SIMPLE AND COMPOUND INTEREST

INTEREST

Interest is the fixed amount paid on borrowed money.

The sum lent is called the **Principal.**

The sum of the principal and interest is called the **Amount.**

Interest is of two kinds:

(i) Simple interest (ii) Compound interest

(i) Simple interest: When interest is calculated on the original principal for any length of time, it is called simple interest.

❖ Simple interest = $\frac{\text{Principal} \times \text{Time} \times \text{Rate}}{100}$

i.e. S.I. = $\frac{P \times R \times T}{100}$

❖ Amount = Principal + Interest

i.e. $A = P + I = P + \frac{PRT}{100} = P\left[1 + \frac{RT}{100}\right]$

❖ Principal(P) = $\frac{100 \times \text{S.I.}}{R \times T}$

❖ Rate(R) = $\frac{100 \times \text{S.I.}}{T \times P}$

❖ Time(T)= $\frac{100 \times \text{S.I.}}{P \times R}$

❖ If rate of simple interest differs from year to year, then

S.I.=P× $\frac{(R1+R2+R3+\cdots)}{100}$

Example 1:

Find the amount to be paid back on a loan of Find the amount to be paid back on a loan of `18,000 at 5.5% per annum for 3 years

Solution:

P=`18000, R=5.5%, T=3 years

S.I. = $\frac{P \times R \times T}{100} = \frac{18000 \times 5.5 \times 3}{100}$ = Rs.2970

Amount = P + I = 18000 + 2970 = Rs.20970

Example 2:

In how many years will a sum of money triple itself, at 25% per annum simple interest.

Solution:

Let the sum of money be `.P. So, A=3P and

S.I. = A - P = 3P - P = 2P

R=25%

$\therefore$ T= $\frac{100 \times \text{S.I.}}{P \times R} = \frac{100 \times 2P}{P \times 25}$ = 8 years

Example 3:

What rate per cent annum will produce `250 as simple interest on `6000 in 2.5 years

Solution:

P=`6000; Time (T) = 2.5 years; S.I. =`250

$\therefore$ Rate = $\frac{\text{S.I.} \times 100}{P \times T} = \frac{250 \times 100}{6000 \times 2.5} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}\%$

Example 4:

To buy furniture for a new apartment, Sylvia Chang borrowed `5000 at 11% simple interest for 11 months. How much interest will she pay?

Solution:

From the formula, I = Prt, with P = 5000, r=.11, and t = 11/12(in years). The total interest she will pay is

I=5000(.11) (11/12) =504.17

Or `504.17

(ii) Compound interest: Money is said to be lent at compound interest when at the end of a year or other fixed period, the interest that has become due is not paid to the lender, but is added to the sum lent, and the amount thus obtained becomes the principal in the next year or period. The process is repeated until the amount for the last period has been found. Hence, When the interest charged after a certain specified time period is added to form new principal for the next time period, the interest is said to be compounded and the total interest accurse is compounded and the total interest accrued is compound interest.

❖ C.I. = $p\left[\left(1 + \frac{r}{100}\right)^n - 1\right]$;

❖ Amount(A) = $P\left(1 + \frac{r}{100}\right)^n$

Where n is number of time period.

❖ If rate of compound interest differs from year to year, then

Amount = $P\left(1+\frac{r_1}{100}\right)\left(1+\frac{r_2}{100}\right)\left(1+\frac{r_3}{100}\right)$.....

Example 5:

If `60000 amounts to `68694 in 2 years then find the rate of interest.

Solution:

Given: A = `68694

P = `60000

n = 2 years

r=?

∴ $A = P\left(1+\frac{r}{100}\right)^n$

∴ $68694 = 60000\left(1+\frac{r}{100}\right)^2$

⇨ $\frac{68694}{60000} = \left(1+\frac{r}{100}\right)^2$

⇨ $\frac{11449}{10000} = \left(1+\frac{r}{100}\right)^2$

⇨ $1+\frac{r}{100} = \sqrt{\frac{11449}{10000}} = \sqrt{1.1449}$

⇨ $1+\frac{r}{100} = 1.07$

⇨ $\frac{r}{100} = 1.07 - 1 = 0.07$

∴ r = 0.07× 100 = 7%

Example 6:

In how many years, the sum of □10000 will become `10920.25 if the rate of compound interest is 4.5% per annum?

Solution:

A = `10920.25

P =`10000

Rate of interest = 4.5%

Time (n) = ?

∴ $A = P\left(1+\frac{r}{100}\right)^n$

∴ $10920.25 = 10000\left(1+\frac{4.5}{100}\right)^n$

$\frac{10920.25}{10000} = \left(1+\frac{0.9}{20}\right)^n = \left(\frac{20.9}{20}\right)^n = 436.81/400$

$\left(\frac{20.9}{20}\right)^n = \left(\frac{20.9}{20}\right)^n = \left(\frac{20.9}{20}\right)^n$

Hence `10000 will become `10920.25 in 2 years at 4.5%.

❖ **Compound interest – when interest is completed annually but time is in fraction**

If time = $t\frac{p}{q}$ years, then

$$\boxed{A=P\left(1+\frac{r}{100}\right)^t\left(\frac{\frac{p}{q}r}{100}\right)}$$

Example 7:

Find the compound interest on □8000 at 15% per annum for 2 year 4 months, compound annually.

Solution:

Time = 2 years 4 months = $2\frac{4}{12}$ years = $2\frac{1}{3}$years

Amount = `$\left[8000\left\{\left(1+\frac{15}{100}\right)\right\}^2\left(1+\frac{\frac{1}{3}\times 15}{100}\right)\right]$

=`$\left(8000\times\frac{23}{20}\times\frac{23}{20}\times\frac{21}{20}\right)$ = `11109

∴ C.I. = ` (11109 - 8000) = `3109

⇨ **Compound interest – when interest is calculated half-yearly**

Since r is calculated half-yearly therefore the rate per cent will become half and the time will become twice, i.e.,

Rate per cent when interest is paid half-yearly = $\frac{r}{2}$%

and time = 2×time given in years

Hence,

$$\boxed{A=P\left(1+\frac{r}{2\times 100}\right)^{2n}}$$

Example 8:

What will be the compound interest on `4000 in 4 years at 8 per cent annum. If the interest is calculated half-yearly.

Solution:

Given: P = Rs.4000, r = 8%, n = 4years

Since interest is calculated half-yearly, therefore ,

r = $\frac{8}{2}$% = 4% and n = 4×2 = 8 half years

$\therefore$ A=4000$\left(1+\frac{4}{100}\right)^8$ =4000× $\left(\frac{26}{25}\right)^8$

= 4000× 1.3685 = 5474.2762

Amount = `5474.28

∴ Interest = Amount - Principal

= `5474.28 - `4000 = `1474.28

❖ **Compound Interest-when interest is calculated quarterly**

Since 1 year has 4 quarters, therefore rate of interest will become $\frac{1}{4}$th of the rate of interest per annum, and the time period will be 4 times the time given in years

Hence, for quaterly interest

$$\boxed{A=P\left(1+\frac{r/4}{100}\right)^{4\times n} = P\left(1+\frac{r}{400}\right)^{4n}}$$

Example 9:

Find the compound interest on `25625 for 12 months at 16% per annum, compound quaterly.

Solution:

Principal(P)= `25625

Rate(r) = 16% =$\frac{16}{4}$% = 4%

Time = 12 months = 4 quaters

A = 25625$\left(1+\frac{4}{100}\right)^4$ = 25625$\left(\frac{26}{25}\right)^4$

25625× $\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}\times\frac{26}{25}$= 29977.62

C.I. =A-P = 29977.62-25625 = `4352.62

❖ **Difference between compound Interest and Simple Interest**

When T=2

(i) C.I.-S.I. = P$\left(\frac{R}{100}\right)^2$

(ii) C.I.-S.I. = $\frac{R\times S.I.}{2\times 100}$

When T=3

(i) C.I.-S.I. = $\frac{PR^2}{10^4}\left(\frac{300\times R}{100}\right)$

(ii) C.I.-S.I. = $\frac{S.I.}{3}\left[\left(\frac{R}{100}\right)^2+3\left(\frac{R}{100}\right)\right]$

When C.I. is compound annually, the ratio of S.I. to C.I. at the same rate per annum and for the same period is given

by $\frac{S.I.}{C.I.} = \frac{r1}{100\left[\left(1+\frac{r}{100}\right)^i-1\right]}$

Example 10:

The difference between compound interest and simple interest on a certain amount of money at 5% per annum for 2 years is `15. Find the sum:

(a) `4500 (b) `7500

(c) `5000 (d) `6000

Solution:

(d) Let the sum be `100.

Therefore, SI=$\frac{100\times5\times2}{100}$ = `10

and CI=100$\left(1+\frac{5}{100}\right)^2$ - 100

= 100×$\frac{21\times21}{20\times20}$ − 100 = `$\frac{41}{4}$

Difference of CI and SI = $\frac{41}{4}$ − 10=$\frac{1}{4}$

If the difference is $\frac{1}{4}$, the sum = 100

⇨ If the difference is `15, the sum

= 400× 15 =Rs.6000

❖ **POPULATION FORMULA**

The original population of a town is P and the annual increase is R%, then the population in years is p$\left(\frac{R}{100}\right)^n$ and if the annual decrease is P $\left(1+\frac{R}{100}\right)^n$ R%, then the population in a year is given by a change of sign in the formula i.e P$\left(1-\frac{R}{100}\right)^n$

Example 11:

If the annual increase in the population of a town is 4% and the present population is 15625 what will be the population in 3 years.

Solution:

15625$\left(1+\frac{4}{100}\right)^3$

Required population: 15625$(1.04)^3$=17576

NOTE:

- A certain sum is lent out on a certain rate of interest for a certain period. Again the same sum is out on x% higher rate of interest for y% higher period. Then the % increase in S.I is given by $\left(x+y+\frac{xy}{100}\right)\%$
- P is lent out at the rate of R_1% and P_2 is lent out at the rate of R_2%. Then over all rate of interest will be $R=\frac{P_1R_1+P_2P_2}{P_1+P_2}$
- $\frac{1}{x_1}$ part of the principal is lent out on R_1% rate of interest,
- $\frac{1}{x_2}$ part of the principal is lent out on R_2% rate of interest,.....,
- $\frac{1}{x_n}$ part on R_n% rate of interest. The over all rate of interest on whole sum is equal to

$$\left(\frac{1}{x_1}\times R_1+\frac{1}{x_2}\times R_2+\cdots+\frac{1}{x_n}\times R_n\right)$$

EFFECTIVE RATE

If `1 is deposited at 4% compounded quaterly, a calculator can be used to find that at the end of one year, the compound amount is ` 1.0406, an increase of 4.06% over the original `1. The actual in the money is somewhat higher than the stated increase of 4%. To differentiate between these two numbers, 4% is called the nominal or stated rate of interest, while 4.06% is called the effective rate. To avoid confusion between stated rates and effective rates, we shall continue to use r for the stated rate and we will use r_e for the effective rate.

Example 12:

Find the effective rate corresponding to a stated rate of 6% compound semiannually.

Solution:

A calculator shows that `100 at 6% compounded semiannually will grow to

$A=100\left(1+\frac{.06}{2}\right)^2 = 100(1.03)^2=\$ 106.09$

Thus, the actual amount of compound interest is

`106.09 - `100=`6.09. Now if you earn `6.09 interest on

`100 in 1 year with annual compounding, your rate is 6.09/100=.0609=6.09%

Thus, the effective rate is r_e = 6.09%

NOTE:

In the preceding example we found the effective rate by dividing compound interest for 1 year by the original principal. The same thing can be done with any principal P and rate r compounded m times per year.

Effective rate $=\frac{\text{compound interest}}{\text{principal}}$

$r_e=\frac{\text{compound amont}-\text{principal}}{\text{principal}}$

$=\frac{P\left(1+\frac{r}{m}\right)^m-P}{P}=\frac{P\left[\left(1+\frac{r}{m}\right)^m-1\right]}{P}$

$=r_e=\left(1+\frac{r}{m}\right)^m-1$

Example 13:

A bank pays interest of 4.9% compounded monthly. Find the effective rate.

Solution:

Use the formula given above with r=.049 and m= 12.

The effective rate is $r_e=\left(1+\frac{.049}{12}\right)^{12}-1$

=1.050115575-1=.0501 or 5.01%

- Present worth of `p due n years hence

  Present worth$=\frac{p}{\left(1+\frac{r}{100}\right)^n}$

- Equal annual instalement to pay the borrowed amount

  Let the value of each instalement = `x

  Rate = r% and time = n years

  Then, Borrowed Amount

  $=\frac{x}{\left(1+\frac{r}{100}\right)}+\frac{x}{\left(1+\frac{r}{100}\right)^2}+\ldots..+\frac{x}{\left(1+\frac{r}{100}\right)^n}$

Example 14:

Subash purchased a refrigerator on the terms that he is required to pay `1,500 cash down payment followed by `1,020 at the end of first year, `1,003 at the end of second year and `990 at the end of third year. Interest is charged at the rate of 10% per annum. Calculate the cash price:

Solution:

Cash down payment = `1500

Let `x becomes `1020 at the end of first year.

Then, 1020 = x$\left(1+\frac{10}{100}\right)$

or x = $\frac{1020\times100}{110}$ = `927.27

Similarly, 1003 = y$\left(1+\frac{10}{100}\right)^2$

or y=$\frac{1003\times20\times20}{22\times22}$ = `828.92

and z = $\frac{990\times20\times20}{22\times22\times22}$ = `743.80

Hence, CP = 1500+927.27+828.92+743.80

= 3999.99 or `4000

Example 15:

The difference between the interest received from two different banks on `500 for 2 yrs is `2.5. Find the difference between their rates.

Solution:

$I_1 = \frac{500\times2\times r_1}{100} = 10r_1$

$I_2 = \frac{500\times2\times r_2}{100} = 10r_2$

$I_1 - I_2 = 10r_1 - 10r_2 = 2.5$

Or, $r_1 - r_2 = \frac{2.5}{10} = 0.25\%$

Short-cut method:

When $t_1 = t_2$,

$(r_1 - r_2) = \frac{I_d\times100}{sum\times t} = \frac{2.5\times100}{500\times2} = 0.25\%$

Example 16:

At what rate per cent compound interest does a sum of money becomes nine-fold in 2 years?

Solution:

Let the sum be `x and the of compound interest be r% per annum; then

9x =x$\left(1+\frac{r}{100}\right)^2$ or 9 = $\left(1+\frac{r}{100}\right)^2$

Or, 3=1+$\frac{r}{100}$; or,$\frac{r}{100}$ = 2.∴ r = 200%

Short-cut method:

The general formula of compound interest can be changed to the following form:

If a certain sum becomes 'm' times, the rate of compound interest is equal to $100\left[(m)^{1/t}-1\right]$

In this case, r = $100\left[(9)^{1/t}-1\right]$

=100(3-1) = 200%

Example 17:

The simple interest on a certain sum of money at 4% per annum for 4 yrs is 80 more than the interest on the same sum for 3 yrs at 5% per annum. Find the sum.

Solution

Let the sum be `x, then at 4% rate for 4 yrs the simple interest

=$\frac{x\times4\times4}{100}$= `$\frac{4x}{25}$

At 5% rate for 3 yrs the simple interest

=$\frac{x\times5\times3}{100}$ = `$\frac{3x}{20}$

Now, we have,$\frac{4x}{25}$ - $\frac{3x}{20}$ = 80

Or $\frac{16x-15x}{100}$ = 80 ∴ x= `8000

Altornate Method:

For this type of question sum

Sum=difference $\times \frac{100}{[r_2t_1 = r_2t_2]}$

$\frac{80\times100}{4\times4-3\times5}$ = `8000

Example 18:

Some amount out of `7000 was lent at 6% per annum and the remaining at 4% per annum. If the total simple interest from both the fractions in 5 yrs was `1600, find the sum lent at 6% per annum.

Solution:

Suppose `x was lent at 6% per annum.

Thus, $\frac{x\times6\times5}{100} + \frac{(7000-x)\times4\times5}{100} = 1600$

Or, $\frac{3x}{10} + \frac{7000-x}{5} = 1600$

Or, 3x+14000-2x = 16000

∴ x=16000-14000 = Rs.2000

By Method of Alligation: Overall rate of interest

$=\frac{1600-100}{5\times7000} = \frac{32}{7}\%$

6%　　　　4%

32/7%

4/7%　　　　10/7%

∴ratio of two amounts = 2:5

∴amount lent at 6% = $\frac{7000}{7} \times 2$ = `2000

Example 19:

As n amount of money grows upto □4840 in 2 yrs and upto `5324 in 3 yrs on compound interest. Find the rate percent

Solution:

We have,

P+CI of 3 yrs = `5324.......(i)

P+CI of 2 yrs = `4840.......(ii)

Subtracting (ii) from (i), we get

CI of 3rd year = 5324-4840 = `484.

Thus, the CI calculated in the third year which is `484 is basically the amount of interest on the amount generated after 2 years which is `4840.

Alternate method:

$$\frac{\text{Difference of amount after n yrs and (n + 1)yrs} \times 100}{\text{Amount after n yrs}}$$

In this, n=2.

$$\therefore \text{ rate} = \frac{\text{Difference of amount after 2 yrs and 3yrs} \times 100}{\text{Amount after 2 yrs}}$$

$$=\frac{(5324-4840)}{4840} \times 100 = \frac{484\times100}{4840} = 10\%$$

Example 20:

Find the compound interest on `18,750 in 2 yrs the rate of interest being 4% for the first year and 8% for the second year.

Solution:

After first year the amount

$=18750\left(1 + \frac{4}{100}\right) = 18750\left(\frac{104}{100}\right)$

After 2nd year the amount = $18750\left(\frac{104}{100}\right)\left(\frac{108}{100}\right)$

$=18750\left(\frac{26}{25}\right)\left(\frac{27}{25}\right) = 21060$

∴ CI = 21060-18,750 = `2310.