CLOCK

A clock has two hands: Hour hand and Minute hand. The minute hand (M.H.) is also called the long hand and the hour hand (H.H.) is also called the short hand.

The clock has 12 hours numbered from 1 to 12.

Also, the clock is divided into 60 equal minute divisions. Therefore, each hour number is separated by five minute divisions. Therefore,

Important Points-

❖ One minute division = $\frac{360}{60}$ = 6° apart, i.e. In one minute, the 60 minute hand moves 6°.

❖ One hour division = 6° × 5 = 30° apart, i.e. In one hour, the hour hand moves 30° apart.

Also, in one minute, the hour hand moves $= \frac{30°}{60°} = \frac{1°}{2}$apart.

❖ Since, in one minute, minute hand moves 6° and hour hand moves$\frac{1°}{2}$, therefore, in one minute, the minute hand gains $5\frac{1°}{2}$ more than hour hand.

❖ In one hour, the minute hand gains $5\ \frac{1°}{2} \times 60$ = 330° over the hour hand. i.e. the minute hand gains 55 minutes divisions over the hour hand.

Relative position of the hands -

The position of the M.H. relative to the H.H. is said to be the same, whenever the M.H. is separated from the H.H. by the same number of minute divisions and is on same side (clockwise or anticlockwise) of the H.H.

Any relative position of the hands of a clock is repeated 11 times in every 12 hours.

(a) When both hands are 15 minute spaces apart, they are at right angle.

(b) When they are 30 minute spaces apart, they point in opposite directions.

(c) The hands are in the same straight line when they are coincident or opposite to each other.

- In every hour, both the hand coincide once.
- In a day, the hands are coinciding 22 times.
- In every 12 hours, the hands of clock coincide 11 times
- In every 12 hours, the hands of clock are in opposite direction 11 times.
- In every 12 hours, the hands of clock are at right angles 22 times.
- In every hour, the two hands are at right angles 2 times.
- In every hour, the two hands are in opposite direction once.
- In a day, the two hands are at right angles 44 times.
- If both the hands coincide, then they will again coincide after $65\frac{5}{11}$minutes i.e. in correct clock, both hand coincide at an interval of $65\frac{5}{11}$ minutes.
- If the two hands coincide in time less than $65\frac{5}{11}$ minutes, then clock is too fast and if the two hands

coincide in time more than $65\frac{5}{11}$ minutes, then the clock is too slow.

NOTE:

ANOTHER SHORT-CUT FORMULA FOR CLOCKS

Angle made by Hands $= \left|30H - \frac{11}{2}M\right|$

Where H = Hour
M = minute

EXAMPLE1. At what time between 4 and 5 O' Clock will the hands of a watch
(i) Coincide, and
(ii) Point in opposite directions.

Sol.

(i) At 4 O' clock, the hands are 20 minutes apart. Clearly the minute hand must gain 20 minutes before two hands can be coincident. But the minute-hand gains 55 minutes in 60 minutes.

Let minute hand will gain x minute in 20 minutes.

So, $\frac{55}{20} = \frac{60}{x}$

$\Rightarrow x = \frac{20 \times 60}{55} = \frac{240}{11} = 21\frac{9}{11}$ min.

∴The hands will be together at $21\frac{9}{11}$ min past 4.

(ii) Hands will be opposite to each other when there is a space of 30 minutes between them. This will happen when the minute hand gains (20 + 30) = 50 minutes.

Now, the minute hand gains 50 min in $\frac{50 \times 60}{55}$ or $54\frac{6}{11}$ min.

∴The hands are opposite to each other at $54\frac{6}{11}$ min past 4.

EXAMPLE2. What is the angle between the hour hand and minute hand when it was 5:05 pm.

Sol. 5.05 pm means hour hand was on 5 and minute hand was on 1, i.e. there will be 20 minutes gap.

Angle = 20 × 6°= 120° [∵ 1 minute=6°]

INCORRECT CLOCK

If a clock indicates 6:10, when the correct time is 6:00 it is said to be 10 minute too fast and if it, indicates 5: 50 when the correct time is 6:00, it is said to be 10 minute too slow.

- Also, if both hands coincide at an interval × minutes and x < $65\frac{5}{11}$, then total time gained $= \left(\frac{65\frac{5}{11} - x}{x}\right)$ minutes and clock is said to be 'fast'.
- If both hands coincide at an interval x minutes and x >$65\frac{5}{11}$, then total time lost $= \left(\frac{x - 65\frac{5}{11}}{x}\right)$ minutes and clock is said to be 'slow'.

EXAMPLE3. My watch, which gains uniformly, is 2 min slow at noon on Sunday, and is 4 minutes 48 seconds fast at 2 pm on the following Sunday. When was it correct.

Sol. From Sunday noon to the following Sunday at 2 pm = 7 days 2 hours =170 hours.

The watch gains $\left(2 + 4\frac{48}{60}\right) = 6\frac{4}{5}$ minutes in 170 hours.

∴The watch gains 2 minutes in $\frac{2}{6\frac{4}{5}} \times 170$

=50 hours.

Now, 50 hours = 2 days

2 hours 2 days 2 hours from Sunday noon = 2 pm on Tuesday.

EXAMPLE4. The minute hand of a clock overtakes the hour hand at intervals of 65 minutes of the correct time. How much a day does the clock gain or lose?

Sol. In a correct clock, the minute hand gains 55 min. spaces over the hour hand in 60 minutes.

To be together again, the minute hand must gain 60 minutes over the hour hand.

55 min. are gained in $\left(\frac{60}{55} \times 60\right)$ min = $65\frac{5}{11}$ min.

But, they are together after 65 min.

∴Gain in 65min = $\left(65\frac{5}{11} - 65\right)$ = $\frac{5}{11}$min.

Gain in 24 hours =$\left(\frac{5}{11} \times \frac{60\times24}{65}\right)$ min = $10\frac{10}{143}$ min.

∴The clock gains $10\frac{10}{143}$ minutes in 24 hours.

EXAMPLE5.A man who went out between 5 or 6 and returned between 6 and 7 found that the hands of the watch had exactly changed place. When did he go out?

Sol. Between 5 and 6 to 6 and 7, hands will change place after crossing each other one time. i.e. they together will make 1 + 1 = 2 complete revolutions.

H. H. will move through $2 \times \frac{60}{13}$ or $\frac{120}{13}$ minute divisions.

Between 5 and 6 → $\frac{120}{13}$minute divisions.

At 5, minute hand is 25 minute divisions behind the hour- hand.

Hence it will have to gain $25 + \frac{120}{13}$ minute divisions on the hour-hand =$\frac{445}{13}$ minute divisions on the hour hand.

The minute hand gains $\frac{445}{13}$minute divisions in $\frac{445}{13} \times \frac{12}{11}$ minutes =$\frac{5340}{143}$

= $37\frac{49}{143}$ minutes

∴The required time of departure is $37\frac{49}{143}$ min past 5.

CALENDAR

The solar year consists of 365 days, 5 hrs 48 minutes, 48 seconds. In 47 BC, Julius Ceasar arranged a calendar known as the Julian calendar in which a year was taken as $365\frac{1}{4}$ days and in order to get rid of the odd quarter of a day, an extra day was added once in every fourth year and this was called as leap year or Bissextile. Nowadays, the calendar, which is mostly used, is arranged by Pope Gregory XII and known as Gregorian calendar.

In India, number of calendars were being used till recently. In 1952, the Government adopted the National Calendar based on Saka era with Chaitra as its first month, in an ordinary year, Chaitra 1 falling on March 22 of Gregorian Calendar and in a leap year it falls on March 21.

REMEMBER

- In an ordinary year,
  1 year = 365 days = 52 weeks + 1 day
- In a leap year
  1 year = 366 days =52 weeks + 2 days
  NOTE: First January I A.D was Monday. So we must count days from Sunday.
- 100 years or one century contains 76 ordinary years and 24 leap years.
  ⇒ [76 ×52 weeks+ 76 odd days]
  + [24×52 weeks+ 24 × 2 odd days]
  = (76 + 24) × 52 weeks + (76 + 48) odd days
  = 100 × 52 weeks + 124 odd days
  = 100 × 52 weeks + (17 × 7 + 5) odd days
  = (100 × 52 + 17) weeks+ 5 odd days
  ∴100 years contain 5 odd days.
  Similarly, 200 years contain 3 odd days,
  300 years contain 1 odd days,
  400 years contain 0 odd days.
  Year whose non-zero numbers are multiple of 4 contains no odd days; like 800, 1200, 1600 etc.

The number of odd days in months

The month with 31 days contains (4×7 + 3) i.e. 3 odd days and the month with 30 days contains (4×7+2) i.e. 2 odd days.

NOTE:

- February in an ordinary year gives no odd days, but in a leap year gives one odd day.
- Day of the week related to ODD days –

No. of Days	0	1	2	3	4	5	6
Days	Sun	Mon	Tues	Wed	Thur	Fri	Sat

EXAMPLE6.What day of the week was 15th August 1949?

Sol. 15th August 1949 means
1948 complete years + first 7 months of the year 1949 + 15 days of August.
1600 years give no odd days.
300 years give 1 odd day.
48 years give {48 + 12} = 60 = 4 odd days.
[∵ For ordinary years → 48 odd days and for leap year 1 more day (48 ÷ 4) = 12 odd days; 60 = 7 × 8 + 4]
From 1st January to 15th August 1949
Odd days:
January-3
February - 0
March-3
April-2
May - 3
June-2
July-3
August - 1
17 ⇒ 3 odd days.
∴ 15th August 1949→ 1 + 4 + 3 = 8 = 1 odd day.
This means that 15th Aug. fell on 1st day. Therefore, the required day was Monday.

EXAMPLE7. How many times does the 29th day of the month occur in 400 consecutive years?

Sol. In 400 consecutive years, there are 97 leap years. Hence in 400 cores cattier year. February has the 29th day 97 times and the remaining eleven month, have the 29th day of the 400×1100 = 4400 times

∴ The 29the day month occurs (4400+97) or 4497 times.

EXAMPLE**8. Today's is 5th February. The day of the week is Tuesday. This is leap year. What will be the day of the week on this date after 5 years after 5 years?**

Sol. This is a leap year. So, next 3 years will give one odd day each. Then leap year gives 2 odd days and then again next year give 1 odd day.

Therefore (3+ 2+1) =6 odd days will be there.

Hence the day of the week will be 6 odd day 5 beyond

Tuesday, i.e., it will be Monday.

EXAMPLE**9. What day of the week was 20th June 1837?**

Sol. 20th June 1837 means 1836 complete years + firs 5 months of the year 1837 + 20 days of June.

1600 years give no odd days.

200 years give 3 odd days.

36 years give (36 + 9) or 3 odd days.

1836 years give 6 odd days.

From 1st January to 20^{th} June there are 3 odd days.

Odd days:

January : 3
February : 0
March : 3
April : 2
May : 3
June : 6
17

Therefore, the total number of odd days = (6 + 3) or 2 odd days.

This means that the 20th of June fell on the 2nd day commencing from Monday. Therefore, the required day was Tuesday.

EXAMPLE**10. Prove that the calendar for 1990 will same for 2001 also.**

Sol. It is clear that the calendar for 1990 will serve for 2001 if first January of both the years is the same weekdays. For that the number of odd day s between 31st December 1989 and 31st December 2000 must be zero. Odd days are as given below.

Year	1990	1991	1992	1993	1994
Odd days	1	1	(Leap) 2	1	1
1995	1996	1997	1998	1999	2000
1	(Leap) 2	1	1	1	(Leap) 2

Total number of odd days = 14 days = 2 weeks + odd days.