1) 15 hours
2) 6 hours and 40 minutes
3) 9 hours
4) 9 hours and 36 minutes

43. X men can do a piece of work in y days. How many men are required to do it in z days?
1) zy/x       2) xy/z
3) xz/y       4) z/(xy)

44. If a/b = c/d which of the following is not true?
1) (a + 3c) / (b + 3d) = (a – b) / (c – d)
2) (a + c) / (b + d) = (a – c) / (b – d)
3) (a + b) / (c + d) = (a – b) / (d – c)
4) (a + b) / (c + d) = (b – a) / (c – d)

45. A student is required to get 40% marks to pass. He gets 253 marks and fails by 27 marks. The total marks of the exam is
1) 600       2) 500
3) 700       4) 800

46. A’s income is 20% more than that of B’s. Find by what percent B’s income is less than that of A?
1) 16.67      2) 20
3) 25         4) 10.75

47. The population of a town is increased by 10% in a year and then decreased by 10% in the next year. After the second year what is the population more or less than that of 2 years ago?
1) no increase  2) 1% more
3) 1% less      4) 10% more

48. A radio is sold at Rs. 1150. Repairing charges amounted to Rs. 50. If there is a profit of 15%, the cost price is
1) Rs. 1000    2) Rs. 950
3) Rs. 1050    4) Rs. 1045

49. The cost price of 10 articles is the same as the selling price of 8 articles. The profit is
1) 25%        2) 20%
3) 30%        4) 10%

50. An article was sold at a gain of 8%. If it had been sold for 10 paise more, the gain would have been 18%. The cost price is
1) Rs. 100    2) Re. 1
3) 10 paise   4) Rs.10

51. If AB is a zero matrix, then
1) A = 0 or B = 0
2) A = 0 and B = 0
3) It is not necessary that either A and B should be 0
4) None of the above
52. MICR code is used in
   1) Letters  2) Cheques
   3) Punched Cards  4) Magnetic Tape

53. In evaluating expressions the following priorities are used. Integer / division (/) and multiplication (*) have some priority but higher than addition (+) and subtraction (-) which have same priority. Left associativity is used for operators. Find the value of the following integer arithmetic expression.
   \[(7 \text{ and } 3) \times 5/2 + 10 \times 8 + 2\]
   The mod operator represents modulus operation i.e. remainder of division.
   1) 42
   2) 28
   3) 4
   4) 31

54. In an Indo-American committee, 2/3 of the member are men, and 3/8 of the men are Americans. If 3/5 of the committee members are Indians, What fraction of the members are American woman?
   1) 3/20
   2) 11/60
   3) 3/4
   4) 7/5

55. If the sum of all positive even integers < 1000 is A, what is the sum of all positive odd integers less than 1000?
   1) A - 998
   2) A - 499
   3) A + 1
   4) A + 500

56. A sequence of numbers begin as 1, 1, 1, 2, 2, 3 and repeats this pattern for ever. What is the sum of 141^{st}, 143^{rd} and 145^{th} number?
   1) 4
   2) 5
   3) 6
   4) 7

57. The product of the three hexadecimal numbers \(2^E \times 8^2 \times 10^3\) could be
   i) \(10^8\)
   ii) \(2^{20}\)
   iii) \(8^{11}\)
   iv) \(16^8\)
   The correct answer is
   1) (i) only
   2) (i) and (ii)
   3) (iv) only
   4) (i) and (iv)

58. In Boolean algebra for the given two variables A and B, find the value of AB + A ^ B + ^ AB + ^ A ^ B; ^ refers to NOT of the symbol that follows
   1) 0
   2) 1
   3) A + B
   4) A

59. The following is a FORTRAN program segment
   \[I = 3\]
   \[DO 10 J = 2, 11, 4\]
   \[10 I = I + J\]
   The value of I after this program segment will be
   1) 47
   2) 11
   3) 21
   4) 31

60. K = 0
   \[DO 10 I = 1, 10\]
   \[DO 10 J = 1, 5\]
   If Odd (J) then K = K + 1 else K = K – 1
   10 continue
   Odd (J) is true if J is odd number.
   The value of K after this program segment will be
   1) 0
   2) 10
   3) 15
   4) 5

61. In a C language program segment a, b, c are integers a = 2, b = 1, c = 3
   The statement is print ("%d, %d, %d", a = b + c, b = a + c, c = a + b);
   The printed values are
   1) 4, 5, 3
   2) 4, 7, 11
   3) 8, 5, 3
   4) 4, 11, 7

62. Which of the following is not a programming language?
   1) ADA
   2) LISP
   3) PL/1
   4) BEANS

63. Minimum number of comparisons required to find the largest of a given set of N numbers is
   1) N
   2) N - 1
   3) N + 1
   4) N * N
64. LAST in FIRST out data structure is called
   1) Queue  2) Tree  3) Stack  4) Graph

65. The binary number corresponding to the following decimal expression 10*256 + 12*16 + 9 is
   1) 110101001011  2) 111101001011  3) 101011101001  4) 101011001001

66. A rectangle of 1 cm long and w cm wide is made 3 cm longer. The area has increased by
   1) 3/w  2) 3w  3) 3l  4) 3l + 3w + 9

67. City X is 200 miles east of city Y and city Z is 150 miles north of city Y. What is the shortest distance between X and Z?
   1) 507  2) 175  3) 250  4) 300

68. I had 18 km to reach a place X. After walking a few km at 4 km per hour. I changed my speed to 3 km per hour. If I took 5 hours to reach X at which distance from X, I changed my speed?
   1) 9 km  2) 3 km  3) 12 km  4) 6 km

69. The interest on a certain loan is Rs. 100 in the first year and Rs. 205 in the first two years. The rate of interest is
   1) 5%  2) 4%  3) 10%  4) 6%

70. The difference between S.I. and C.I. for two years on a certain sum of money at 4% is Rs. 160. The sum is
   1) 1.5 lakhs  2) 1 lakh  3) 0.75 lakh  4) 2 lakhs

71. What sum of money will amount to Rs. 1352 in two years at 4% compound interest?
   1) Rs. 1000  2) Rs. 1150  3) Rs. 1250  4) Rs. 1200

72. How many terms of the series 8 + 10 + 12 + … must be taken to make 228?
   1) 12  2) 11  3) 10  4) 13

73. The sum of three numbers in A.P. is 51 and the product of the extremes is 273. The greatest number in the A.P. is
   1) 17  2) 21  3) 13  4) 19

74. The next number in the sequence 2, 5, 10, 17, 28, 41, … is
   1) 58  2) 54  3) 50  4) 56

75. If 4x – x^2 - 2b < 0 for all values of x, then
   1) b > 4  2) b < 2  3) b > 2  4) b > 3

76. The number of mappings from (a, b, c) to (x, y) is
   1) 3  2) 6  3) 8  4) 9

77. If f = { (6, 2), (5, 1) }, g = { (2, 6), (1, 5) } then f o g =
   1) (6, 6) (5, 5)  2) (2, 2) (1, 1)  3) (6, 2) (2, 6) (5, 1) (1, 5)  4) (6, 6) (1, 1)

78. If A = {1, 2, 3} and B = {a, b, c, d}, the number of subsets in the Cartesian product of A and B is
   1) 2^{12}  2) 2^7  3) 12  4) 7

79. The solution of the equation x^{2/3} – 3x^{1/3} + 2 = 0 is
   1) 1, 2  2) 1, 8  3) 2, 6  4) 1, 4

80. f(x) = ax^2 + bx + c, then the solution of f'(x) = 0 is
   1) A.M. of the roots of f(x) = 0  2) G.M. of the roots of f(x) = 0  3) H.M. of the roots of f(x) = 0  4) None of the above
81. Which of the following may be true quadratic equation (α is real)?
1) If α is a root, 1/α is also a root
2) If α is a root, -α is also a root
3) If α is a root, Iα is also a root
4) If α is a root, -Iα is also a root

82. If α and β are the roots of [x² + x + 5] + 6x + 1 = 0 then α + β is
1) 7  
2) -7
3) 5  
4) -5

83. If a + b + c = 0, then one root of the equation ax² + bx + c = 0 is
1) -b/a  
2) -c/a
3) (a + c)/a  
4) (a + b)/a

84. If a and b are positive integers such that a³ - b³ is a prime number, then a³ - b³ is
1) a² + ab + b²  
2) a² + ab + b²
3) a + b  
4) a - b

85. The smallest angle between the hour and minute needles of a clock when the time is 12 hr 30 min is
1) 180°  
2) 165°
3) 196°  
4) 150°

86. The dollating sequence of five alphabets is W, P, Z, A, E. Which will be the first string of the above collating sequence?
1) AZPWW  
2) APAEP
3) ZPAPA  
4) ZAPWE

87. How many numbers between 100 and 300 (inclusive) is divisible by 3?
1) 100  
2) 66
3) 76  
4) 56

88. Which of the following figures has the largest area of the given circumference?
1) Square  
2) Triangle
3) Circle  
4) Ellipse

89. If 2 < r < 8 and 1 < s < 5/2 which of the following expresses all possible values of rs?
1) 1 < rs < 5  
2) 2 < rs < 20
3) 5/2 < rs < 8  
4) 5/2 < rs/20

Directions for Questions (90-95):
Ocean water plays an indispensable role in supporting life. The great ocean basins hold about 300 million cubic miles of water. From this vast amount, about 80,000 cubic miles of water are sucked into the atmosphere each year by evaporation and returned by precipitation and drainage to the ocean. More than 24,000 cubic miles of rain descend annually upon the continents. This vast amount is required to replenish the lakes and stream, springs and water tables on which all flora and fauna are dependent. Thus, the hydrosphere permits organic existence. The hydrosphere has strange characteristics because water has properties unlike those of any other liquid. One anomaly is that water upon freezing expands by about 9 percent, whereas most liquids contract on cooling. For this reason, ice floats on water bodies instead of sinking to the bottom. If the ice sank, the hydrosphere would soon be frozen solidly, except for a thin layer of surface melt water during the summer season. Thus, all aquatic life would be destroyed and interchange of warm and cold currents, which moderates climate, would be notably absent. Another outstanding characteristic of water is that water has a heat capacity which is the highest of all liquids and solids except ammonia. This characteristic enables the oceans to absorb and store vast quantities of heat, thereby often preventing climatic extremes. In addition, water dissolves more substances than any other liquid. It is the characteristic which helps make oceans a great storehouse for minerals which have been washed down from the continents. In several areas of the world these minerals are being commercially exploited. Solar evaporation of salt is widely practiced, potash is extracted from the Dead Sea, and magnesium is produced from sea water along the American Gulf Coast.

90. The author’s main purpose in this passage is to
1) Describe the properties and uses of water
2) illustrate the importance of conserving water
3) Explain how water is used in commerce and industry
4) Compare water with other liquids

91. **According to the passage, fish can survive in the oceans because**
1) they do not need oxygen
2) ice floats
3) evaporation and condensation create a water cycle
4) there are currents in the oceans

92. **Which of the following characteristics of water does the author mention in the passage?**
I. Water expands when it is frozen
II. Water is a good solvent
III. Water can absorb heat.
1) I only 2) II only 3) I and II only 4) I, II and III

93. **According to the passage the hydrosphere is not**
1) responsible for all forms of life
2) able to modify weather
3) a source of natural resources
4) in danger of freezing over

94. **The author’s tone in the passage can best be described as**
1) Dogmatic 2) Dispassionate
3) Speculative 4) Biased

95. **The author organizes the passage by**
1) comparison and contrast
2) juxtaposition of true and untrue ideas
3) general statements followed by examples
4) hypothesis and proof

96. **Pick the odd one out.**
1) Counting sort 2) Bucket sort
3) Shell sort 4) Radix sort

97. **The complexity of merge operation on two sorted arrays of size m and n (given m > n) is**
1) O(m n) 2) O(m + n)
3) O(m/n) 4) O(m)

98. **Pick the odd one cut**
1) Random Value 2) Return Address
3) Local variable space 4) Global variable space

99. **The function for which among the following will not be linear recursion**
1) Factorial
2) Fibonacci series
3) \(a^b\)
4) Some of the natural numbers

100. **How many stack will be needed for the evaluation of a prefix expression?**
1) 1 2) 2
3) 0 4) 3
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1. (3) 
Graph of \( f(x) = q + |\sin x| \) 
\[ f(x) = q + |\sin x| \]
\[ f(x) = q + \sin x, \quad x \geq 0 \]
\[ = q - \sin x, \quad x < 0 \]
\[ \Rightarrow Lf'(0) \neq Rf'(0) \]
\[ \Rightarrow f(x) \text{ is not differentiable at } x = 0 \]
From the above graph for \( f(x) = P + |\sin x| \) 
\[ x = \pm \pi, \pm 2\pi, \pm 3\pi, \ldots \] has some pattern as 
\[ x = 0 \]
\[ \Rightarrow f(x) = p + |\sin x| \text{ is not differentiable an} \]
infinite number of points \( x = 0, \pm \pi, \pm 2\pi, \pm 3\pi, \ldots \).

2. (1) 
f(x) = 2 \log(x - 2) - x^2 + 4x + 1 
\[ f'(x) = \frac{2}{x - 2} - 2x + 4 \]
If \( f(x) \) increases then 
\[ f'(x) > 0 \]
Solving \( y = x - 1 \) and \( y = 3 - x \).
x = 2, y = 1
\[\therefore\text{ Coordinates of B are (2, 1)}\]
Also coordinates of A are (1, 0)
Coordinates of C are (0, 3)
\[AB = \sqrt{(2 - 1)^2 + (1 - 0)^2} = \sqrt{1^2 + 1^2} = \sqrt{2}\]
\[BC = \sqrt{(0 - 2)^2 + (3 - 1)^2} = \sqrt{4 + 4} = 2\sqrt{2}\]
\[\therefore\text{ Area of ABCD} = AB \times BC = 2 \times 2\sqrt{2} = 4\]

5. (1)
In \(1 + (y'')^3 = (y'')^2\)
order = 2
degree = 3

Directions for (6-10)

6. (4)
CC, EE, BB, FF are males
7. (1)
AA is mother of BB.
8. (3)
DD, FF and BB are children of AA.
9. (4)
FF is a brother of BB
BB is the brother of FF
EE is the brother of CC
CC is the brother of EE
10. (3)
EE is the uncle of DD.

Directions (11-15) :
11. (4)
None of the given conclusion follows

12. (3)
Conclusion 3 follows.

13. (2)
Conclusion 2 follows.

14. (4)
Conclusion (4) follows.

15. (4)
Kapil Dev may be a player.

16. (1)
\[1^3 + 2^3 = 9\]
\[2^3 + 3^3 = 35\]
\[3^3 + 4^3 = 91\]
\[4^3 + 5^3 = 189\]
\[5^3 + 6^3 = 341\]
\[6^3 + 7^3 = 559\]
\[\therefore x = 341\]

21. (1)
ydx - xdy + e^{(1/x)}dx = 0
\[\Rightarrow ydx - xdy = -e^{(1/x)}dx\]
\[\Rightarrow xdy - ydx = e^{(1/x)}dx\]
\[\Rightarrow \frac{xdy - ydx}{x^2} = \left(\frac{1}{x^2}\right) e^{(1/x)}dx\]
\[\Rightarrow d\left(\frac{y}{x}\right) = \frac{1}{x^2} e^{(1/x)}dx\]
On integrating
\[ \int \frac{dy}{x} = \int e^{\frac{1}{x^2}} \left( \frac{1}{x^2} \right) \, dx \]
\[ \Rightarrow y = -e^{\frac{1}{x}} + c \]
\[ y = -xe^{\frac{1}{x}} + xc \]
\[ y + xe^{\frac{1}{x}} = xc \]

22. (1)
No. of licence plates = 26P^3 \times 10P^3
= (26 \times 25 \times 24) \times (10 \times 9 \times 8)
= 11232000

23. (3)
No. of bijective functions from an N element set to an N element set = N!

24. (2)
\[ \frac{5x + 2x}{6} = 1 \]
\[ \Rightarrow 7x = 6 \]
\[ \therefore x = \frac{6}{7} \]
Probability of getting 3
\[ = \frac{2x}{6} = \frac{1}{5} \frac{6}{7} = \frac{2}{7} \]
Probability of getting other than 3
\[ = \frac{x}{6} = \frac{1}{7} \]
Probability of getting other than 3
\[ = \frac{x}{6} = \frac{1}{7} \]
Probability of getting odd number
\[ = P(1) + P(3) + P(5) \]
\[ = \frac{1}{7} + \frac{2}{7} + \frac{1}{7} = \frac{4}{7} \]

25. (4)
If A and B are any two arbitrary events then
\[ P(A \cap B) \leq P(A) + P(B) \]

26. (1)
Possible sums
\[ S = \{2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12\} \]
Possible events of getting sum is 2 = (1, 1)

P(2) = \frac{1}{36}
Possible events of getting sum is 3 = (2, 1) and (1)
\[ \therefore P(3) = \frac{2}{36} \]
Similarly
P(4) = \frac{3}{36}
P(5) = \frac{4}{36}
P(6) = \frac{5}{36}
P(7) = \frac{6}{36}
P(8) = \frac{5}{36}
P(9) = \frac{4}{36}
P(10) = \frac{3}{36}
P(11) = \frac{2}{36}
P(12) = \frac{1}{36}

\[ E(S) = 2 \times \frac{1}{36} + 3 \times \frac{2}{36} + 4 \times \frac{3}{36} + 5 \times \frac{4}{36} + 6 \times \frac{5}{36} + 7 \times \frac{6}{36} + 8 \times \frac{5}{36} + 9 \times \frac{4}{36} + 10 \times \frac{3}{36} + 11 \times \frac{2}{36} + 12 \times \frac{1}{36} \]
\[ = \frac{2}{36} + \frac{6}{36} + \frac{12}{36} + \frac{20}{36} + \frac{30}{36} + \frac{42}{36} + \frac{40}{36} + \frac{30}{36} + \frac{22}{36} + \frac{12}{36} \]
\[ = \frac{252}{36} = 7 \]

27. (1)
Possible events of getting sum is 2 = (1, 1)

X (1, j) = 2 \times 1 = 2
Similarly for the second row
X (2, j) = 2 \times 2 = 4
1. Also \[ X(3, j) = 3 \times 2 = 6 \]
   \[ X(4, j) = 4 \times 2 = 8 \]
   \[ X(5, j) = 10 \]
   \[ X(6, j) = 12 \]

   \[ E(X) = 2 \times \frac{6}{36} + 4 \times \frac{6}{36} + 6 \times \frac{6}{36} + 8 \times \frac{6}{36} + 10 \times \frac{6}{36} + 12 \times \frac{6}{36} \]
   \[ = \frac{1}{6} \left[ 2 + 4 + 6 + 8 + 10 + 12 \right] \]
   \[ = 7 \]

   \[ E(X^2) = 2^2 \times \frac{6}{36} + 4^2 \times \frac{6}{36} + 6^2 \times \frac{6}{36} + 8^2 \times \frac{6}{36} + 10^2 \times \frac{6}{36} + 12^2 \times \frac{6}{36} \]
   \[ = \frac{1}{6} \left[ 4 + 16 + 36 + 64 + 100 + 144 \right] \]
   \[ = \frac{364}{6} \]

   \[ \because \text{Variance} = E(X^2) - (E(X))^2 \]
   \[ = \frac{364}{6} - 7^2 \]
   \[ = \frac{364}{6} - 49 \]
   \[ = \frac{364 - 294}{6} \]
   \[ = 70 \]

28. (4)
Consider the convergent series
\[ \log 2 = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \frac{1}{6} + \ldots \]
This can be compared with
\[ a_1 - a_2 + a_3 - a_4 + \ldots (-1)^{n+1}a_n + \ldots \]
Where \( a_n = \frac{1}{n} \)
In the above series, \( a_n \)'s are all positive
Also \( \frac{1}{n} > \frac{1}{n+1} \)
\[ \Rightarrow a_n \to a_{n+1} \]
Also \( \lim_{n \to \infty} \frac{1}{n} = 0 \)
\( \lim_{n \to \infty} a^n = 0 \)
\( a^n \to 0 \)
\[ \therefore (1) \text{ (2) and (3) are true} \]

29. (2)
If a linear programming problem, has an optional solution, then at least one basic feasible solution must be optional.

30. (1)
If A and B are independent events then \( P(A \cap B) = P(A) P(B) \)
If A is independent to itself then
\[ P(A \cap A) = P(A) P(A) \]
\[ \Rightarrow P(A) = P(A) P(A) \]
\[ \Rightarrow 1 = P(A) \]

31. (2)
Let the numbers be x and y then
\[ x + y = 200 \]
\[ \Rightarrow y = 200 - x \]
Product \( = xy = x[200-x] \)
Let \( f(x) = x(200-x) \)
\[ = 200x - x^2 \]
\( f'(x) = 200 - 2x \)
\( f''(x) = -2 \)
For maxima
\[ f'(x) = 0, \quad f''(x) < 0 \]
\[ 200 - 2x = 0 \]
\[ x = 100 \]
\[ f''(100) = -2 < 0 \]
\[ \therefore x = 100 \text{ gives maximum} \]
Where \( x = 100 \)
\[ y = 200 - x \]
\[ = 200 - 100 = 100 \]
Maximum product \( = 100 \times 100 = 10000 \]

32. (1)
\[ \frac{dy}{dx} = 3x^2 \]
\[ \Rightarrow dy = 3x^2 dx \]
\[ \Rightarrow \int dy = \int 3x^2 dx \]
\[ \therefore y = 3 \left( \frac{x^3}{3} \right) + c \]
\[ \Rightarrow y = x^3 + c \]
(1)
(1) passes through (1,-1)
\[ \Rightarrow -1 = (1)^3 + c \]
\[ \Rightarrow -1 = 1 + c \]
\[ \therefore c = -2 \]
\[ \therefore (1) \Rightarrow \]
\[ y = x^3 - 2 \]
33. (1) A semi-group with more than one indempotent element cannot be a group.

34. (4) Maximum curvature = ±π, ±π/2

35. (3) Let the radius be r
Then area = πr² = 25π
⇒ r² = 25 ⇒ r = 5
Equation of the circle = x² + y² = 25
From the above diagram, (5, 5) does not lie on the circle.
∴ The circle does not pass through (5, 5).

36. (1) Number of pupils = Total desks – 3
= d × r – 3

37. (1) Let the length and breadth of the rectangle be l and b.
Then area = lb
New length = \( \frac{150}{100} \) l = \( \frac{3l}{2} \)
Let New breadth be b'
then area \( \frac{3l}{2} \times b' = lb \) (area same)
\( b' = \frac{lb}{\frac{3l}{2}} = \frac{2b}{3} \)
Decrease in breadth = b − \( \frac{2b}{3} \) = \( \frac{b}{3} \)
% decrease in breadth = \( \frac{\frac{b}{3}}{b} \times 100 \)

38. (3) Perimeter = 2πr
Number of revolutions wheel make per 1 km (=1000 meter)
\( = \frac{1000}{2\pi \times 0.5} = \frac{1000}{\pi} \)

39. (4) Average \( = \frac{5+10+15+x}{4} = 20 \)
⇒ 30 + X = 80
⇒ X = 80 – 30
= 50

40. (4) From the given choices 15 and 51 are not prime numbers.
Also \( \frac{255}{17} = 15 \)
∴ largest prime factor = 17

41. (4) LCM of 4 and 6 = 12
Now A : B = 15 : 12
B : C = 12 : 14
∴ A : B : C = 15 : 12 : 14

42. (1) \( \text{Kg per minute} \text{ hectares} \text{ time} \)
\[ \frac{50}{40} \times \frac{6}{4} \times 8 = 15 \]

43. (2) 1 man can do the work in xy days.
∴ No. of men required to complete the work in z days = \( \frac{xy}{z} \)

44. (3) \( \frac{(a+b)}{(c+d)} = \frac{(a-b)}{(d-c)} \) is not true

45. (3) Let the total marks be T
then
\[40\% \text{ of } T - 253 = 27\]
\[\frac{40}{100} \times T = 253 + 27 = 280\]
\[\therefore T = \frac{280 \times 10}{4} = 700\]

46. (1) 
**Formula:**
If A is R\% more than B, then B is less than A. by
\[\left[\frac{R}{100 + R} \times 100\right] \%\]
\[\therefore \text{ Required percentage }\]
\[= \frac{20}{100 + 20} \times 100\]
\[= \frac{20}{120} \times 100 = 16.67\%\]

47. (3)
Let the population be x.
After first year population
\[= \frac{110}{100} x\]
After two years population
\[= \frac{90}{100} \left(\frac{110}{100} x\right) = \frac{99}{100} x\]
% less = \[\frac{\frac{x}{100} \times 99x}{x} \times 100 = \frac{99}{100} \times 100 = 1\%\]
\[\therefore \text{ After 2 years there is 1\% less than that of 2 years ago.}\]

48. (2)
\[\therefore \text{ Selling price } = Rs. 1150\]
Total cost price = Cost price + Repairing charge
Now,
Total cost price = \[\frac{100}{100 + \text{profit } \%} \times \text{S. P.}\]
\[= \frac{100}{115} \times 1150 = 1000\]
Cost price + Repairing charge
\[= 1000\]
\[\Rightarrow \text{ Cost price } = 1000 - 50 = Rs. 950\]

49. (1)
Let the cost price of 1 article be Rs. 1
Cost price of 8 articles = Rs. 8
Selling price of 8 articles
\[= \text{ Cost price of 10 article } = Rs. 10\]
\[\therefore \text{ Profit of 8 articles }\]
\[= 10 - 8 = 2\]
\[\therefore \text{ Profit } \% = \frac{\text{ profit } \times \text{ cost price}}{100}\]
\[= \frac{2}{8} \times 100 = 25\%\]

50. (2)
Let the cost price be Rs. x
\[\frac{108x}{100} + 10 = \frac{118x}{100}\]
\[\frac{108x}{100} = 10\]
x = 100 paise = Re. 1

51. (3)
Let
\[A = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}\]
\[B = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\]
Then clearly A ≠ 0, B ≠ 0
Now,
\[AB = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 0 \\ 0 & 0 \end{pmatrix}\]
\[\therefore AB = 0\]
Therefore AB is a zero matrix, then it is not necessary that either A and B should be zero.

54. (1)
Let the total members be x
No. of men = \[\frac{2x}{3}\]
\[\therefore \text{ No. of women } = x - \frac{2x}{3} = \frac{x}{3}\]
Male Americans = \[\frac{3}{8} \times \frac{2x}{3} = \frac{x}{4}\]
No. of Indians = \[\frac{3x}{5}\]
\[\therefore \text{ No. of Americans } = x - \frac{3x}{5} = \frac{2x}{5}\]
\[\therefore \text{ American women } = \frac{2x}{5} - \frac{x}{4}\]
i.e., $\frac{3}{20}$ of the Committee members are American women.

55. **(4)**

Given $2 + 4 + 6 + 8 + \ldots + 988 = A$

Now

$$1 + (2 + 1) + (4 + 1) + (6 + 1) + (8 + 1) + \ldots + (988 + 1) = A + 500$$

$$\Rightarrow 1 + 3 + 5 + 7 + 9 + \ldots + 999 = A + 500$$

56. **(1)**

Given sequence is

1, 1, 2, 2, 3, 1, 1, 2, 2, 3, 1, 1, 2, 2, 3, \ldots

The sequence repeats after every six terms

Clearly $6 \times 23 = 138$

\[ \therefore \text{The sequence repeats in } 139^{\text{th}} \text{ term.} \]

57. **(3)**

Length = 1 cm

Breadth = w cm

Area = $1 \times w = w$ cm$^2$

New length = $1 + 3 = 4$ cm

New area = $4w = 4w$ cm$^2$

Increase = $4w - w = 3w$

68. **(4)**

Let the speed is changed at B and BX = d.

then $AB = 18 - d$

Now $\frac{18 - d}{4} + \frac{d}{3} = 5$

$\Rightarrow 54 + d = 60$

$\therefore d = 6\text{ km}$

69. **(1)**

\[ \frac{P \times 1 \times R}{100} = 100 \quad \ldots (1) \]

Also \[ \frac{(P + 100) \times 1 \times R}{100} = 105 \quad \ldots (1) \]

\[ (1) \div (2) \]

\[ \Rightarrow \frac{P}{P + 100} = \frac{100}{105} \]

\[ \Rightarrow 5P = 10000 \]

\[ \therefore P = 2000 \]

\[ \therefore (1) \Rightarrow \frac{100 \times R}{2000 \times R} = 100 \]

\[ \therefore R = \frac{100}{20} = 5\% \]

70. **(2)**

Formula:

The difference between S.I. and C.I. for two years

\[ = P \left( \frac{R}{100} \right)^2 \]

Given Difference $= 160$

$R = 4\%$

\[ \therefore P \left( \frac{4}{100} \right)^2 = 160 \]

\[ \Rightarrow P \left( \frac{1}{25} \right)^2 = 160 \]

\[ \Rightarrow P \left( \frac{1}{625} \right) = 160 \]

\[ \Rightarrow P = 160 \times 625 \]

\[ = 100000 \]

\[ = \text{Rs. 1 lakh} \]

71. **(3)**
Amount \[= P \left( 1 + \frac{R}{100} \right)^2\]
\[= 1352\]
\[\Rightarrow P \left( 1 + \frac{4}{100} \right)^2 = 1352\]
\[P \left( \frac{26}{25} \right)^2 = 1352\]
\[P = \frac{625}{676} \times 1352 = Rs. 1250\]

72. (1)
Sn = \[\frac{n}{2} [2a + (n - 1)d]\]
Sn = 228
a = 8
d = 2
\[\therefore 228 = \frac{n}{2} [2 \times 8 + (n - 1)2]\]
\[= N[8 + (n - 1)]\]
\[= 8n + n^2 - n\]
\[= n^2 + 7n\]
\[\Rightarrow n^2 + 7n - 228 = 0\]
\[\Rightarrow (n - 12)(n + 19) = 0\]
\[\Rightarrow n = 12 (n \text{ positive})\]

73. (2)
Let the three terms of A.P. are a – d, a, a + d
sum \[= (a – d) + a + (a + d) = 51\]
\[\Rightarrow 3a = 51\]
\[\therefore a = 17\]
Also product of extrems
\[= 273\]
\[\Rightarrow (a – d)(a + d) = 273\]
\[\Rightarrow a^2 - d^2 = 273\]
\[\Rightarrow 17^2 - d^2 = 273\]
\[\Rightarrow 289 - d^2 = 273\]
\[\therefore d^2 = 289 - 273 = 16\]
\[\therefore d = 4\]
Greatest number \[= a + d\]
\[= 17 + 4 = 21\]

74. (4)
\[6^2 + 5 = 41\]
\[7^2 + 7 = 56\]

75. (3)
\[4x - x^2 - 2b < 0\]
\[\Rightarrow x^2 + 4x + 2b > 0\]
\[\Rightarrow (x^2 - 4x + 4) - 4 + 2b > 0\]
\[\Rightarrow (x-2)^2 + 2b - 4 > 0\]
\[\Rightarrow (x-2)^2 is \text{ always positive}\]
\[\therefore (1) \text{ is greater than zero.}\]
\[\text{if } 2b - 4 > 0\]
\[\Rightarrow b - 2 > 0\]
\[\Rightarrow b > 0\]

76. (3)
Number of mappings = \[2^3 = 8\]

78. (1)
No. of subsets of a set having n elements = \[2^n\]
No. of elements of \(A \times B\) = \[3 \times 4 = 12\]
\[\therefore \text{ No. of subsets of } A \times B = 2^{12}\]

79. (2)
\[x^{2/3} - 3x^{1/3} + 2 = 0 \quad \ldots (1)\]
Let \(y = x^{1/3}\)
\[\Rightarrow x = y^3\]
\[\therefore (1) \Rightarrow y^2 + 3y + 2 = 0\]
\[\Rightarrow (y - 2)(y - 1) = 0\]
\[\Rightarrow y = 1, 2\]
If \(y = 1\) then \(x = y^3 = 1^3 = 1\)
y = 2 then \(x = y^3 = 2^3 = 8\)
\[\therefore \text{ Solutions are } x = 1, 8\]

80. (1)
f(x) \[= ax^2 + bx + c\]
f'(x) \[= 2ax + b\]
Let be solution of \(f'(x)\)
f'(x) = 0
2ax + b = 0
\[x = \frac{-b}{2a}\]
\[\therefore \delta = \frac{-b^2}{2a}\]
Now let \(\alpha\) and \(\beta\) be roots of
\[ax^2 + bx + c = 0\]
The sum
\[ \alpha + \beta = \frac{-b}{2} \]
\[ \frac{\alpha + \beta}{2} = \frac{-b}{2a} = \delta \]
\[ \therefore \text{Arithmetic mean of the roots of } f(x) = 0 \]
\[ = \text{Solution of } f'(x) = 0.3 \]

81. (4)
If \( i \alpha \) is a root, then \(-i \alpha\) is also a root.

82. (2)
\[ x^2 + x + 5 + 6x + 1 = 0 \]
\[ x^2 + 7x + 6 = 0 \]
\[ \Rightarrow \alpha + \beta = -7 \]

83. (2)
When \( x = \frac{-c}{a} \)
then
\[ a \left( \frac{-c}{a} \right)^2 - b \left( \frac{-c}{a} \right) + c = a \times \frac{c^2}{a^2} + \frac{bc}{a} + c \]
\[ = \frac{c^2}{a} + \frac{bc}{a} + c \]
\[ = \frac{c^2 + bc + ac}{a} \]
\[ = \frac{c(c + b + a)}{a} \]
\[ = \frac{c \times 0}{0} = 0 \]
\[ [a + b + c = 0] \]
\[ \therefore \frac{-c}{a} \text{ is a root.} \]

84. (1)
\[ a^3 - b^3 \text{ is a prime number.} \]
but
\[ a^3 - b^3 = (a - b)(a^2 + ab + b^2) \ldots \ldots (1) \]
Since \( a^3 - b^3 \) is a prime number implies one of the factors \( a - b \) (or) \( a^2 + ab + b^2 \) is equal to 1.
Since a and b are positive integers.
\[ \Rightarrow a^2 + ab + b^2 \neq 1 \]
\[ \therefore a - b = 1 \]
\[ \therefore (1) \Rightarrow \]
\[ a^3 - b^3 = a^2 + ab + b^2 \]

Example :
\[ 3^3 - 2^3 = 27 - 8 = 19 \] is a prime number
Now \[ 3^3 - 2^3 = 3^2 + 3 \times 2 + 2^2 \]

85. (2)
\[ \text{Angle traced by the hour hand in 12 hours} = 360^\circ \]
\[ \therefore \text{Angle traced it in 12 hrs.} \]
\[ 30 \text{ min} = 30 \text{ min} = \frac{1}{2} \text{hr.} \]
\[ = \frac{360}{12} \times \frac{1}{2} = 15^\circ \]
\[ \text{Angle traced by minute hand in 60 min.} = 360 \]
\[ \therefore \text{Angle traced by it in} \]
\[ 30 \text{ min} \]
\[ = \frac{360}{60} \times 30 = 180^\circ \]
\[ \therefore \text{Required angle} = 180 - 15 = 165^\circ \]

86. (4)
Except (4) in all other strings, some letter repeats.

87. (2)
\[ \text{Required number} = \left\lfloor \frac{300 - 100}{3} \right\rfloor \text{where } \lfloor . \rfloor \text{ is the greatest integer function} \]
\[ = \left\lfloor \frac{200}{3} \right\rfloor = 66 \]

88. (3)
Circle has the greatest area.

89. (2)
\[ 2 < r < 8 \]
\[ 1 < s < \frac{5}{2} \]
\[ \Rightarrow 2 \times 1 < rs < 8 \times \frac{5}{2} \]
\[ \Rightarrow 2 < rs < 20 \]

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