Numerical Methods

Previous Year Questions & Detailed Solutions

1. The rate of convergence in the Gauss-Seidal method is ________ as fast as in Gauss Jacobi’s method
   1) thrice 2) half-times 3) twice 4) three by two times

2. In application of Simpson’s 1/3 rd rule, the interval h for closer approximation should be
   1) small 2) odd and large 3) large 4) even and small

3. Which of the following is a step by step method?
   1) Taylor’s method 2) Picards method 3) Adams Bashforth method 4) Euler’s method

4. The interpolation polynomial from the data

<table>
<thead>
<tr>
<th>x</th>
<th>0</th>
<th>1</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>5</td>
<td>4</td>
<td>8</td>
</tr>
</tbody>
</table>

1) $x^2 - 2x + 5$  2) $x^2 + 2x + 5$
3) $2x^2 - 2x + 5$  4) $2x^2 - 5x + 5$

5. The value of the integration $\int_a^{a+2h} f(x) dx$ is approximately equal to
   1) $\frac{h}{2}[f(a)+3f(a+h)+f(a+2h)]$
   2) $\frac{h}{3}[f(a)+4f(a+h)+f(a+2h)]$
   3) $\frac{h}{3}[f(a)+2f(a+h)+f(a+2h)]$
   4) $\frac{h}{2}[f(a)+4f(a+h)+f(a+2h)]$

6. The finite difference approximation for $\frac{d^2y}{dx^2}$ at $x = x_0$ is
   1) $\frac{1}{h^2}[y(x_0-h)-2y(x_0)+y(x_0+h)]$
   2) $\frac{1}{h^2}[y(x_0-h)+2y(x_0)+y(x_0+h)]$

3) $\frac{1}{h^2}[y(x_0-h)-y(x_0)+y(x_0+h)]$
4) $\frac{1}{h^2}[y(x_0-h)+y(x_0)+y(x_0+h)]$

7. For the following data

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Y</td>
<td>-1</td>
<td>3</td>
<td>7</td>
<td>11</td>
</tr>
</tbody>
</table>

The straight line $y=mx+c$ by the method of least square is
   1) $y=-2x-1$  2) $y=x-1$
   3) $y=-2x$  4) $y=2x-1$

8. The velocity $v$ (km/min) of a train which starts from rest, is given at fixed intervals of time $t$ (min) as follows:

<table>
<thead>
<tr>
<th>t</th>
<th>2</th>
<th>4</th>
<th>6</th>
<th>8</th>
<th>10</th>
<th>12</th>
<th>14</th>
<th>16</th>
<th>18</th>
<th>20</th>
</tr>
</thead>
<tbody>
<tr>
<td>v</td>
<td>10</td>
<td>18</td>
<td>25</td>
<td>29</td>
<td>32</td>
<td>30</td>
<td>11</td>
<td>5</td>
<td>2</td>
<td>0</td>
</tr>
</tbody>
</table>

The approximate distance covered by Simpson’s 1/3 rule is
   1) 306.3  2) 309.3  3) 310.3  4) 307.3

9. Find the cubic polynomial by Newton’s forward difference which takes the following:

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>1</td>
<td>2</td>
<td>1</td>
<td>10</td>
</tr>
</tbody>
</table>

then $f(4)$ is
   1) 40  2) 41  3) 39  4) 42

10. The first derivative $\frac{dy}{dx}$ at $x=0$ for the given data

<table>
<thead>
<tr>
<th>X</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(x)</td>
<td>2</td>
<td>1</td>
<td>2</td>
<td>5</td>
</tr>
</tbody>
</table>

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Numerical Methods

11. Error in Simpson’s \( \frac{1}{3} \) rule is of the order
   1) \(-h^2\) 2) \(h^3\) 3) \(h^4\) 4) \(\frac{2h^3}{3}\)

12. If \(\alpha, \beta\) and \(\gamma\) are the roots of \(x^3 + px^2 + qx + l = 0\), then the equation whose roots are \(-\frac{1}{\alpha}, -\frac{1}{\beta}, -\frac{1}{\gamma}\) is
   1) \(x^3 + qx^2 – px – l = 0\)
   2) \(x^3 + qx^3 – px + 1 = 0\)
   3) \(x^3 - qx^2 – px - l = 0\)
   4) \(x^3 - qx^3 + px + l = 0\)

13. In Jacobi’s iteration method to get the \((r+1)\)th iterates, we use the
   1) values of the \(r\)th iterates
   2) values of the \((r-1)\)th iterates
   3) values of the \((r+1)\)th iterates
   4) latest available values

14. If \(h\) is the length of the intervals, then the error in the trapezoidal rule is of order
   1) \(h\) 2) \(h^2\) 3) \(h^3\) 4) \(h^4\)

15. If \(E\) is the translation operator, then the central difference operator \(\delta\) is
   1) \(E^{1/2} + E^{-1/2}\) 2) \(E^{1/2} - E^{-1/2}\)
   3) \(\frac{3}{2} (E^{1/2} + E^{-1/2})\) 4) \(\frac{1}{2} (E^{1/2} + E^{-1/2})\)

16. If \(\Delta f(x) = f(x + h) - f(x)\), \(E f(x) = f(x + h)\) and \(\delta f(x) = (x + \frac{h}{2}) - f(x - \frac{h}{2})\) then \(\Delta \delta\) is
   1) \(\delta^2\) 2) \(\delta\) 3) \(\delta^{1/2}\) 4) \(\delta^{1/2} + \delta^{-1/2}\)

17. Expression for \(\left(\frac{d^2y}{dx^2}\right)_x=x_n\) using Newton’s
   backward interpolation
   \[
   \frac{1}{h^2} \left[ \nabla^2 f_n + \nabla^3 f_n + \frac{11}{12} \nabla^4 f_n + \cdots \right]
   \]
   1) \(\frac{1}{h^2} \left[ \nabla^2 f_n + \Delta^2 f_n + \Delta^4 f_n + \cdots \right] f_n\)

18. By Taylor’s series method solution of
   \(y' = x^2 + y^2; y(0) = 1\) is
   1) \(y = l + x - x^2\) 2) \(y = 1 - x + x^2\)
   3) \(y = x + x^2 + x^3\) 4) \(y = l + x + x^2 + \frac{4}{3} x^3\)

19. Error in the Trapezoidal rule is of the order
   1) \(-h\) 2) \(h^3\) 3) \(h^4\) 4) \(h^2\)

20. A river is 80 ft wide, the depth \(d\) in feet at a distance \(x\) ft. from one bank is given by the following table:

<table>
<thead>
<tr>
<th>(x)</th>
<th>0</th>
<th>10</th>
<th>20</th>
<th>30</th>
<th>40</th>
<th>50</th>
<th>60</th>
<th>70</th>
<th>80</th>
</tr>
</thead>
<tbody>
<tr>
<td>(y)</td>
<td>0</td>
<td>4</td>
<td>7</td>
<td>9</td>
<td>12</td>
<td>15</td>
<td>14</td>
<td>8</td>
<td>3</td>
</tr>
</tbody>
</table>

Approximately the area of cross section by Simpson’s \(1/3\) rule is
   1) \(712\) sq.ft. 2) \(680\) sq.ft. 3) \(710\) sq.ft. 4) \(610\) sq.ft.

21. The first derivative \(\frac{dy}{dx}\) at \(x = 4\) for the data.

<table>
<thead>
<tr>
<th>(X)</th>
<th>0</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(f(x))</td>
<td>1</td>
<td>-2</td>
<td>-3</td>
<td>-2</td>
<td>1</td>
</tr>
</tbody>
</table>

1) \(-4\) 2) \(4\) 3) \(16\) 4) \(-16\)

22. If \(x_0\) and \(x_1\) are two points, \(f(x_0)\) and \(f(x_1)\) are of opposite signs, then the root \(x_2\) by regular falsi method is equal to
   1) \(\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}\)
   2) \(\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}\)
   3) \(\frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_1) - f(x_0)}\)
   4) \(\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_0) - f(x_1)}\)

23. The Newton-Raphson iteration scheme to find an approximate real root of the equation \(f(x) = 0\) is
   1) \(x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}\)
   2) \(x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\)

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30. What is the highest degree of a polynomial whose definite integration using Simpson’s \(\frac{1}{3}\) rule gives the exact value?
   1) 1  
   2) 2  
   3) 3  
   4) 4

31. The global error arising in using Runge-Kutta method of fourth order to solve an ordinary differential equation is of
   1) \(0(h^2)\)  
   2) \(0(h^3)\)  
   3) \(0(h^4)\)  
   4) \(0(h^5)\)

32. Simpson’s one-third rule integration is exact for all polynomial of degree not exceeding
   1) 2  
   2) 3  
   3) 4  
   4) 5

33. With usual notations, the value of \((1+\Delta)(1-V)\) is
   1) 0  
   2) 2  
   3) 1  
   4) 4

34. The modified Newton-Raphson formula of finding the root of the equation \(f(x)=0\) is
   1) \(x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\)
   2) \(x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\) 
   3) \(x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}\) 
   4) \(x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}\)

35. The value of \(\int_0^1 \frac{dx}{1+x^2}\) Trapezoidal rule with \(h = 0.2\) is
   1) 0.687322  
   2) 0.877323  
   3) 0.727333  
   4) 0.783732

36. Simpson’s one-third rule of integration is exact for all polynomial of degree not exceeding
   1) 2  
   2) 3  
   3) 4  
   4) 5
37. Which of the following formula is a particular case of Runge-Kutta formula of the second order?
   1) Taylor series formula
   2) Picard’s formula
   3) Euler’s modified formula
   4) Milne’s predictor formula

38. The Newton Raphson method of finding the root of the equation \( f(x) = 0 \) is by using the formula
   1) \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \)
   2) \( x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)} \)
   3) \( x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)} \)
   4) \( x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \)

39. The value of \( \int_0^2 \frac{dx}{1+x} \) by Trapezoidal Rule taking \( n=2 \) is
   1) 0.645
   2) 0.641
   3) 0.583
   4) 0.621

40. Trapezoidal rule for the evaluation of the integral \( \int_a^b f(x) \, dx \) requires the Interval to be divided into
   1) only an even number of intervals
   2) only an odd number of intervals
   3) any number of intervals
   4) a minimum of ten intervals

41. If \( \int_0^0.6 f(x) \, dx = k[f(0) + 4f(0.3) + f(0.6)] \) then the value of \( k \) when Simpson’s rule is applied is
   1) 0.1
   2) 0.2
   3) 0.3
   4) 0.4

42. The Newton - Raphson formula to find an approximate value of \( \sqrt{a} \) is
   1) \( x_{n+1} = \frac{1}{2} \left( x_n + \frac{a}{x_n} \right) \)
   2) \( x_{n+1} = \frac{1}{2} \left( x_n - \frac{a}{x_n} \right) \)
   3) \( x_{n+1} = \left( x_n + \frac{a}{x_n} \right) \)
   4) \( x_{n+1} = x_n - \frac{a}{x_n} \)

43. The iteration formula given by Newton-Raphson method to find root of the equation \( x \sin x + \cos x = 0 \) is
   1) \( x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n} \)
   2) \( x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n} \)
   3) \( x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n + 2 \sin x_n} \)
   4) \( x_{n+1} = x_n - x_n \cos x_n + 2 \sin x_n \)

44. The interpolating polynomial for the data
   \[
   \begin{array}{c|cccc}
   x & -1 & 0 & 1 & 2 \\
   \hline
   f & 1 & 1 & 1 & -5 \\
   \end{array}
   \]
   1) \( 1 + x - x^3 \)
   2) \( 1 + x + x^2 \)
   3) \( 1 + x - x^2 \)
   4) \( x + x^2 + x^3 \)

45. Given that \( f(1) = 1 \), \( f(1.5) = 2.875 \), \( f(2) = 7 \), \( f(2.5) = 14.125 \) and \( f(3) = 25 \), the value of \( \int_1^3 f(x) \, dx \) obtained by Simpson’s \( \frac{1}{3} \) rule is
   1) 17.50
   2) 18.00
   3) 18.25
   4) 19.125

46. A root of \( x^3 - 4x - 9 = 0 \) using bisection method in two stages is
   1) 2.75
   2) 0.5361
   3) 0.7969
   4) 0.7123

47. The lagrange’s interpolation polynomial curve passing through the points \( (0, 1) \), \( (1, 0) \) and \( (3, 10) \) is
   1) \( x^2 - 2x + 1 \)
   2) \(-4x^2 + 3x + 1 \)
   3) \( 2x^2 - 3x + 1 \)
   4) \( x^2 - x + 1 \)

48. What is the highest degree polynomial whose definite integration using Simpson’s \( \frac{1}{3} \) -rule gives the exact value?
   1) 1
   2) 2
   3) 3
   4) 4
49. Milne’s predictor value of \( y(0, 4) \) given that \( y' = y - \frac{2x}{y} \), \( y(0) = 1 \)
\( y'(0.1) = 0.9, \ y'(0.2) = 0.8, \ y'(0.3) = 0.7 \) is
1) \( \frac{23}{15} \)
2) \( \frac{33}{25} \)
3) \( \frac{36}{25} \)
4) \( \frac{59}{50} \)

50. The recurrence formula for finding the reciprocal of the \( n \)th root of \( M \), by Newton’s method is
1) \( x_{k+1} = \frac{[n-1]x_k^n + M}{nx_k^n-1} \)
2) \( x_{k+1} = \frac{x_k[n+1-Mx_k^n]}{n} \)
3) \( x_{k+1} = \frac{x_k[n-1+Mx_k^n]}{n} \)
4) \( x_{k+1} = \frac{1}{n} \left( \frac{1}{x_k} + Mx_k \right) \)

51. The value of \( \int_0^1 (1 + x^2)^{-1} \) by Simpson’s one-third rule with two strips is
1) \( \frac{31}{60} \)
2) \( \frac{47}{30} \)
3) \( \frac{31}{40} \)
4) \( \frac{47}{60} \)

52. Which of the following formulas is a particular case of Runge-Kutta formula of second order?
1) Taylor series formula
2) Picard’s formula
3) Euler’s modified formula
4) Milne’s predictor formula

DETAILED SOLUTIONS

1. (3) The rate of convergence of Gauss-Seidel method is roughly twice that of Gauss-Jacobi.

2. (4) In application of Simpson’s \( \frac{1}{3} \)rd rule, the interval \( h \) for closer approximation should be even and small.

3. (4) Euler’s method is a step by step method.

4. (1) Consider \( f(x) = x^2 - 2x + 5 \)
then \( f(0) = 0^2 - 2(0) + 5 = 5 \)
f(1) = \( 1^2 - 2(1) + 5 = 4 \)
f(3) = \( 3^2 - 2(3) + 5 = 8 \)
\( \because f(x) = x^2 - 2x + 5 \) is the required polynomial.

5. (2) \( \int_a^{a+2h} f(x) \, dx = \frac{h}{3} [f(a) + 4f(a + h) + f(a + 2h)] \)

6. (1) \( \frac{d^2y}{dx^2} (x = x_0) = \frac{f_{i-2} - 2f_{i-1} + f_{i-3}}{h^2} \)
\( \frac{1}{h^2} [y(x_0 - h) - 2y(x_0) + y(x_0 + h)] \)

7. (4)

<table>
<thead>
<tr>
<th>( x )</th>
<th>0</th>
<th>2</th>
<th>4</th>
<th>6</th>
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</thead>
<tbody>
<tr>
<td>( y )</td>
<td>-1</td>
<td>3</td>
<td>7</td>
<td>11</td>
</tr>
</tbody>
</table>

Consider \( y = 2x - 1 \)
x = 0 \( \implies y = 2(0) - 1 = -1 \)
x = 2 \( \implies y = 2(2) - 1 = 3 \)
x = 4 \( \implies y = 2(4) - 1 = 7 \)
x = 6 \( \implies y = 2(6) - 1 = 11 \)
\( \therefore y = 2x - 1 \) is the required straight line.

8. (*) \( \int y \, dx = \frac{h}{3} \left( y_0 + y_n + 4(y_1 + y_3 + ... + y_n) + 2(y_2 + y_4 + ... + y_n - 2) \right) + 4(18+29+20+5)+2(25+32+11+2)] \)
\( = \frac{2}{3} \left( 10+0+4(18+29+20+5)+2(25+32+11+2) \right) \)
\( = 292 \)

9. (2) \( f(x) = y_0 + p \Delta f(x_0) + \frac{p(p-1)}{2!} \Delta^2 f(x_0) + \frac{p(p-1)(p-2)}{3!} \Delta^3 f(x_0) \)
where \( p = \frac{x-x_0}{h} \)
\( \Delta f(x_0) = 1 \)
\( \Delta^2 f(x_0) = -2 \)
\[ \Delta^3 f(x_0) = 12 \]
\[ p = x \]
\[ f(x) = 1 + x + \frac{x(x-1)}{2!} (2) \]
\[ + \frac{x(x-1)(x-2)(12)}{3!} \]
\[ = 1 + x - x (x - 1) + 2x(x-1)(x-2) \]
\[ : f(4) = 1 + 4 - 4 \times 3 + 2 \times 4 \times 3 \times 2 \]
\[ = 5 - 12 + 48 \]
\[ = 41 \]
\[ \alpha, \beta, \gamma \text{ are roots of } \]
\[ x^3 + px^2 + qx + 1 = 0 \]
\[ \Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma} \text{ are roots of } \]
\[ \left( \frac{1}{x} \right)^3 + p \left( \frac{1}{x} \right)^2 + q \left( \frac{1}{x} \right) + 1 = 0 \]
\[ \Rightarrow x^3 + qx^2 + px + 1 = 0 \]
\[ \delta = E^{1/2} - E^{-1/2} \]
\[ \delta f(x) = (E^{1/2} - E^{-1/2}) f(x) \]
\[ \Delta = E - 1; \Delta = 1 - E^{-1} \]
\[ \Delta V = (E - 1) (1 - E^{-1}) \]
\[ = E - 1 - 1 + E^{-1} \]
\[ = E + E^{-1} - 2 \]
\[ d\delta^2 = (E^{1/2} - E^{-1/2})^2 \]
\[ = E^2 - 2E^{1/2} E^{-1/2} + E^{-1} \]
\[ = E^2 + E^{-1} \]

10. (2)

<table>
<thead>
<tr>
<th>x</th>
<th>f(x)</th>
<th>\Delta y</th>
<th>\Delta^2 y</th>
<th>\Delta^3 y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>2</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>-1(\Delta y_0)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>2(\Delta^2 y_0)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2</td>
<td></td>
<td>0(\Delta^3 y_0)</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>5</td>
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</table>

First derivative by Newton’s forward difference formula
\[ y = \frac{1}{h} \left[ \Delta y_0 - \frac{1}{2} \Delta^2 y_0 + \frac{1}{3} \Delta^3 y_0 - \cdots \right] \]
\[ = \frac{1}{h} \left[ -1 - \frac{1}{2}(2) + \frac{1}{3}(0) \right] \]
\[ = -1 - 1 = -2 \]

11. (3)
Error in Simpson’s \( \frac{1}{3} \) rule is of order \( h^4 \)

12. (3)

Results:
Let \( a_1, a_2, \ldots, a_n \) be the roots of the equation
\[ x^n + p_1 x^{n-1} + p_2 x^{n-2} + \cdots + p_n = 0 \]
then the roots of
\[ x^n - p_1 x^{n-1} + p_2 x^{n-2} - \cdots (-1)^n p_n = 0 \]
are \(-a_1, -a_2, \ldots, -a_n\)

Given

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17. (4)
Standard formula
\[ \left( \frac{d^2y}{dx^2} \right)_{x=x_n} = \frac{1}{h^2} \left[ \Delta^2 - \Delta^3 + \frac{11}{12} \Delta^4 - \cdots \right] f_n \]

18. (4)
Taylor’s series
\[ y = y_0 + \frac{h}{1!} y_0' + \frac{h^2}{2!} y_0'' + \frac{h^3}{3!} y_0''' + \cdots \]

Given \( y(0) = 1 \)
\[ \Rightarrow y_0 = 1; x_0 = 0 \]
\[ y' = x^2 + y^2 \text{ (Given)} \]
\[ y_0' = 0 + 1 = 1 \]
\[ y'' = 2x + 2yy' \]
\[ y_0'' = 2(0) + 2.1.1 = 2 \]
\[ y''' = 2 + 2(yy'' + (y')^2) \]
\[ = 2 + 2(1.2 + 1^2) \]
\[ = 2 + 2(2 + 1) \]
\[ = 8 \]

Putting \( h - x \) then
\[ y = 1 + \frac{x}{1!} + \frac{x^2}{2!} 2 + \frac{x^3}{3!} 8 \]
\[ = 1 + x + x^2 + \frac{4}{3} x^3 \]

19. (4) Error in the Trapezoidal rule is of the order \( h^2 \).

20. (3)
Simpson’s rule
\[ \int_{x_0}^{x_0 + nh} f(x) \, dx = \frac{h}{3} \left[ y_0 + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 + \cdots) + y_n \right] \]
\[ = \frac{h}{3} \left[ y_0 + y_n + 4 \text{(sum of odd ordinates)} + 2 \text{(sum of even ordinates)} \right] \]
\[ \therefore \text{Cross Section Area} \]

21. (2)

<table>
<thead>
<tr>
<th>( x )</th>
<th>( y = f(x) )</th>
<th>First difference</th>
<th>Second difference</th>
<th>Third difference</th>
<th>Fourth difference</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-2</td>
<td>2</td>
<td>-1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>-3</td>
<td>2</td>
<td>1</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>-2</td>
<td>0</td>
<td></td>
<td>2</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

\[ \nabla y_0 = 3; \nabla^2 y_0 = 2 \]
\[ \nabla^3 y_0 = \nabla^4 y_0 = \cdots = 0 \]

Newton’s formula for Backward difference
\[ f'(x) = \frac{1}{h} [\nabla y_0 + \frac{1}{2} \nabla^2 y_0 + \frac{1}{3} \nabla^3 y_0 \cdots] \]
\[ = \frac{1}{h} \left[ 3 + \frac{2}{2} \right] \]
\[ = 4 \]

22. (2)
By regular - falsi method the root is given by
\[ x = \frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)} \]

23. (2)
The Newton - Rapson Iteration scheme to find an approximate real root of the equation \( f(x) = 0 \) is
24. (4)
Power method is used to determine numerically largest eigen value and the corresponding eigen vector of a matrix A.

25. (1)
Let \( x_0 = 0 \); \( y_0 = 1 \)
\( x_1 = 1 \); \( y_1 = 2 \)
\( x_2 = 3 \); \( y_2 = 10 \)
\[ y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1 + \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2 \]
\[ = \frac{(x-1)(x-3)}{-1x-3} \times 1 + \frac{(x-0)(x-3)}{1x-2} \times 2 + \]
\[ + \frac{3 \times 2}{3x^2} \times 10 \]
\[ = \frac{x^2 - 4x + 3}{3} \times \frac{5x^2 - 5x}{3} \]
\[ = \frac{3x^2 + 3}{3} = x^2 + 1 \]

26. (3)
\[ \frac{d^2y}{dx^2} = \frac{1}{h^2} [\nabla^2 y_1 + (r + 1)\nabla^2 y_1 + \ldots] \]
\[ = \frac{1}{h^2} [\nabla^2 y_1] \text{ approximately} \]
\[ = \frac{1}{h^2} [\nabla y_{i+1} - \nabla y_i] \]
\[ = \frac{1}{h^2} [y_{i+2} - y_{i+1} - (y_{i+1} - y_i)] \]
\[ = \frac{1}{h^2} [y_{i+2} - 2y_{i+1} + y_i] \]
which is also equal to \( \frac{d^2y}{dx^2} = \frac{1}{h^2} (y_{i+1} - 2y_i + y_{i-1}) \)

27. (2)
\[ \int_a^{a+2h} f(x)dx = \frac{h}{3} [f(a) + 4f(a + h) + f(a + 2h)] \]
is called Simpson’s \( \frac{1}{3} \) rule.

28. (3)
Since only 3 values of x are given. Assume \( y = f(x) = a_0 + a_1x + a_2x(x-1) \)
put \( x=0, y=1 \)
\[ \Rightarrow 1 = a_0 + a_1(0) + a_2(0) \]
\[ \Rightarrow a_0 = 1 \]
put \( x - 1, y=0 \)
\[ \Rightarrow 0 = a_0 + a_1 + a_2(0) \]
\[ \Rightarrow 0 = a_0 + a_1 \]
\[ \Rightarrow 0 = 1 + a_1 \Rightarrow a_1 = -1 \]
put \( x=3, y=10 \)
\[ \Rightarrow 10 = a_0 + 3a_1 + 6a_2 \]
\[ \Rightarrow f(x) = 1 - x + 2x(x-1) \]
\[ = 1 - x + 2x^2 - 2x \]
\[ = 2x^2 - 3x + 1 \]

29. (3)
Paricular case of R.K. formula of second order is Euler’s modified formula.

30. (2)
Simpson’s one-third rule is
\[ \int_{x_0}^{x_0+nh} ydx = \frac{h}{3} [(y_0 + y_n) + 4(y_1 + y_2 + \ldots + y_{n-1}) + 2(y_2 + y_4 + \ldots + y_{n-2})] \]
In this case we neglect all differences above the second.
So \( y \) will be a polynomial of second degree only, i.e., \( y = ax^2 + bx + c \)

31. (3)
The error in R.K. method of fourth order to solve an ordinary differential equation is of order \( 0(h^4) \).

32. (1)
Required degree = 2

33. (3)
Formula
\[ (1 + \Delta)(1 - \nabla) = 1 \]
34. (3)
Newton – Rapson formula
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

35. (4)

<table>
<thead>
<tr>
<th>X</th>
<th>( y = \frac{1}{1 + x^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>0.2</td>
<td>0.9615</td>
</tr>
<tr>
<td>0.4</td>
<td>0.8621</td>
</tr>
<tr>
<td>0.6</td>
<td>0.7353</td>
</tr>
<tr>
<td>0.8</td>
<td>0.6098</td>
</tr>
<tr>
<td>1</td>
<td>0.5</td>
</tr>
</tbody>
</table>

By Trapezoidal Rule,
\[
\int_0^1 \frac{dx}{1 + x^2} = \frac{h}{2} [y_0 + y_5 + 2(y_1 + y_2 + y_3 + y_4)]
\]
\[
= \frac{0.2}{2} [1.5 + 2(3.1687)]
\]
\[
= 0.1 \times 7.8374
\]
\[
= 0.78374
\]

36. (1)
Simpsons \( \frac{1}{3} \) rule of Integration is exact for all polynomial of degree not exceeding 2.

37. (3)
Particular case of Runge Kutta formula of second order is Euler’s modified formula.

38. (2)
Newton Raphson formula of finding the root of \( f(x) = 0 \) is
\[ x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)} \]

39. (1)

40. (3)
Any number of intervals.

41. (1)
\( h = \) interval length
\[
= 0.3
\]
By Simpsons \( \frac{1}{3} \) rule
\[
k = \frac{h}{3} = \frac{0.3}{3}
\]
\[
= 0.1
\]

42. (1)
Let \( x = \sqrt{a} \)
\[
\Rightarrow x^2 - a = 0
\]
Let \( f(x) = x^2 - a \)
then \( f'(x) = 2x \)
\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]
\[
= x_n - \left( \frac{x_n^3 - a}{2x_n} \right)
\]
\[
= \frac{1}{2} \left[ 2x_n^2 - x_n^3 + a \right]
\]
\[
= \frac{1}{2} \left[ x_n + \frac{a}{x_n} \right]
\]

43. (1)
Newton Raphson Method
\[
x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}
\]
\[
f(x) = x \sin x + \cos x
\]
f(x) = x \cos x + \sin x - \sin x \\
= x \cos x \\
∴ By Newton Raphson method \\
x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n}

44. (1) 
Since only 4 values of x are given assume f(x) to be a polynomial of degree 3 
Let f(x) = a_0 + a_1 x + a_2 x (x-1) + a_3 x (x-1) (x-2) 
put x = -1, f(x) = 1 
⇒ 1 = a_0 - a_1 + 2a_2 - 6a_3 
∴ a_0 = a_1 + 2a_2 - 6a_3 
\(\cdots (1)\) 
x = 0 then f(x) = 1 
∴ f(x) = a_0 + a_1 x + a_2 x (x-1) + a_3 x (x-1) (x-2) 
put x = 0 then f(x) = 1 
∴ f(x) = a_0 + a_1 
⇒ a_1 = 0 
\(\cdots (2)\) 
put x = 2 
then -5 = a_0 + 2a_1 + 2a_2 
∴ a_0 + 2a_2 = -5 
2a_2 = -6 
a_2 = -3 
\(\cdots (3)\) 
put the values of a_0, a_1, a_2 in (1) 
1 - 0 - 6 - 6a_3 = 1 
6a_3 = -6 
a_3 = -1 
∴ f(x) = 1 + 0x + (-3)x(x-1) + x(x-1)(x-2) 
= 1 - 3x^2 + 3x - x(x^2 - 3x + 2) 
= 1 - 3x^2 + 3x - x^3 + 3x^2 - 2x 
= 1x - x^3 
∴ f(x) = 1 + x - x^3 

45. (2) 
x_0 = 1 ; \; y_0 = 1 
x_1 = 1.5 ; \; y_1 = 2.875 
x_2 = 2 ; \; y_2 = 7 
x_3 = 2.4 ; \; y_3 = 14.125 
x_4 = 3 ; \; y_4 = 25 
Simpson’s \(\frac{1}{3}\) formula 

46. (1) 
If a function f(x) is continuous between a and b and f(a) and f(b) are of opposite sign then there exists at least one root between a and b. 
f(x) = x^3 - 4x - 9 
f(2) = 8 - 8 - 9 = -9 = -ve 
f(3) = 27 - 12 - 9 = 6 = +ve 
Hence, the root lies between 2 and 3 
By first approximation 
x_0 = \frac{3+2}{2} = 2.5 
Again f(2.5) = 15.625 - 10 - 9 = -ve 
Hence, the root lies between 2.5 and 3 
Second approximation 
\[\frac{2.5 + 3}{2} = 2.75\] 

47. (3) 
Since only 3 values of x are given assume f(x) to be a polynomial of degree 2 
Let f(x) = a_0 + a_1 x + a_2 x (x-1) 
when x=0 f(x) = 1 
⇒ 1 = a_0 + a_1 + 2a_2 
⇒ a_0 = a_1 + 2a_2 
\(\cdots (1)\) 
when x=1; \; f(x) = 0 
⇒ 0 = a_0 + a_1 + a_2 
⇒ a_0 = -a_1 - a_2 
\(\cdots (2)\) 
when x=3, f(x)=10 
⇒ 10 = a_0 + 3a_1 + 6a_2 
⇒ a_0 = 10 - 3a_1 - 6a_2 
\(\cdots (3)\) 
Now f(x) = 1 - x + 2x(x-1) 
= 1 - x + 2x^2 - 2x 
= 2x^2 - 3x + 1 
∴ Required polynomial 

\[\frac{h}{3} [y_0 + y_n + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots)]\] 
\[= \frac{0.5}{3} [1 + 25 + 4(1.875 + 1.125) + 2(7)]\] 
\[= 18.00\]
\[ y_{n+1,p} = y_{n-3} + \frac{4h}{3} [2y_{n-2} - y_{n-1} + 2y_n] \]

Put \( n = 3 \)

\[ y_{4,p} = y_0 + \frac{4h}{3} [2y_1 - y_2 + 2y_3] \]

\[ = 1 + \frac{4}{3} (0.1) (2 \times 0.9 - 0.8 + 2 \times 0.7) \]

\[ = 1 + \frac{0.4}{3} (1.8 - 0.8 + 1.4) \]

\[ = 1 + \frac{0.4}{3} (2.4) \]

\[ = 1 + 0.32 \]

\[ = 1.32 \approx \frac{33}{25} \]

51. (4) \( h = \frac{1}{2} \)

<table>
<thead>
<tr>
<th>X</th>
<th>( y = \frac{1}{1+x^2} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1</td>
</tr>
<tr>
<td>( \frac{1}{2} )</td>
<td>( \frac{4}{5} )</td>
</tr>
<tr>
<td>1</td>
<td>( \frac{1}{2} )</td>
</tr>
</tbody>
</table>

\[ \int_0^1 \frac{1}{1+x^2} \, dx = \frac{h}{3} [y_0 + y_2 + 4y_1] \]

\[ = \frac{1}{6} \left[ 1 + \frac{1}{2} + \frac{16}{5} \right] \]

\[ = \frac{1}{6} \left[ \frac{10 + 5 + 32}{10} \right] \]

\[ = \frac{47}{60} \]

52. (3)

Euler’s modified formula is the particular case of Runge - Kutta formula.