# GEOMETRY

## INTRODUCTION

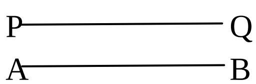
**Line:** A line has length. It has neither width nor thickness. It can be extended indefinitely in both directions.

**Ray:** A line with one end point is called a ray. The end point is called the origin.


**Line segment:** A line with two end points is called a segment.

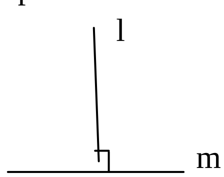


**Parallel lines:** Two lines, which lie in a plane and do not intersect, are called parallel lines. The distance between two parallel lines is constant.




We denote it by PQ||AB.

**Perpendicular lines:** Two lines, which lie in a plane and intesect each other at right angles are called perpendicular lines.




We denote it by l m.

**PROPERTIES**

Three or more points are said to be collinear if they lie on a line, otherwise they are said to be non-collinear.

Two or more lines are said to be coplanar if they lie in the same plane, otherwise they are said to be non-coplanar.

A line, which intersects two or more given coplanar lines in distinct points, is called a transversal of the given lines.

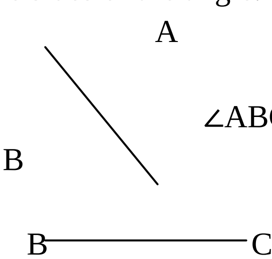
A line which is perpendicular to a line segment, i.e. intersect at 900 and passes through the midpoint of die segment is called the perpendicular bisector of the segment

Every point on the perpendicular bisector of a segment is equidistant from the two endpoints of the segment.

If two lines are perpendicular to the same line, they are parallel to each other.

Lines which are parallel to the same line are parallel to each other.

**Angles:** An angle is the union of two non-collinear rays with a common origin. The common origin is called the vertex and the two rays are the sides of the angle.



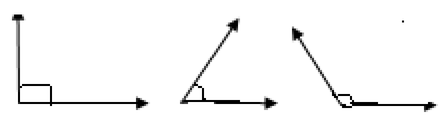

∠ABC with vertex B

**Congruent angles:** Two angles are said to be congruent, denoted by ≅if it divides the interior of the angle into two angles of equal measure.

**TYPES OFANGLE**

1. A right angle is an angle of 90° as shown in [fig. (a)].
2. An angle less than 90° is called an acute angle [fig. (b)]. An angle greater than 90° but less than 180° is called an obtuse angle (fig (c)].
3. An angle of 180° is a straight line [fig. (d)].
4. An angle greater than 180° but less than 360° is called a reflex angle [fig.(e)].

(a)



(a) (b)
(c)

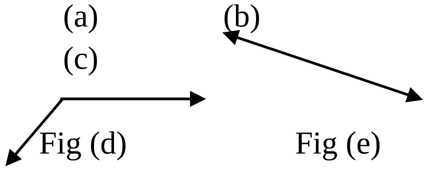
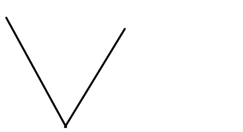


Fig (d) Fig (e)

**PAIRS OF ANGLES**

Adjacent angles; Two angles are called adjacent angles if they have a Common side and their interiors are disjoint

Q R

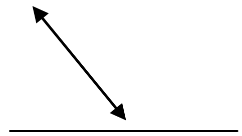


P S

∠QPR is adjacent to ∠RPS

**Linear Pair:** Two angles are said to form a linear pair if they have common side and their other two sides are opposite rays. The sum of the measures of the angles is 180°.

N

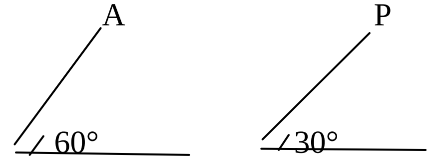


A M` B

∠AMN + ∠BMN =180°

**Complementary angles:** Two angles whose sum is 90°, are complementary, each one is the complement of the other.

A P

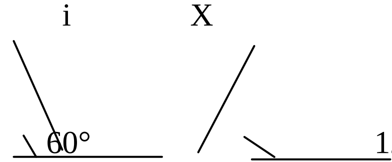



B N Q R

∠ABC + ∠PQR = 90°

**Supplementary angles:** Two angles whose sum is 180° are supplementary, each one is the supplement of the other.

i X

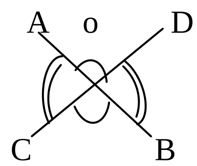



M N Y

C

∠LMN+∠XYZ = 60°+ 120°-180°

**Vertically Opposite angles:** Two angles are called vertically opposite angles if their sides form two pairs of opposite rays. Vertically opposite angles are congruent




$\angle AOD$ - $\angle COB$ and $\angle AOC$ - $\angle BOD$

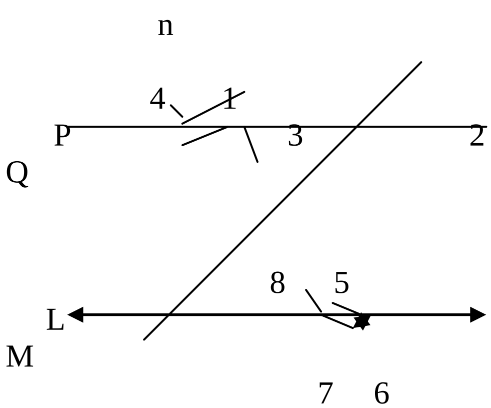
**Corresponding angles:** Here, PQ||LM and n is transversal.

Then, $\angle 1$ and $\angle 5$, $\angle 2$ and $\angle 6$, $\angle 3$ and $\angle 7$ and $\angle 4$ and $\angle 8$ are corresponding angles.

When two lines are intersected by a transversal, they form four pairs of corresponding angles.

The pairs of corresponding angles thus formed are congruent.

i.e. $\angle 1 = \angle 5$; $\angle 2 = \angle 6$; $\angle 4 = \angle 8$; $\angle 3 = \angle 7$




**Alternate angles:** In the above figure, $\angle 3$ and $\angle 3$, $\angle 2$ and $\angle 8$ are Alternative angles.

When two lines are itnersected by a transversal, they form two pairs of alternate angles.

The pairs of alternate angles thus formed are congruent, i.e.

$\angle 3$- $\angle 3$ and$\angle 2 = \angle 8$

**Interior angles**: In the above figure, $\angle 2$ and $\angle 5$, $\angle 3$ and $\angle 8$ are interior angles.

When two lines are intersected by a transversal, they form two pairs of interior angles.

The pairs of interior angles thus formed are

supplementary. i.e. $\angle 2$+$\angle 5$-$\angle 3$ + $\angle 8$ = 180°

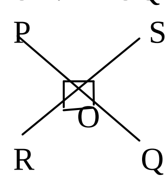
**Example 1:**

In figure given below, lines PQ and RS intersect each other at point O. If $\angle POR$; $\angle ROQ$ = 5:7, find all the angles.

**Solution:**

$\angle POR + \angle ROQ = 180°$ (Linear pair of angles)

But $\angle POR : \angle ROQ = 5:7$ (Given)




$\angle POR = \frac{5}{12} \times 180° = 75°$

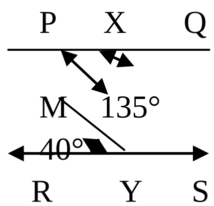
Similarly, $\angle ROQ = \frac{7}{12} \times 180° = 105°$

Now, $\angle POS = \angle ROQ = 105°$ (Vertically opposite angles)

and $\angle SOQ = \angle POR = 75°$ (Vertically opposite angles)

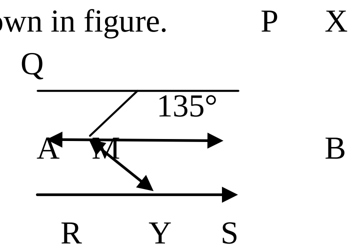
**Example 2:**

In fig. if PQ||RS, ∠MXQ = 135° and∠MYR - 40°, find ∠XMY.




**Solution:**

Here, we need to draw a line AB parallel to line PQ, through point M as shown in figure.




Now, AB||PQ and PQ||RS ⇒AB||RS

Now, ∠QXM+∠XMB =180°

(∴ AB||PQ, interior angles on the same side of the is equal to die ratio of the transversal)

But ∠QXM- 135° = 135° + ∠XMB = 180°

∴∠XMB = 45° … (i)

Now, ∠BMY = ∠MYR (∴ AB||RS, alternate angles)

∠BMY = 40° … (ii)

Adding (i) and (ii) we get

∠XMB + ∠BMY = 45° + 40°

i.e. ∠XMY = 85°

**Example 3:**

An angle is twice its complement Find the angle.

**Solution:**

If die complement is x, the angle = 2x

2x + x = 90°

⇒ 3x = 90° ⇒ x = 30°

The angle is 2 × 30°-60°

**Example 4:**

The supplement of an angle is one-fifth of itself Determine the angle and its supplement.

**Solution:**

Let the measure of the angle be x°. Then the measure of its supplementary angle is 180° - x°,

It is given that

⇒ 5 (180° - x) = x

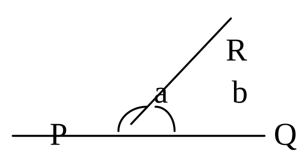
⇒ 900 – 5x = x ⇒ 900 = 5x +x

⇒ 900 = 6x ⇒ 6x = 900 ⇒ x = $\frac{900}{6}$ = 150

Supplementary angle is 180°-150° = 30°

**Example 5:**

In figure, ∠POR and ∠QOPform a linear pair. If a- b =80°, find the values of a and b.




**Solution:**

$\because \angle POR$ and $\angle QOR$ for a linear pair

$\therefore \angle POR + \angle QOR = 180°$

or a+ b = 180° … (i)

But a – b = 80° … (ii) [Given]

Adding eqs. (i) and (ii), we get

2a = 260° $\therefore a = \frac{260}{2} = 130°$

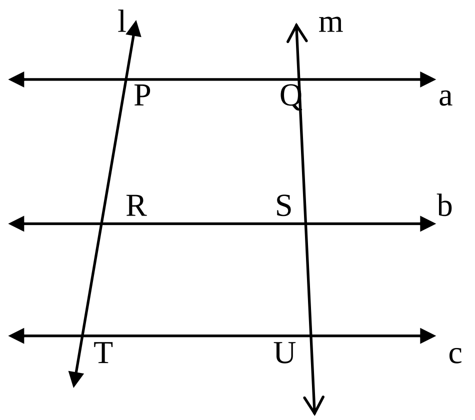
Substituting the value of a in (1) we get

130° + b = 180°

B = 180° - 130° = 50°

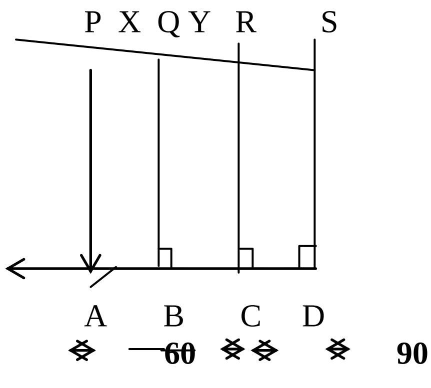
**PROPORTIONALITY THEOREM**

The ratio of intercepts made by three parallel lines on a transversal is equal to the corresponding intercepts made on any other transversal $\frac{PR}{RT} = \frac{QS}{SU}$




**Example 6:**

In the figure, if PS = 360, find PQ, QR and RS.




**Solution:**

PA, QB, RC and SD are perpendicular to AD. Hence, they are parallel. So the intercepts are proportional.

$$\therefore \frac{AB}{BD} = \frac{PQ}{QS} \Rightarrow \frac{60}{210} = \frac{x}{360 - x}$$

$$\Rightarrow \frac{2}{7} = \frac{x}{360 - x} \Rightarrow x = \frac{720}{9} = 80$$

∴ PQ = 80

So, QS = 360 – 80 = 280

Again, $\frac{BC}{CD} = \frac{QR}{RS}$

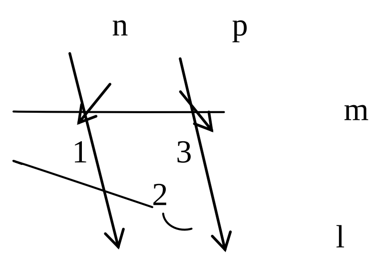
$$\therefore \frac{90}{120} = \frac{y}{280 - y} \Rightarrow \frac{3}{4} = \frac{y}{280 - y}$$

$$\Rightarrow y = 120$$

∴ QR = 120 and SR = 280 – 120 = 160

**Example 7:**

In figure if l||m, n||p and ∠1 = 85° find ∠2.




**Solution:**

∴n||p and m is transversal

∴∠1 - ∠ 3 - 85° (Corresponding angles)

Also, m||l & p is transversal

∠2 + ∠3 = 180° (∴ Consecutive interior angles)

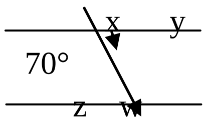
⇒∠2 + 85° = 180°

⇒ ∠2 = 180° - 85°

⇒∠2 = 95°

**Example 8:**

From the adjoining diagrams, calculate ∠x,∠y,∠z and ∠w.




**Solution:**

∠y = 70°

∠x+70- 180°

..... (vertical opp. angle)

∴∠x = 180 – 70 = 110°

.... (adjacent angles on a st line or linear pair)

∠z = 70° (corresponding angles)

∠z+∠w = 180° (adjacent angles on a st. line or liner pair)

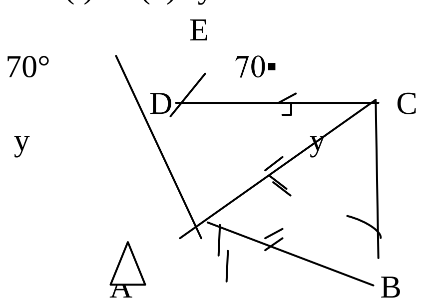
∴70 + ∠w = 180°

∴∠w = 180° - 170° = 110°

**Example 9:**

From the adjoining diagram

Find(l)∠x (2)∠y




**< CBA = 90°**

**Solution:**

∠x = ∠EDC= 70°

(corresponding angles)

Now, ∠ADB = x = 70°

[AD = DB]

In ΔABD,

∠ABD = 180 - ∠x-∠x

=180 - 70 - 70 = 40°

⇒∠BDC-∠ABD = 40°(alternate angles)

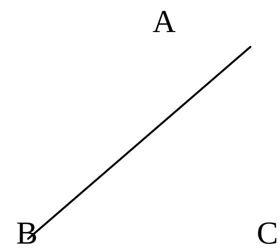
∠y = 40°

**TRIANGLES**

The plane figure bounded by the union of three lines, which Join three non-collinear points. Is called a triangle. A triangle la denoted by the symbol Δ.

The three non-collinear points, are called the vertices of the triangle.

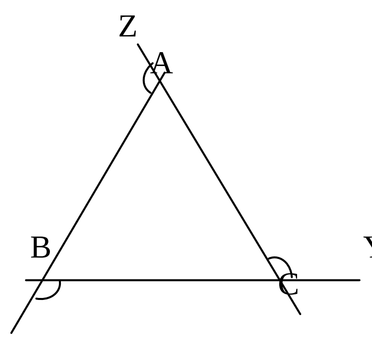
In ΔABC, A, B and C are the vertices of the triangle; AB, BC, CA are the three sides, and ∠A, ∠B, ∠C are the three angles.




**Sum of interior angles:** The sum of the three Interior angles of a triangle is 180°.

∠A + ∠B + ∠C = 180°

**Exterior angles and interior angles**



X

(i) The measure of an exterior angle is equal to the sum of the measures of the two interior opposite angles of the triangle.
$\therefore \angle ACY = \angle ABC + \angle BAC$
$\angle CBX = \angle BAC + \angle BCA$ and $\angle BAZ = \angle ABC + \angle ACB$

(ii) The sum of an interior angle and adjacent exterior angle is 180°.
i.e. $\angle ACB + \angle ACY = 180°$
$\angle ABC + \angle CBX = 180°$ and $\angle BAC + \angle BAZ = 180°$

**Example 10:**

If the ratio of three angles of a triangle is 1:2:3, find the angles.

**Solution:**

Ratio of the three angles of a $\Delta$ = 1:2:3
Let the angles be x, 2x and 3x.
$\therefore x+2x+3x=180°$
$\therefore 6x = 180°$
Hence the first angle = x = 30°
The second angle = 2x = 60°
The third angle = 3x = 90°

## CLASSIFICATION OF TRIANGLES

**Based on sides:**

**Scalene triangle:** A triangle in which none of the three sides is equal is called a scalene triangle.

**Isosceles triangle:** A triangle in which at least two sides are equal is called an isosceles triangle.

**Equilateral triangle:** A triangle in which all the three sides are equal is called an equilateral triangle. In an equilateral triangle, all the angles are congruent and equal to 60°.

**Based on angles:**

**Right triangle:** If any of a triangle is a right angle i.e., 90° then the triangle is a right-angled triangle.

**Acute triangle:** If all the three angles of a triangles are acute, i.e., less than 90°, then die triangle is an acute angled triangle.

**Obtuse triangle:** If any one angle of a triangle is obtuse, i.e., greater than 90°, then the triangle is an obtuse-angled triangle.

## SOME BASIC DEFINITIONS

1. **Altitude (height) of a triangle:** The perpendicular drawn from the vertex of a triangle to the opposite side is called an altitude of the triangle.
2. **Median of a triangle:** The line drawn from a vertex of a triangle to the opposite side such that it bisects the side, is called the median of the triangle.A median bisects the area of the triangle.
3. **Orthocentre:** The point of intersection of the three altitudesof a triangle is called the orthocentre. The angle made by any side at the orthocentre = 180°- the opposite angle to the side.
4. **Centroid:** The point of intersection of the three medians of a triangle is called the centroid. The centroid divides each median in the ratio 2:1.

5. **Circumcentre:** The point of intersection of the perpendicular bisectors of the sides of a triangle is called the circumcentre.
6. **Incentre:** The point of intersection of the angle bisectors of a triangle is called the incentre.

   (i) Angle bisector divides the opposite sides in the ratio of remaining sides

   **Example:** $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$

   (ii) Incentre divides the angle bisectors in the ratio (b+ c):a, (c + a):band(a + b):c

**CONGRUENCY OF TRIANGLES**

Two triangles are congruent if the sides and angles of one triangle are equal to the corresponding sides and angles of the other triangle.

(i) **SAS Congruence rule:** Two triangles are congruent if twosides and the Included angle of one triangle are equal to the sides and tile included angle of the other triangle.

(ii) **ASA Congruence rule:** Two triangles are congruent if twoangles and the included side of one triangle are equal to two angles and the included side of other triangle.

(iii) **AAS Congruence rule:** Two triangles are congruent if any two pairs of angles and one pair of corresponding sides are equal.

(iv) **SSS Congruence rule:** If three sides of one triangle are equal to the three sides of another triangle, then the two triangles are congruent.

(v) **RHS Congruence role:** If in two right triangles, the hypotenuse and one side of the triangle are equal to the hypotenuse and one side of the other triangle, then the two triangles are congruent

**SIMILARITY OF TRIANGLES**

For a given correspondence between two triangles, if the corresponding angles are congruent and their corresponding sides are in proportion, then the two triangles are said to be similar.

Similarity is denoted by ~.

(i) **AAA Similarlity:** For a given correspondence between two triangles, if the two angles of one triangle are congruent to the corresponding two angles of the other triangle, then the two triangle are similar.

**(ii)** **SSS Similarlity:** If the corresponding sides of two triangles are proportional, their corresponding angles are equal and hence the triangles are similar.

**(iii)** **SAS Similarlity:** If one angle of a triangle is equal to one angle of the other and the sides including these angles are proportional, the triangles are similar.

**PROPERTIES OF SIMILAR TRIANGLES**

1. If two triangles are similar,

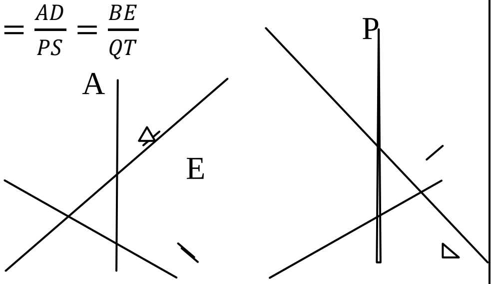
   Ratio of sides = Ratio of height = ratio of Median = Ratio of angle bisectors = Ratio of in radii = Ratio of circumradii

If ΔABC ~ ΔPQR

$\frac{AB}{PQ} = \frac{AD}{PS} = \frac{BE}{QT}$

P

A



E

T

B D C Q

S R

The ratio of the areas of two similar triangles is equal to the ratio of the squares of the corresponding sides.

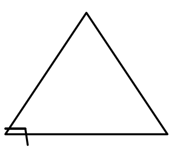
If ΔABC ~ ΔPQR, then

$$\frac{Ar(\Delta ABC)}{Ar(\Delta PQR)} = \frac{(AB)^2}{(PQ)^2} = \frac{(BC)^2}{(QR)^2} = \frac{(AC)^2}{(PR)^2}$$

**PYTHACORAS THEOREM**

In a right triangle, the square of the hypotenuse is equal to the sum of the squares of the other two sides.

C



B A

If a right triangle ABC right angled at B. Then,

By Pythagoras theorem, $AC^2 = AB^2 + BC^2$

**BASIC PROPORTION THEOREM (BPT)**

If aline is drawn parallel to one side of a triangle to intersect the other two sides in distinct points, the other two sides are divided in the same ratio.

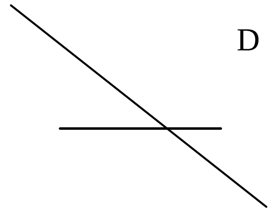
If ΔABC in which a line parallel to BC intersects AB to D and AC at E. Then,

A

By BPT, $\frac{AD}{DB} = \frac{AE}{EC}$

D

E



B

C

**MID-POINTTHEOREM**

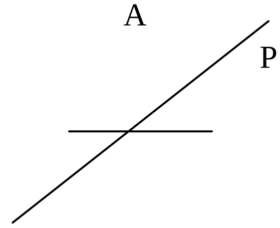
The line segment joining the mid-points of two sides of a triangle is parallel to the third side and half of it.

In ΔABC, if P and Q are the mid-points of AB and AC then PQ || BC and PQ = $\frac{1}{2}$BC

A

P

Q



B

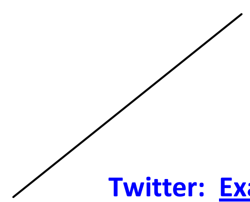
C

**INEQUALITIES INATRIANGLE**

(i) If two sides of a triangle are unequals, the angle oppositeto the longer side is larger. Conversely,

In any triangle, the side opposite to the larger angle is longer.

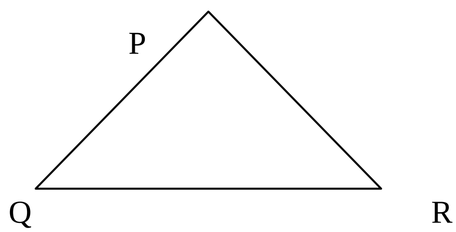
A



B C

If AB > AC then ∠C >∠B

(ii) The sum of any two side of a triangle is greater than the third aide.



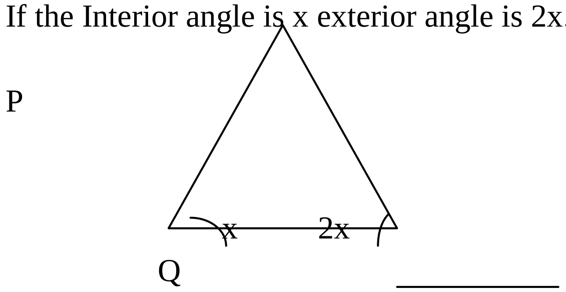

PQ+PR> QR; PQ+QR> PR and QR+PR> PQ

**Example10:**

The interior and its adjacent exterior angle of a triangle are in the ratio 1:2. What is the sum of the other two angles of the triangle?

**Solution**

If the Interior angle is x exterior angle is 2x.




∵x+2x = 180°

⇒3x = 180°

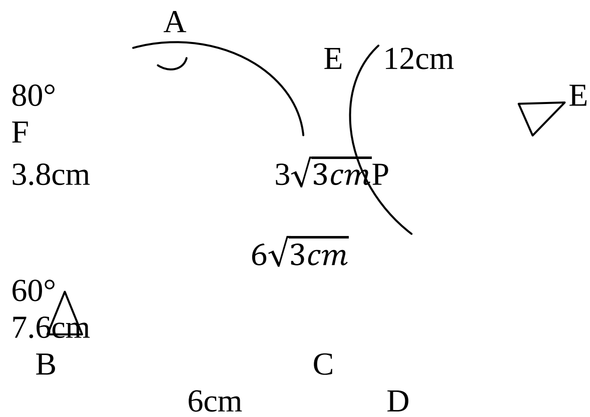
⇒x = 60°

∴ Exterior angle = 120°

Hence sum of the other two angles of triangle = 120° (Exterior angle is the sum of two opposite interior angles)

**Example 11:**

In figure, find ∠F.




**Solution:**

In triangles ABC and DEF, we have

$\frac{AB}{DF} = \frac{3.8}{7.6} = \frac{1}{2}$

Similarly, $\frac{BC}{FE} = \frac{6}{12} = \frac{1}{2}$ and $\frac{AC}{DE} = \frac{3\sqrt{3}}{6\sqrt{3}} = \frac{1}{2}$, i.e.

in the two triangles, sides are proportional.

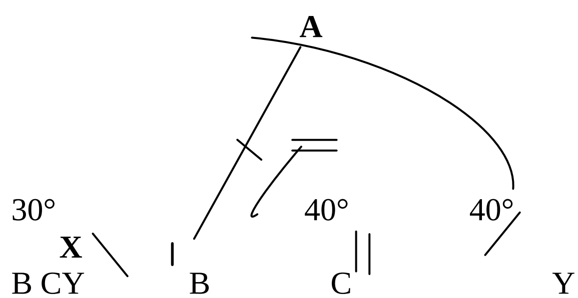
∴Δ ABC ~ΔDEF (by SSS Similarly)

∴∠B = ∠F (Corresponding angles are equal) But ∠B = 60° (Given)

∴∠F = 60°

**Example 12:**

In the given figure, find ∠BAC and ∠XAY.




**Solution:**

∠AXB =∠XAB - 30° (∵BX = BA)

$\angle ABC = 30° + 30° = 60°$ (Exterior angle)

$\angle CYA = \angle YAC = 40°$ ($\because$ CY=CA)

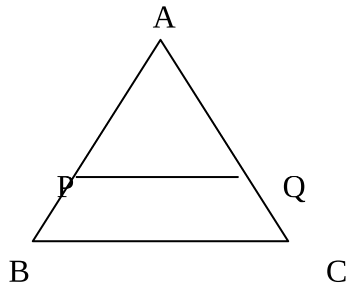
$\angle ACB = 40° + 40° = 80°$ (Exterior angle)

$\angle BAC = 180° - (60° + 80°) = 40°$ (Sum of all angles of a triangle Is 180°.

$\angle XAY = 180-(30+40) = 110°$

**Example 13:**

In the fig., PQ || BC, AQ = 4 cm, PQ = 6 cm and BC = 9cm. Find QC




**Solution:**

By BPT, $\frac{AQ}{QC} = \frac{PQ}{BC}$

$\frac{4}{QC} = \frac{6}{9} \Rightarrow QC = 6$ cm

**Example 14:**

Of the triangles with sides 11, 5, 9 or with sides 6, 10, 8; which is a right triangle?

**Solution:**

(Longest side)$^2 = 11^2$ - 121;

$5^2+9^2 = 25 + 81 = 106$

$\therefore 11^2 \neq 5^2 + 9^2$

So, it is not a right triangle

Again,(longest side)$^2 = (10)^2 = 100$;

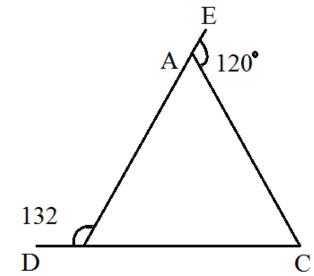
$6^2 + 8^2 = 36 + 64 = 100$

$10^2 = 6^2 + 8^2$

$\therefore$It is a right triangle.

**Example 15:**

In figure, $\angle DBA = 132°$ and $\angle EAC = 120°$. Show that AB>AC.




**Solution:**

As DBC is a straight line,

$132° + \angle ABC = 180°$

$= \angle ABC = 180° - 132° = 48°$

For $\Delta$ ABC,

$\angle EAC$ is an exterior angle

$120° = \angle ABC + \angle BCA$

(ext. $\angle$= sum of two opp. interior $\angle$ s)

$\Rightarrow 120° = 48° + \angle BCA$

$\Rightarrow \angle BCA = 120° - 48° = 72°$

Thus, we find that $\angle BCA > \angle ABC$

$\Rightarrow$ AB > AC (side opposite to greater angle is greater)

**Example 16:**

From the adjoining diagram, calculate

(i) AB (ii)AP (iii)ar$\Delta$APC: ar$\Delta$ABC

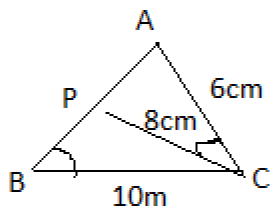
**Solution:**

In $\Delta$APC and $\Delta$ABC

$\angle ACP = \angle ABC$

$\angle A = \angle A$

$\Rightarrow \Delta ACP \sim \Delta ABC$




$\Rightarrow \frac{AP}{AC} = \frac{PC}{BC} = \frac{AC}{AB}$

$\therefore \frac{AP}{6} = \frac{8}{10} = \frac{6}{AB}$

$\Rightarrow AP = 6\times\frac{8}{10} = 4.8$ and $AB = \frac{60}{8} = 7.5$

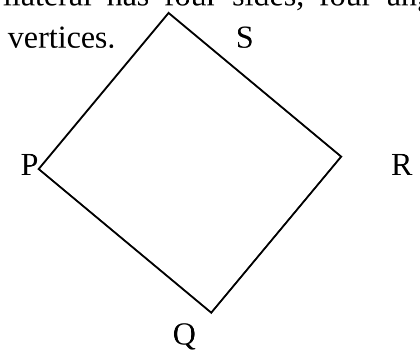
$\Rightarrow AP = 4.8$ cm and $AB = 7.5$ cm

$\frac{\Delta ACP}{\Delta ABC} = \frac{CP^2}{BC^2} = \frac{8^2}{10^2} = 0.64$

**QUADRILATERALS**

A figure formed by joining four points is called a quadrilateral.

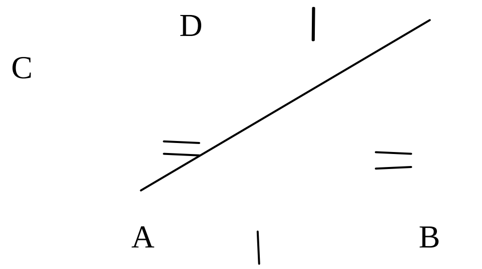
A quadrilateral has four sides, four angles and four vertices.




In quadrilateral PQRS, PQ, QR, RS and SP are the four sides; P, Q, R and S are four vertices and ∠P + ∠Q + ∠R and ∠S are four angles The sum of the angles of a quadrilateral is 360▪ ∠P + ∠Q +∠R+∠S = 360▪

**TYPES OF QUADRILATERALS:**

1. **Parallelogram:** A quadrilateral whose opposite sides are parallel is called parallelogram.




**Properties:**

(i) Opposite sides are parallel and equal.

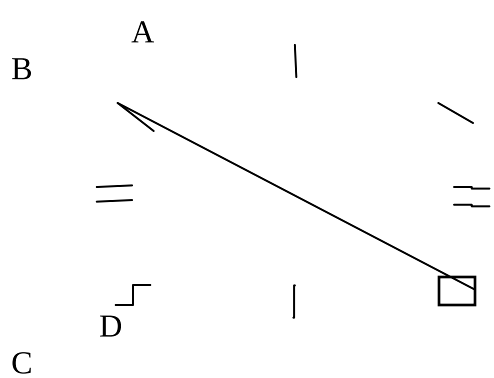
(ii) Opposite angles are equal.

(iii) Diagonals bisect each other.

(iv) Sum of any two adjacent angles is 180°.

(v) Each diagonal divides the parallelogram into two triangles of equal area.

2. **Rectangle:** A parallelogram, in which each angle is a right angle, i.e., 90° is called a rectangle.



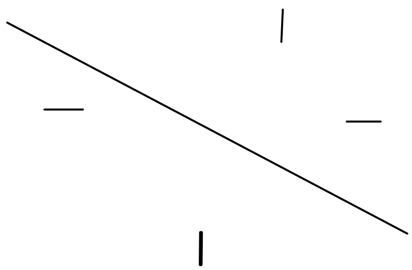

**Properties:**

(i) Opposite sides are parallel and equal.

(ii) Each angle is equal to 90°

(iii) Diagonals are equal and bisect each other.

3. **Rhombus:** A parallelogram in which all sides are congruent (or equal) is called a rhombus.



**Properties:**

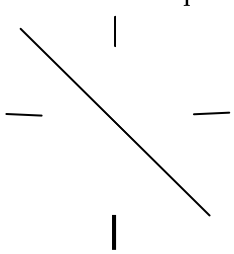
(i) Opposite sides are parallel.

(ii) All sides are equal.

(iii) Opposite angles are equal.

(iv) Diagonals bisect each other at right angle.

**4. Square:** A rectangle in which all sides are equal is called a square.



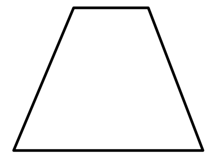
**Properties:**

(i) All sides are equal and opposite sides are parallel.

(ii) All angles are 90°

(iii) The diagonals are equal and bisect each other at right angle.

5. **Trapezium:** A quadrilateral is called a trapezium if two of the opposite sides are parallel but the other two sides are not parallel.



**Properties:**

(i) The segment joining the mid-points of the non-parallel sides is called the median of the trapezium.

**Median = $\frac{1}{2}\times$ sum of the parallel sides**

**Example 17:**

The angle of quadrilateral are in the ratio 3: 5: 9: 13. Find all the angles of the quadrilateral.

**Solution:**

Let the angles of quadrilateral are 3x, 5x, 9x, 13x.

$\therefore 3x + 5x + 9x + 13x = 360°$

(Sum of the angles of quadrilateral)

$\Rightarrow 30x = 360°$

$\Rightarrow x = 12°$

Hence angles of quadrilateral are:

$3x = 3\times12° = 36°$

$5x = 5\times 12° = 60°$

$9x = 9\times 12° = 108°$

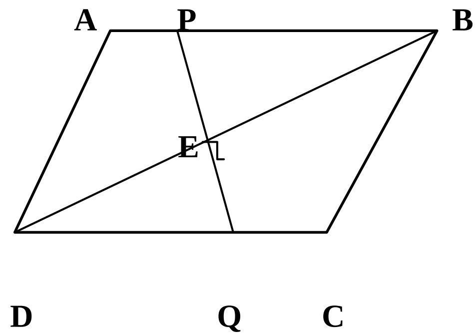
$13x = 13\times 12° = 156°$

**Example 18:**

ABCD is a parallelogram. E is the mid point of the diagonal DB. DQ = 10cm, DB = 16cm. Find PQ.

**Solution:**

$\angle EDQ = \angle EBP$ (Alternate angles)




∴∠DEQ = ∠BEP (opposite angles)

∴ ΔDEQ ≅ ΔBEP (By ASA congruency)

∴ PE = EQ

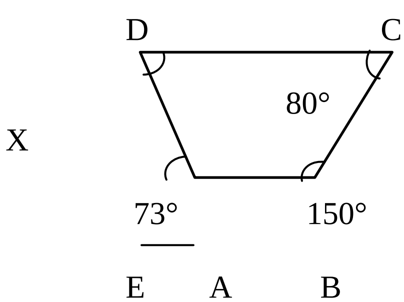
$(EQ)^2 = (DQ)^2 – (DE)^2$

$= 10^2 – 8^2 = 100 – 64 = 36$

∴ EQ = 6 cm and PQ = 12 cm.

**Example 19:**

Use the information given in figure to calculate the value of x.




**Solution:**

Since, EAB is a straight line

∴∠DAE + ∠DAB = 180°

⇒ 73° + ∠DAB = 180°

i.e., ∠DAB = 180° - 73° = 107°

Since, the sum of the angles of quadrilateral ABCD is 360°

⇒ 292° + x = 360°

and, x =360° - 292° = 68°

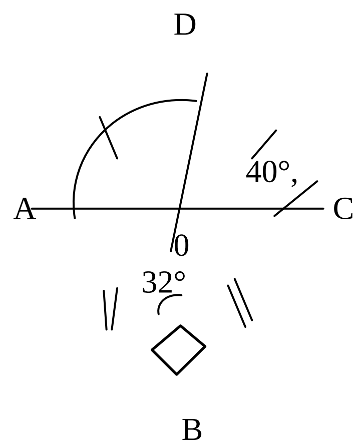
**Example 20:**

In the adjoining kite, diagonals intersect at O. If ∠ABO = 32° and ∠OCD = 40°, find

(i) ∠ABC

(ii) ∠ADC

(iii) ∠BAD




**Solution:**

Given, ABCD is a kite.

(i) As diagonal BD bisects ∠ABC,

∠ABC = 2 ∠ABO = 2 × 32° = 64°

(ii) ∠DOC = 90°

[diagonals intersect at right angles]

∠ODC + 40° + 90° = 180°

[Sum of angles in OCD]

⇒∠ODC = 180° - 40° - 90° = 50°

As diagonal BD bisects ∠ADC,

∠ADC = 2 ∠ODC = 2× 50° = 100°

(iii) As diagonal BD bisects ∠ABC

∠OBC = ∠ABO = 32°

∠BOC = 90° [diagonals intersect at right angles]

∠OCB + 90° + 32° = 180° [sum of angles in Δ OBC]

⇒∠OCB = 180° - 90° - 32° = 58°

∠BCD = ∠OCD + ∠OCB = 40° + 58° = 98°

∴∠BAD = ∠BCD = 98° [In kite ABCD, ∠A = ∠C)

**POLYGON**

A plane figure formed by three or more non-collinear points joined by line segments is called a polygon.

A polygon with 3 sides is called a triangle.

A polygon with 4 sides is called a quadrilateral.

A polygon with 5 sides is called a pentagon.

A polygon with 6 sides is called a hexagon.

A polygon with 7 sides is called a heptagon.

A polygon with 8 sides is called an octagon.

A polygon with 9 sides is called a nonagon.

A polygon with 10 sides is called a decagon.

**Regular polygon:** A polygon in which all its sides and angles are equal, is called a regular polygon.

Sum of all interior angles of a regular polygon of side n is given by (2n – 4) 90°.

Hence, angle of a regular polygon = $\frac{(2n-4)90°}{n}$

Sum of an interior angle and its adjacent exterior angle is 180°

Sum of all exterior angles of a polygon taken in order is 360°.

**Example 21:**

The sum of the measures of the angles of regular polygon is 2160°. How many sides does it have?

**Solution:**

Sum of all angles = 90° (2n - 4)

⇒ 2160 = 90 (2n – 4)

2n = 24 + 4

∴ n = 14

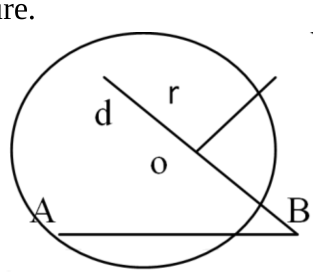
Hence the polygon has 14 sides.

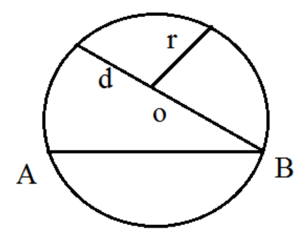
**CIRCLE:**

The collection of all points in a plane, which are at a fixed distance from a fixed point in the plane, is called a circle.

The fixed point is called the centre of the circle and the fixed distance is called the radius r.

Chord: A chord is a segment whose endpoints lie on the circle, AB is a chord in the figure.



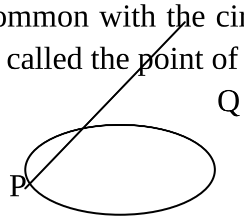

**Diameter:** The chord, which passes through the centre of the circle, is called the diameter

(d) of the circle. The length of the diameter of a circle is twice the radius of the circle.

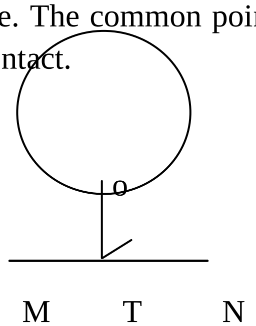
$$\boxed{d = 2r}$$

**Secant:** A secant is a line, which intersects the circle in two distinct points.

**Tangent:** Tangent is a line in the plane of a circle and having one and only one point common with the circle. The common point is called the point of contact.



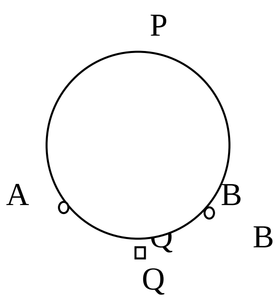

p q is a secant




MN is a tangent. T is the point of contact.

**Semicircle:** Half of a circle cut off by a diameter is called the semicircle. The measure of a semicircle is 180°.

**Are:** A piece of a circle between two points is called an arc. A minor are is an arc less than the semicircle and a major arc is an arc greater than a semicircle.

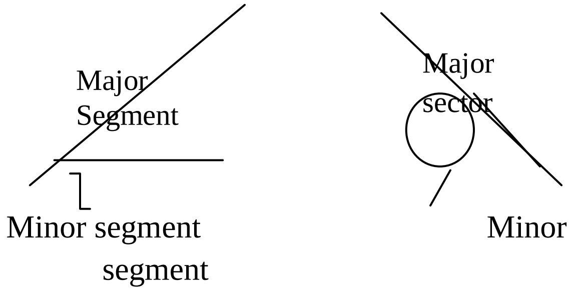



$\widehat{AQB}$ is a minor arc and $\widehat{APB}$ is a major arc.

**Circumference:** The length of the complete circle is called its circumference (C).

$$c = 2\pi r$$

**Segment:** The region between a chord and either of its arcs is called a segment.




**Sector:** The region between an arc and the two radii, joining the centre to the endpoints of the arc is called a sector.

## ■ REMEMBER

- ❖ Equal chords of a circle subtend equal angles at the centre.
- ❖ The perpendicular from the centre of a circle to a chord bisects the chord.
- ❖ Equal chords of a circle are equidistant from the centre.
- ❖ The angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.
- ❖ Angles in the same segment of a circle are equal.
- ❖ Angle in a semicircle is a right angle.
- ❖ The tangent at any point of a circle is perpendicular to the radius through the point of contact.
- ❖ The length of tangents drawn from an external point to a circle are equal.

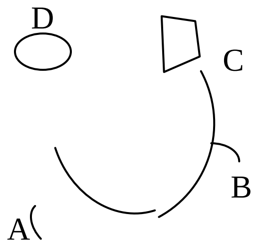
**CYCLIC QUADRILATERAL**

If all the four vertices of a quadrilateral lies on a circle then the quadrilateral is said to be cyclic quadrilateral.

❖ The sum of either pair of the opposite angles of a cyclic quadrilateral is 180°.

i.e. ∠A + ∠C = 180°

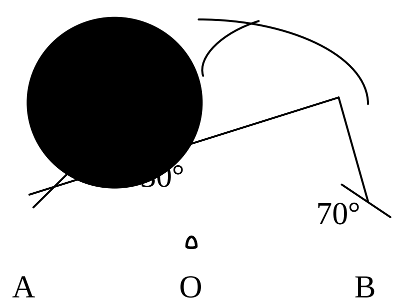
∠B + ∠D = 180°




❖ Conversely, if the sum of any pair of opposite angles of quadrilateral is 180°, then the quadrilateral must be cyclic.

**Example 22:**

In the adjoining figure, C and D are point on a semi-circle described on AB as diameter. If ∠ABC = 70° and ∠CAD = 30°, calculate ∠BAC and ∠ACD.




**Solution:**

∠ACB = 90° [Angle in a semi-circle]

In ΔABC, ∠BAC + ∠ACB + ∠ABC = 180° [Sum of the ∠ s of Δ is 180°]

⇒∠BAC + 90° + 70° = 180°

⇒∠BAC = (180° - 160°) = 20°

Now, ABCD being a cyclic quadrilateral, we have

∠ABC + ∠ADC = 180°

(Opposite ∠s of a cyclic quad. are supplementary]

⇒ 70° +∠ADC = 180°

⇒∠ADC = (180° - 70°) = 110°

Now, in Δ ADC, we have

∠CAD + ∠ADC + ∠ACD = 180°

(Sum of the ∠s of a Δ is 180°)

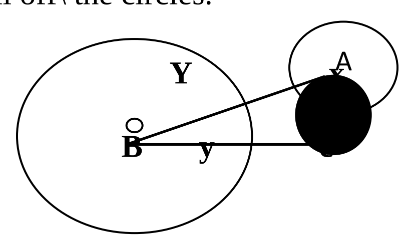
⇒ 30° + 110° + CD = 180°

⇒∠ACD = (180° - 140°) =40°

Hence, ∠BAC = 20° and ∠ACD = 40°

**Example 23:**

With the vertices of ΔABC as centres, three circles are described, each touching the order two externally. If the sides of the triangle are 9 cm, 7 cm and 6 cm, find the radii off\ the circles.




**Solution:**

Let AB = 9 cm, BC = 7 cm and CA = 6 cm

Let x, y, z be the radii of circle with centres A, B, C respectively.

Then, x + y = 9, y + z = 7 and z + x = 6

Adding, we get 2 (x + y + z) = 22

⇒ x + y + z = 11

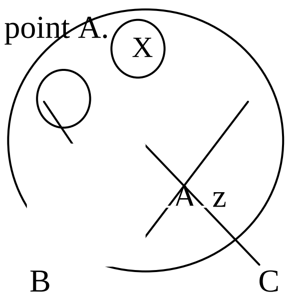
∴ x = [(x + y + z) – (y + z)] = (11 – 7) cm = 4cm.

Similarly, y = (11 – 6) cm = 5 cm and z = (11 – 9) cm = 2cm.

Hence, the radii of circles with centres A, B, C are 4 cm, 5 cm, and 2 cm respectively.

**Example 24:**

In the adjoining figure, 2 circles with centres Y and Z touch each other externally at point A.




Another circle, with centre X, touches the other 2 circles internally at Band C. IfXY=6cm, YZ =9cmandZX =7cm, then find the radii of the circles.

**Solution:**

Let X, Y, Z be the radii of the circle, centres X, V, Z respectively YAZ, XYB, XZC are straight lines (Contact of circles)

XY=X-Y-=6 ..... (1)

XZ=X-Z=7 ..... (2)

YZ=Y+Z=9 ..... (3)

⇒ (1) + (2) + (3)

2X = 22 ⇒X = 11, Y = 5, Z = 4

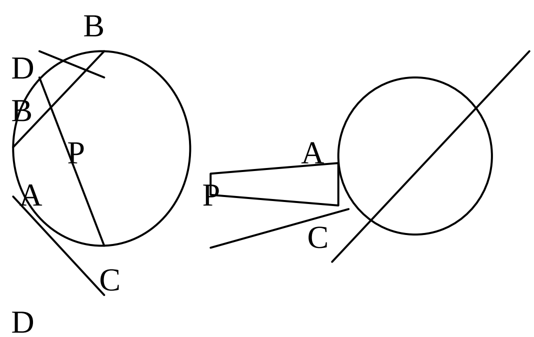
The radius of the circle, centre X, is 11 cm.

The radius of the circle, centre Y, is 5 cm.

The radius of the circle, centre Z, is 4 cm.

**SOME IMPORTANTTHEOREMS**

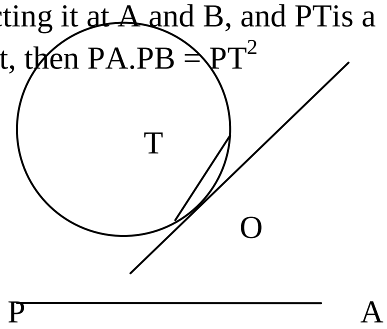
1. If two chords of a circle intersect inside or outside the circle, then the rectangle formed by the two parts of one chord is equal in area to the rectangle formed by the two parts of the other.




2. Two chords AB and CD of a circle such that they interest each other at a point P lying inside (fig. (i)) or outside (fig. (ii)) the circle.

| |
|---|
| PA.PB = PC.PD |

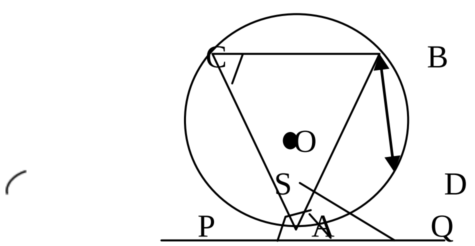
3. If PAB is a secant to a circle intersecting it at A and B, and PTis a Tangent, then PA.PB = PT²




B

**4. Alternate segment theorem:**

If a line touches a circle and from the point of contact a chord is drawn, the angle which this chord makes with the given line are equal respectively to the angles formed in the corresponding alternate segments.




PQ is a tangent to a circle with centre O at a point A, A B chord andC,D are points in the two segments of the circle formed by the chord AB. Then,

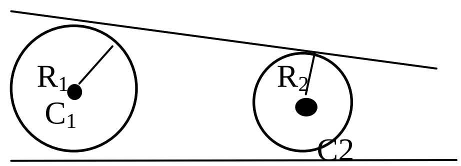
∠BAQ = ∠ACB

∠BAP = ∠ADB

**COMMON TANGENTS FORA PAIR OFCIRCLE**

(A) Length of direct common tangent

$$L_1 = \sqrt{(C_1C_2)^2 - (R_1R_2)^2}$$



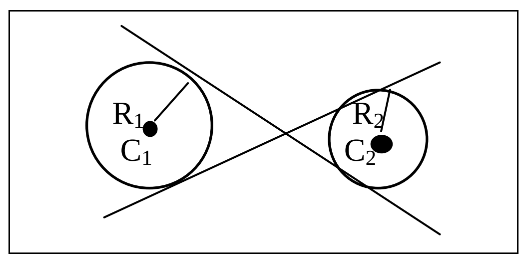

where $C_1C_2$= Distance between the centres

(B) Length Of transverse common tangent

$L_{12} = \sqrt{(C_1C_2)^2 - (R_1 + R_2)^2}$ ;

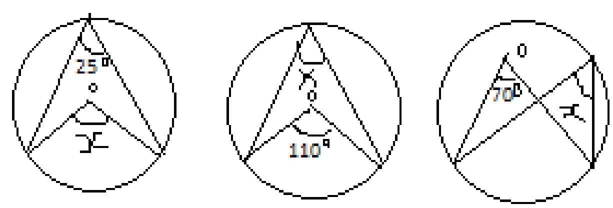
where $C_1C_2$= Distance between the centres, and

$R_1$ and $R_2$ be the radii of the two circles.




**Example 25:**

Find the angle marked as x in each of the following figures where O is the centre of the circle.




(a) (b) (c)

**Solution:**

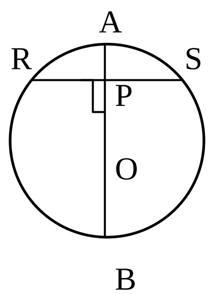
We know that the angle subtended by an arc at the centre is double the angle subtended by it at any point on the remaining part of the circle.

(a) $x = 2 \times 25° = 50°$ (b) $x = \frac{1}{2} \times 110° = 55°$

(c) $x = \frac{1}{2} \times 70° = 35°$

**Example 26:**

In the figure, RS = 12 cm and radius of the circle is 10 cm. Find PB.




**Solution:**

RP = PS = 6cm

$OS^2 = PO^2 + PS2$

$10^2 = PO^2 + 6^2$

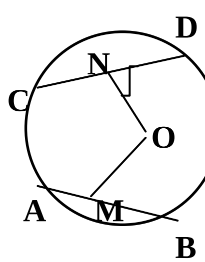
$PO^2 = 100 - 36 = 64$

PO = 8cm

$\therefore$ PB = PO + OB = 8 + 10 = 18 cm

**Example 27:**

In the figure, AB = 16 cm, CD = 12cm and OM = 6 cm. Find ON.




**Solution:**

$MB = \frac{1}{2} \times AB = 8cm$ (perpendicular from the centre of the circle bisects the chord)

$OB^2 = OM^2 + MB^2$

$\Rightarrow OB^2 = 6^2 + 8^2 = 36 + 64 = 100$

$\Rightarrow$ OB = 10 cm

OB = OD = 10cm (Radii)

$OD^2 = ON^2 + ND^2$

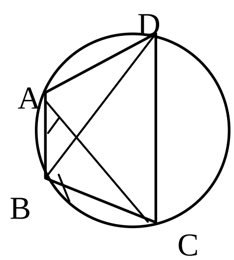
$10^2 = ON^2 + 6^2$

$\therefore ON^2 = 100 - 36 = 64$

Hence ON = 8 cm

**Example 28:**

In figure, ABCD is a cyclic quadrilateral in which AC and BD are its diagonals. If ∠DBC = 55° and ∠BAC = 45° find ∠BCD




**Solution:**

∠CAD = ∠DBC = 55° (Angles in the same segment)

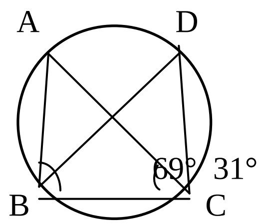
$\therefore$∠DAB = ∠CAD + ∠BAC = 55° +45° = 100°

But ∠DAB + ∠BCD = 180° (Opposite angles of a cyclic quadrilateral)

$\Rightarrow$ BCD = 180° - 100 = 80°

**Example 29:**

In figure, ∠ABC = 69°, ∠ACB = 31°, find ∠BDC.




**Solution:**

In $\Delta ABC$, $\angle ABC + \angle ACB + \angle BAC = 180°$

$\Rightarrow 69° + 31° + \angle BAC = 180°$

$\Rightarrow \angle BAC = 180° - 100°$

$\therefore \angle BAC = 80°$

But $\angle BAC = \angle BDC$

(Angles in the same segment of a circle are equal)

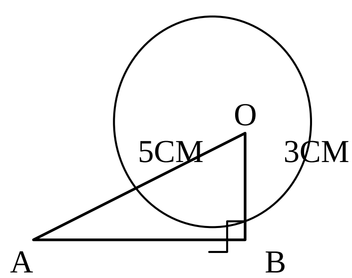
Hence $\angle BDC = 80°$

**Example 30:**

Find the length of the tangent from a point which is at a distance of 5cm from the centre of the circle of radius 3 cm.

**Solution:**

Let AB be the tangent $\Delta ABO$ is a right triangle at B.




By Pythagoras theorem,

$OA^2 = AB^2 + BO^3$

$\Rightarrow 5^2 = AB^2 + 32$

$\Rightarrow 25 = AB^2 + 9$

$\Rightarrow AB^2 = 25 - 9 = 16$

$\therefore AB = 4$

Hence, length of the tangent is 4 cm

.

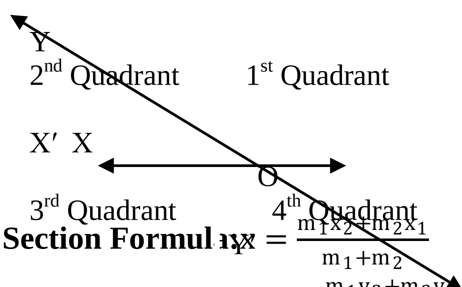
**COORDINATE GEOMETRY**

The Cartesian Co-ordinate System: Let$X'OX$ and $YOY'$ betwo perpendicular straight lines meeting at fixed point O then $X'OX$ is called X axis $Y'OY$ is called the axis of y or y axis point ‘O’is called the origin. X axis is known as abscissa and y - axis is known as ordinate.

**Distance Formula:** The distance between two points whoseco-ordinates are given:

$$\sqrt{(x_2 + x_1)^2 + (y_2 + y_1)^2}$$

Distance from origin: $\sqrt{(x - 0)^2 + (y - 0)^2}$




**Section Formula:** $x = \frac{m_1x_2+m_2x_1}{m_1+m_2}$

(Internally division) $y = \frac{m_1y_2+m_2y_1}{m_1+m_2}$

These points divide's the line segment in the ratio $m_1 : m_2$.

**TRIANGLE**

Suppose ABC be a triangle such that the coordinates of its vertices are $A(x_1, y_2)$, $B(x_2, y_2)$ and $C(x_3, y_2)$. Then, area of the triangle

$$= \frac{1}{2}[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]$$

**Centroid of triangle:** The coordinates ofthe centroid are $\left(\frac{x_1+x_2+x_3}{3}, \frac{y_1+y_2+y_3}{3}\right)$

**Example 31:**

Find the distance between the point

P $(a\cos\alpha, a\sin\alpha)$ and Q$(a\cos\beta, a\sin\beta)$ .

**Solution:**

$$d_2 = (a\cos\alpha - a\cos\beta)^2 + (a\sin\alpha - a\sin\beta)^2$$

$$= a^2(\cos\alpha - \cos\beta)^2 + a^2(\sin\alpha - \sin\beta)^2$$

$$a^2\left\{2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}\right\}^2 +$$

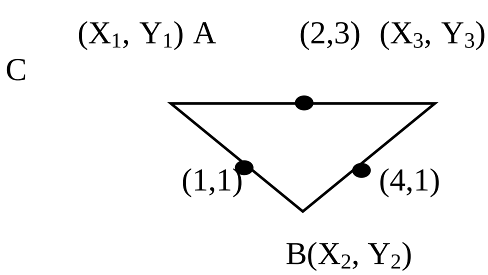
$$a^2\left\{2\cos\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}\right\}^2$$

$$= 4a^2\sin^2\frac{\alpha-\beta}{2}\left\{\sin^2\frac{\alpha+\beta}{2} + \cos^2\right\}$$

$$= 4a^2\sin^2\frac{\alpha-\beta}{2} \Rightarrow d = 2a\sin\frac{\alpha-\beta}{2}$$

**Example 32:**

The coordinates of mid-points of the sides of a triangle are (1, 1), (2, 3) and (4,1). Find the coordinates of the centroid.




**Solution:**

Let the coordinates of the vertices be $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_1, y_2)$.

Then, we have

$x_1 + x_2 = 2$, $x_2 + x = 8$, $x_3 + x_1 = 4$

and, $y_1 + y_2 = 2$, $y_2 + y_3 = 2$, $y_3 + y_1 = 6$

From the above equations, we have $x_1 + x_2 + x_3 = 7$ and $y_1 + y_2 + y_3 = 5$

Solving together, we have $x_1 = -1$, $x_2 = 3$, $x_3 = 5$ and $y_1 = 3$, $y_2 = -1$, $y_3 = 3$

Therefore the coordinates of the vertices are (-1,3), (3, -l) and (5,3).

Hence, the centroid is $\left(\frac{-1+3+5}{3}, \frac{3-1+3}{3}\right)$ i.e. $\left(\frac{7}{3}, \frac{5}{3}\right)$

**Example 33:**

If distance between the point (x, 2) and (3,4) is 2, then the value of x =

**Solution:**

$$2 = \sqrt{(x-3)^2 + (2-4)^2}$$

$$\Rightarrow 2 = \sqrt{(x-3)^2 + 4}$$

Squaring both sides

$4 = (x-3)^2 + 4 \Rightarrow x - 3 = 0 \Rightarrow x = 3$

**Example 34:**

Find the coordinates of a point which divides the line segment joining each of the following points in the given ratio:

(a) (2,3) and (7,8) in the ratio 2:3 internally

(b) (-1,4) and (0, -3) in the ratio 1:4 internally.

**Solution:**

(a) Let A(2, 3) and B(7, 8) be the given points.

Let P(x, y) divide AB in the ratio 2:3 internally.

Using section formula, we have,

$x = \frac{2\times7+3\times2}{2+3} = \frac{20}{5} = 4$

and $y = \frac{2\times8+3\times3}{2+3} = \frac{25}{5} = 5$

∴ P (4, 5) divides AB in the ratio 2:3 internally.(b) Let A (-1, 4) and B (0, -3) be the given points. Let P(x, y) divide AB in the ratio l: 4 internally

Using section formula, we have

$$x = \frac{1\times0+4\times(-1)}{1+4} = -\frac{4}{5}$$

and $y = \frac{1\times(-3)+4\times4}{1+4} = \frac{13}{5}$

$\therefore P\left(-\frac{4}{5}, \frac{13}{5}\right)$ divides AB in the ratio 1:4 internally.

**Example 35:**

Find the mid-point of the line-segment joining two point (3, 4) and (5, 12).

**Solution:**

Let A(3, 4) and B(5, 12) be the give points.

Let C(x, y) be the mid-point of AB. Using mid-point formula, we have, $x = \frac{3+5}{2}$ = 4 y = $\frac{4+12}{2}$ = 8

∴ C(4, 8) are the co-ordinates of the mid-point of the line segment joining two points (3, 4) and (5, 12).

**Example 36:**

The co-ordinates of the mid-point of a line segment are (2, 3). If co-ordinates of one of the end points of the line segment are (6, 5), find the co-ordiants of the other end point.

**Solution:**

Let other the end point be A(x, y)

It is given that C(2, 3) is the mid point

∴ We can write, $2 = \frac{x+6}{2}$ and $3 = \frac{y+5}{2}$

or 4 = x + 6 or 6 = y + 5

or x = -2 or y = 1

∴ A (-2, 1) be the co-ordinates of the other end point.

**Example 37:**

The area of a triangle is 5. Two of its vertices are (2, 1) and (3, -2). The third vertex lines on y = x + 3. Find the third vertex.

**Solution:**

Let the third vertex be$(x_3, y_3)$, area of triangle

$= \frac{1}{2}|[x_1(y_2 - y_3 + x_2(y_3 - y_1) +$ x3(y1−y2)

As $x_1 = 2, y_z = 1, x_2 = 3, y_2 = -2$,

Area of Δ = 5

$\Rightarrow 5 = \frac{1}{2}|2(-2 - y_3) +$ 3y3−1+x3(1+2)

$\Rightarrow 10 = |3x_3 + y_3 - 7| \Rightarrow 3x_3 + y_3 - 7 = \pm10$

Taking positive sign

$3x_3 + y_3 - 7 = 10 \Rightarrow 3x_3 + y_3 = 17$ ….(i)

Taking negative sign

$3x_3 + y_3 - 7 = -10 \Rightarrow 3x_3 + y_3 = -3$ ...(ii)

Given that $(x_3, -y_3)$ lies on y = x + 3

So, $-x_3 + y_3 = 3$ ….(iii)

Solving Esq. (i) and (iii), $x_3 = \frac{7}{2}, y_3 = \frac{13}{2}$

Solving Esq. (ii) and (iii), $x_3 = \frac{-3}{2}, y_3 = \frac{3}{2}$.

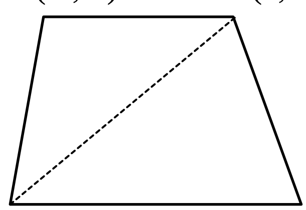
So the third vertex are $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(\frac{-3}{2}, \frac{3}{2}\right)$

**Example 38:**

Find the area of quadrilateral whose vertices, taken in order, are A(-3, 2), B(5, 4), C(7, -6) and D(-5, -4).

**Solution:**

Area of quadrilateral = Area of ΔABC + Area of ΔACD




So, Area of ΔABC $= \frac{1}{2}|(-3)(4+6+5-6-2+7(2-4)$

So area 08 Δ ACD 1 -30-40-141

$=\frac{1}{2}\Big|-3(-6+4)+7(-4-2)+$

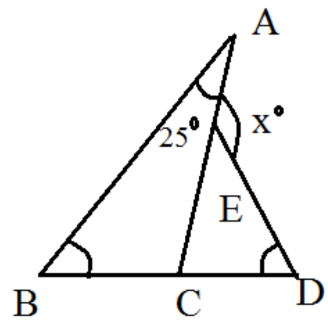
$-5(2+=121-841=42$ squnits 6)

$= \frac{1}{2}|+6-42-40| = \frac{1}{2}|-76| = 38$ sq. units

So, Area of quadrilateral ABCD = 42 + 38 = 80 sq. units.

**Example 39:**

In the figure, find the value of x°.




**Solution:**

In the Δ ABC, ∠A + ∠B + ∠ACB = 180°

⟹ 25° + 35° + ∠ACB = 180°

⟹∠ACB = 120°

Now, ∠ACB + ∠ACD = 180° (linear pair)

or 120° +∠ACD = 180°

or ∠ACD = 60° = ∠ECD

Again in the ΔCDE, CE is produced to A.

Hence, ∠AED = ∠ECD + ∠EDC

⟹ x = 60° + 60° = 120°.

**Example 40:**

Find the equation of the circle whose diameter is the line joining the points (-4, 3) and (12, 1). Find the intercept made by it on the y-axis.

**Solution:**

The equation of the required circle is

(x + 4) (x – 12) + (y -3) (y + 1) = 0

On the y-axis, x = 0

⟹ - 48 + $y^2$ - 2y -3 = 0 ⟹$y^2$ - 2y - 51 = 0

⟹ y = 1 ± $\sqrt{52}$

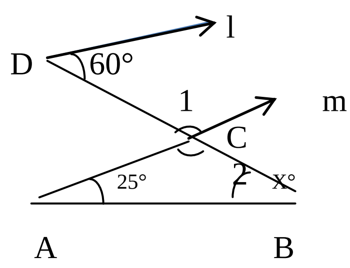
Hence the intercept on the y-axis

= $2\sqrt{52} = 4\sqrt{13}$

**Example 41:**

In figure, if l||m, then find the value of x.

**Solution:**




As l||m and DC is transversal

∴∠D + ∠1 = 180°

60° + ∠1 = 180°

∠1 = 120°

Here, ∠2 = ∠1 = 120°

(vertically opposite angles)

In the ΔABC

∠A + ∠B + ∠C = 180°

25° + x° + 120° = 180°

or x = 35°

**Example 42:**

M and N are points on the sides PQ and PR respectively of a ΔPQR. For each of the following cases state whether MN is parallel to QR:

(a) PM = 4, QM = 4.5, PN = 4, NR = 4.5

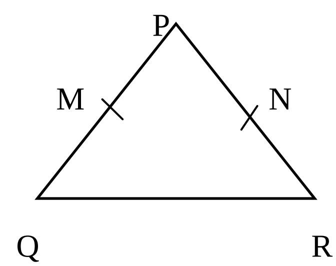
(b) PQ = 1.28, PR = 2.53, PM = 0.16, PN = 0.32

**Solution:**

(a) The triangle PQR is isosceles

⇒ MN || QR by converse of Proportionally theorem

(b) Again by converse of proportionally theorem, MN || OR




**Example 43:**

The point A divides the join the points (-5, 1) and (3, 5) in the ratio k: l and coordinates of points B and C are (1, 5) and (7, -2) respectively. If the area of Δ ABC be 2 units, then find the value (s) of k.

**Solution:**

A ≡ $\left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$, Area of Δ ABC = 2 units

$\Rightarrow \frac{1}{2}\left[\frac{3k-5}{k+1}(5+2) + 1\left(-2-\frac{5k+1}{k+1}\right) +\right.$ $75k+1k+1-5 = \pm 2$

⇒ 14k – 66 = ± 4 (k + 1) ⇒ k = 7 or 31/9