Previous Year Questions & Detailed Solutions

1. The rate of convergence in the Gauss-Seidal method is________as fast as in Gauss Jacobi'smethod
   1) thrice 2) half-times
   3) twice 4) three by two times
2. In application of Simpsons $\frac{1}{3}$rd rule, theinterval h for closer approximation should be
   1) small 2) odd and large
   3) large 4) even and small
3. Which of the following is a step by step method?
   1) Taylor's method
   2) Picards method
   3) Adams Bashforth method
   4) Eulers method
4. The interpolation polynomial from the data

x	0	1	3
f(x)	5	4	8

   1) $x^2 - 2x + 5$ 2) $x^2 + 2x + 5$
   3) $2x^2 - 2x + 5$ 4) $2x^2 - 5x + 5$
5. The value of the integration $\int_a^{a+2h} f(x)dx$is approximately equal to
   1) $\frac{h}{2}[f(a)+3f(a+h)+f(a+2h)]$
   2) $\frac{h}{3}[f(a)+4f(a+h)+f(a+2h)]$
   3) $\frac{h}{3}[f(a)+2f(a+h)+f(a+2h)]$
   4) $\frac{h}{2}[f(a)+4f(a+h)+f(a+2h)]$
6. The finite difference approximation for $\frac{d^2y}{dx^2}$ at $x = x_0$ is
   1) $\frac{1}{h^2}(y(x_0-h)-2y(x_0)+y(x_0+h)]$
   2) $\frac{1}{h^2}[y(x_0-h)+2y(x_0)+y(x_0+h)]$
   3) $\frac{1}{h^2}[y(x_0-h)-y(x_0)+y(x_0+h)]$
   4) $\frac{1}{h^2}[y(x_0-h)+y(x_0)+y(x_0+h)]$
7. For the following data

X	0	2	4	6
Y	-1	3	7	11

   the straight line y=mx+c by the method of least square is
   1) y=-2x-l 2) y=x-l
   3) y=l-2x 4) y=2x-l
8. The velocity v (km/min) of a train which starts from rest, is given at fixed intervals of time t(min) as follows:

t	2	4	6	8	10	12	14	16	18	20
v	10	18	25	29	32	20	11	5	2	0

   The approximate distance covered by Simpson's $\frac{1}{3}$ rule is
   1) 306.3 2) 309.3
   3) 310.3 4) 307.3
9. Find the cubic polynomial by Newton's forward difference which takes the following:

X	0	1	2	3
f(x)	1	2	1	10

   then f(4) is
   1) 40 2) 41
   3) 39 4) 42
10. The first derivative $\frac{dy}{dx}$ at x=0 for the given data

X	0	1	2	3
f(x)	2	1	2	5

1) 2 2) -2
3) -1 4) 1

11. Error in Simpson's $\frac{1}{3}$ rule is of the order
1) $-h^2$ 2) h^3
3) h^4 4) $\frac{2h^3}{3}$

12. If α, β and γ are the roots of $x^3 + px^2 + qx+1 = 0$, then the equation whose roots are $\frac{-1}{\alpha}, \frac{-1}{\beta}, \frac{-1}{\gamma}$ is
1) $x^3 + qx^2 - px - l=0$
2) $x^3+qx^3-px+1=0$
3) $x^3-qx^2+px-l =0$
4) $x^3- qx^2+px+l=0$

13. In Jacobi's iteration method to get the (r+ 1)thiterates, we use the
1) values of the rthiterates
2) values of the (r -1)thiterates
3) values of the (r+ 1)th iterates
4) latest available values

14. If h is the length of the intervals, then the error in the trapezoidal rule is of order
1) h 2) h^2
3) h^3 4) h^4

15. If E is the translation operator, then the central difference operator δ is
1) $E^{1/2} + E^{-1/2}$ 2) $E^{1/2} - E^{-1/2}$
3) $\frac{1}{2}(E^{1/2} + E^{-1/2})$ 4) $-\frac{1}{2}(E^{1/2} + E^{-1/2})$

16. If $\Delta f(x) = f(x + h)-f(x)$, $E f(x) =f(x + h)$ and $\delta f(x)=\left(x+\frac{h}{2}\right) - f\left(x-\frac{h}{2}\right)$ then $\Delta\nabla$ is
1) δ^2 2) δ
3) $\delta^{1/2}$ 4) $\delta^{1/2} + \delta^{-1/2}$

17. Expression for $\left(\frac{d^2y}{dx^2}\right)_{x=x_n}$ using Newton's backward interpolation
1) $\frac{1}{h^2}\left[\nabla^2 f_n + \nabla^3 f_n + \frac{11}{12}\nabla^4 f_n + \cdots\right]$
2) $\frac{1}{h^2}[\nabla^2 - \Delta^2 - \Delta^4 - \cdots]f_n$
3) $\frac{1}{h^2}[\Delta^2 + \Delta^3 + \Delta^4 + \cdots]f_n$
4) $\frac{1}{h^2}\left[\Delta^2 - \Delta^3 - \frac{11}{12}\Delta^4 - \cdots\right]f_n$

18. By Taylor's series method solution of $y'=x^2 + y^2$; $y(0) = 1$ is
1) $y=l+x - x^2$ 2) $y = 1-x+x^2$
3) $y=x+ x^2+x^3$ 4) $y=l+x+x^2+\frac{4}{3}x^3$

19. Error in the Trapezoidal rule is of the order
1) -h 2) h^3
3) h^4 4) h^2

20. A river is 80 ft wide, the depth d in feet at a distance x ft. from one bank is given by the following table:

x	0	10	20	30	40	50	60	70	80
y	0	4	7	9	12	15	14	8	3

Approximately the area of cross section by Simpson's 1/3 rule is
1) 712 sq.ft. 2) 680 sq.ft.
3) 710 sq.ft. 4) 610 sq.ft.

21. The first derivative $\frac{dy}{dx}$ at x= 4 for the data.

X	0	1	2	3	4
f(x)	1	-2	-3	-2	1

1) -4 2) 4
3) 16 4) –16

22. If x_0 and x_1 are two points, $f(x_0)$ and $f(x_1)$ are of opposite signs, then the root x_2 by regular falsi method is equal to
1) $\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$ 2) $\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$
3) $\frac{x_1 f(x_0) - x_0 f(x_1)}{f(x_1) - f(x_0)}$ 4) $\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_0) - f(x_1)}$

23. The Newton-Raphson iteration scheme to find an approximate real root of the equationf(x) = 0 is
1) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$ 2) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

3) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$ 4) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

24. Power method is used to find
1) all eigen values only
2) all eigen vectors only
3) all eigen values and eigen vectors
4) numerically largest eigen value and the corresponding eigen vector

25. Lagrangean interpolation polynomial which passes through the points (0, 1), (1, 2), (3, 10) is
1) $x^2 + 1$ 2) x^2-1
3) x^3+1 4) x^3-1

26. $\frac{d^2y}{dx^2}$ is approximately equal to
1) $\frac{y_{i+1}-y_i-1}{2h}$ 2) $\frac{y_{i+1}+y_i-1}{2h}$
3) $\frac{y_{i-1}-2y_i+y_{i+1}}{h^2}$ 4) $\frac{y_{i-1}+2y_i+y_{i+1}}{h^2}$

27. The scheme
$\int_a^{a+2h} f(x)dx = \frac{h}{3}[f(a) + 4f(a + h)+f(a + 2h)$ is called
1) Trapezoidal rule
2) Simpson's 1/3 rule
3) Simpson's 3/8 rule
4) Weddle's rule

28. The Lagrange's interpolation polynomial curve passing through the points (0, 1) (1, 0) and (3, 10) is
1) x^2-2x+1 2) $-4x^2+ 3x + 1$
3) $2x^2 – 3x +1$ 4) $x^2 – x +1$

29. Which of the following formulas is a particular case of Runge-Kutta formula of second order?
1) Taylar series formula
2) Picard's formula
3) Euler's modified formula
4) Milne's predictor formula

30. What is the highest degree of a polynomial whose definite integration using Simpson's $\frac{1}{3}$ rule gives the exact value?
1) 1 2) 2
3) 3 4) 4

31. The global error arising in using Runge - Kutta method of fourth order to solve an ordinary differential equation is of
1) $0(h^2)$ 2) $0(h^3)$
3) $0(h^4)$ 4) $0(h^5)$

32. Simpson's one-thid rule integration is exact for all polynomial of degree not exceeding
1) 2 2) 3
3) 4 4) 5

33. With usual notations, the value of $(1+\Delta)(1-\nabla)$ is
1) 0 2) 2
3) 1 4) 4

34. The modified Newton-Raphson formula of finding the root of the equation f(x)=0 is
1) $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$
2) $x_{n+1} = x_n - \left\{\frac{f(x_n)}{\frac{f'(x_n)-f^n(x_n)f(x_n)}{2f'(x_n)}}\right\}$
3) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
4) $x_{n+1} = x_n + \left[\frac{f'(x_n)}{f(x_n)-\frac{f''(x_n)f(x_n)}{2f'(x_n)}}\right]$

35. The value of $\int_0^1 \frac{dx}{1+x^2}$ Trapezoidal rule with h = 0.2 is
1) 0.687322 2) 0.877323
3) 0.727333 4) 0.783732

36. Simpson's one-third rule of integration is exact for all polynomial of degree not exceeding
1) 2 2) 3
3) 4 4) 5

37. Which of the following formula is a particular case of Runge - Kutta formula of the second order?
1) Taylor series formula
2) Picard's formula
3) Euler's modified formula
4) Milne's predictor formula

38. The Newton Raphson method of finding the root of the equation f(x)=0 is by using the formula
1) $x_{n+1} = x_n - \frac{f'(x_n)}{f(x_n)}$
2) $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$
3) $x_{n+1} = x_n + \frac{f'(x_n)}{f(x_n)}$
4) $x_{n+1} = x_n + \frac{f(x_n)}{f'(x_n)}$

39. The value of $\int_0^2 \frac{dx}{1+x}$ by Trapezoidal Ruletaking n=2 is
1) 0.645 2) 0.641
3) 0.583 4) 0.621

40. Trapezoidal rule for the evaluation of the integral $\int_a^b f(x)dx$ requires the Interval to be divided into
1) only an even number of intervals
2) only an odd number of intervals
3) any number of intervals
4) a minimum of ten intervals

41. If $\int_0^{0.6} f(x)dx$ - k[f(0) + 4f(0.3)+f(0.6)] then thevalue of k when Simpson's rule is applied is
1) 0.1 2) 0.2
3) 0.3 4) 0.4

42. The Newton - Raphson formula to find an approximate value of $\sqrt{a}$ is
1) $x_{n+1} = \frac{1}{2}\left(x_n + \frac{a}{x_n}\right)$
2) $x_{n+1} = \frac{1}{2}\left(x_n - \frac{a}{x_n}\right)$
3) $x_{n+1} = \left(x_n + \frac{a}{x_n}\right)$
4) $x_{n+1} = x_n - \frac{a}{x_n}$

43. The iteration formula given by Newton-Raphson method to find to root of the equation x sin x+cos x=0 is
1) $x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n}$
2) $x_{n+1} = x_n - \frac{x_n \cos x_n}{x_n \sin x_n + \cos x_n}$
3) $x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n + 2\sin x_n}$
4) $x_{n+1} = x_n - x_n \cos x_n + 2\sin x_n$

44. The interpolating polynomial for the data

x	-1	0	1	2
f	1	1	1	-5

1) $1 + x - x^3$ 2) $1 + x + x^2$
3) $1 + x - x^2$ 4) $x + x^2 + x^3$

45. Given that f(l) = l, f(1.5) = 2.875, f(2) = 7, f(2.5) = 14.125 and f(3) = 25, the value of $\int_1^3 f(x)dx$ obtained by Simpson's $\frac{1}{3}$ rule is
1) 17.50 3) 18.25
2) 18.00 4) 19.125

46. A root of x^3- 4x -9 = 0 using bisection method in two stages is
1) 2.75 2) 0.5361
3) 0.7969 4) 0.7123

47. The lagrange's interpolation polynomial curve passing through the points (0, 1) (1, 0) and (3, 10) is
1) $x^2 - 2x + 1$ 2) $-4x^2 + 3x + 1$
3) $2x^2 - 3x + 1$ 4) $x^2 - x + 1$

48. What is the highest degree polynomial whose definite integration using Simpson's $\frac{1}{3}$ –rule gives the exact value?
1) 1 2) 2
3) 3 4) 4

49. Milne's predictor value of y(0, 4) given that $y' = y - \frac{2x}{y}$, y(0) = 1
y'(0.1)=0.9.
y'(0.2) = 0.8, y'(0.3) =0.7 is
1) $\frac{23}{15}$ 2) $\frac{33}{25}$
3) $\frac{36}{25}$ 4) $\frac{59}{50}$

50. The recurrence formula for finding the reciprocal of the n^{th} root of M, by Newton's method is
1) $x_{k+1} = \frac{[(n-1)x_k^n + M]}{n x_k^{n-1}}$
2) $x_{k+1} = \frac{x_k[n+1-Mx_k^n]}{n}$
3) $x_{k+1} = \frac{x_k[n-1+Mx_k^n]}{n}$
4) $x_{k+1} = \frac{1}{n}\left(\frac{1}{x_k} + Mx_k\right)$

51. The value of $\int_0^1 (1+x^2)^{-1}$ by simpson's one-third rule with two strips is
1) $\frac{31}{60}$ 2) $\frac{47}{30}$
3) $\frac{31}{40}$ 4) $\frac{47}{60}$

52. Which of the following formulas is a particular case of Runge - Kutta formula of second order?
1) Taylor series formula
2) Picard's formula
3) Euler's modified formula
4) Milne's predictor formula

DETAILED SOLUTIONS

1. (3)
The rate of convergence of Gauss - Seidel method is roughly twice that of Gauss - Jacobi.

2. (4)
In application of Simpsons $\frac{1}{3}$ rd rule, the intervalh for closer approximation should be even and small.

3. (4)
Euler's method is a step by step method.

4. (1)
Consider $f(x) = x^2-2x+5$
then $f(0) = 0^2-2(0)+5 = 5$
$f(l) = l^2-2(l)+5 = 4$
$f(3) = 3^2-2(3)+5 = 8$
$\therefore f(x) = x^2 -2x+5$ is the requiredpolynomial.

5. (2)
$\int_a^{a+2h} f(x)dx = \frac{h}{3}[f(a) + 4f(a+h) + f(a+2h)]$

6. (1)
$\frac{d^2y}{dx^2}(x = x_0) = \frac{f_1 - 2f_0 + f_{-1}}{h_2}$
$\frac{1}{h_2}[y(x_0 - h) - 2y(x_0) + y(x_0 + h)]$

7. (4)

x	0	2	4	6
y	-l	3	7	11

Consider y = 2x-l
$x = 0 \Longrightarrow y = 2(0)-1 = -1$
$x = 2 \Longrightarrow y=2(2)-l = 3$
$x = 4 \Longrightarrow y=2(4)-l = 7$
$x = 6 \Longrightarrow y= 2(6)-1 = 11$
$\therefore$y=2x-l is the required straight line.

8. (*)
$\int ydx = \frac{h}{3}[(y_0+y_n) + 4(y + y_3 + ... + y_n) + 2(y_2 + y_4 + \ldots y_{n-2})]$
$= \frac{2}{3}[(10+0)+4(18+29+20+5)+2(25+32 + 11+2)]$
= 292

9. (2)
$f(x) = y_0 + p\Delta f(x_0) + \frac{p(p-1)}{2!}\Delta^2 f(x_0)$
$+ \frac{p(p-1)(p-2)}{3!}\Delta^3 f(x_0)$
where $p = \frac{x - x_0}{h}$
$\Delta f(x_0) = 1$
$\Delta^2 f(x_0) = -2$

$\Delta^3 f(x_0) = 12$

p = x

$f(x) = 1 + x + \frac{x(x-1)}{2!}(-2)$

$+ \frac{x(x-1)(x-2)(12)}{3!}$

=1 + x – x (x - 1)+2x(x-1) (x-2)

∴f(4) = 1+4 -4 ×3 + 2 × 4 × 3 × 2

= 5-12+48

= 41

10. (2)

x	f(x)	Δy	$\Delta^2 y$	$\Delta^3 y$
0	2			
		$-1(\Delta y_0)$		
1	1		$2\ (\Delta^2 y_0)$	
		1		$0\ (\Delta^3 y_0)$
2	2		2	
		3		
3	5			

First derivative by Newton's forward difference formula

$y = \frac{1}{h}\left[\Delta y_0 - \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 - \cdots\right]$

$= \frac{1}{1}\left[-1 - \frac{1}{2}(2) + \frac{1}{3}(0)\right]$

= -1-1=-2

11. (3)

Error in Simpson's $\frac{1}{3}$ rule is of order h^4

12. (3)

Results:

Let $a_1, a_2, ..., a_n$ be the roots of the equation

$x^n + p_1 x^{n-1} + p_2 x^{n-2} + ... p_n = 0$

than the roots of

$x^n - p_1 x^{n-1} + p_2 x^{n-2} - ... (-1)^n P_n = 0$

are $-a_1, -a_2, ... -a_n$

Given

α, β, γ are root of $x^3 + px^2 + qx + 1 = 0$

$x^3 + px^2 + qx + 1$

$\Rightarrow \frac{1}{\alpha}, \frac{1}{\beta}, \frac{1}{\gamma}$ are roots of

$$\left(\frac{1}{x}\right)^3 + p\left(\frac{1}{x}\right)^2 + q\left(\frac{1}{x}\right) + 1 = 0$$

$\Rightarrow x^3 + qx^2 + px + 1 = 0$

$\therefore \frac{-1}{\alpha}, \frac{-1}{\beta}, \frac{-1}{\gamma}$ are roots of $x^3 - qx^2 + px - l = 0$

13. (1)

For the system of equations

$a_1 x + b_1 y + c_1 z = d_1$

$a_2 x + b_2 y + c_2 z = d_2$

$a_3 x + b_3 y + c_3 z = d_3$

Then the $(r+1)^{th}$ iteration in Jacobi's method is

$$x_{r+1} = \frac{1}{a_1}(d_1 - b_1 y_r - c_1 z_r)$$
$$y_{r+1} = \frac{1}{b_1}(d_2 - a_2 x_r - c_2 z_r)$$
$$z_{r+1} = \frac{1}{c_3}(d_3 - a_3 x_r - b_3 y_r)$$

To find $(r+1)^{th}$ iteration we use r iterations.

14. (2)

Error in the trapezoidal rule is of the order h^2.

15. (2)

$\delta f(x) = (E^{1/2} - E^{-1/2}) f(x)$

16. (1)

$\delta = E^{½} - E^{-1/2}$ e

$\Delta = E - 1; \Delta = 1 - E^{-1}$

where E = f(x+h)

$E^{-1} = f(x-h)$

$E^{1/2} = f(x + h/2); E^{-1/2} = f(x - h/2)$

$\Delta\nabla = (E - 1)(1 - E^{-1})$

$= E - 1 - 1 + E^{-1}$

$= E + E^{-1} - 2$ ... (1)

$d\delta^2 = (E^{1/2} - E^{-1/2})2$

$= E - 2E^{1/2} E^{-1/2} + E^{-1}$

$= E^{-2} + E^{-1}$ ...(2)

(1) = (2)

$\therefore \Delta\nabla = \delta^2$

17. (4)

Standard formula

$$\left(\frac{d^2y}{dx^2}\right)_{x=x_n}$$

$$\frac{1}{h^2}\left[\Delta^2 - \Delta^3 + \frac{11}{12}\Delta^4 - \cdots\right]f_n$$

18. (4)

Taylor's series

$y = y_0 + \frac{h}{1!}y_0' + \frac{h^2}{2!}y_0'' + \frac{h^3}{3!}y_0''' + ...$

Given y(0) =1

$\Longrightarrow y_0 = 1$; $x_0 = 0$

$y' = x^2 + y^2$ (Given)

$y_0' = 0+1 = 1$

$y'' = 2x + 2yy'$

$y_0'' = 2(0)+2.1.1$

= 2

$y''' = 2+ 2(yy'' + (y')2)$

$= 2+ 2(1.2 + 1^2)$

= 2+2(2 + l)

= 8

putting h - x then

$y = 1 + \frac{x}{1!} + \frac{x^2}{2!}2 + \frac{x^3}{3!}8$

$= l + x + x^2 + \frac{4}{3}x^3$

19. (4)Error in theTrapezoidal rule is of the order h^2.

20. (3)

Simpson's $\frac{1}{2}$ rule

$= \int_{x_0}^{x_0+nh} f(x)dx$

$=\frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots) + 2(y_2 + y_4 +$ ...+yn

$= \frac{h}{3}$ [y_0+y_n+4(sum of odd ordinates) +2sum of even ordinates)]

∴Cross Section Area

$= \frac{10}{3}$[(0+3+4(4+9+15+8)+2(7+12+14)]

$= \frac{10}{3}$ [3+144+66]

= 710 sq.ft.

21. (2)

x	y=f(x)	First difference	Second difference	Third difference	Fourth difference
0	1				
		-3			
1	-2		2		
		-1		0	
2	-3		2		
		1			0
3	-2			0	
			2		
		3			
4	1				

$$\nabla y_0 = 3; \nabla^2 y_0 = 2$$

$$\nabla^3 y_0 = \nabla^4 y_0 = \cdots = 0$$

Newtons formula for Backward difference

$f'(x) = \frac{1}{h}[\nabla y_0 + \frac{1}{2}\nabla^2 y_0 + \frac{1}{3}\nabla^3 y_0 ...]$

$= \frac{1}{1}\left[3 + \frac{2}{2}\right]$

= 4

22. (2)

By regular - falsi method the root is given by

$$\frac{x_0 f(x_1) - x_1 f(x_0)}{f(x_1) - f(x_0)}$$

23. (2)

The Newton - Rapson Iteration scheme to find an approximate real root of the equation f(x)=0 is

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

24. (4)

Power method is used to determine numerically largest eigen value and the corresponding eigen vector of a matrix A.

25. (1)

Let $x_0 = 0$; $y_0 = 1$

$x_1 = 1$; $y_1 = 2$

$x_2 = 3$; $y_2 = 10$

$$y = \frac{(x-x_1)(x-x_2)}{(x_0-x_1)(x_0-x_2)}y_0 + \frac{(x-x_0)(x-x_2)}{(x_1-x_0)(x_1-x_2)}y_1$$

$$+ \frac{(x-x_0)(x-x_1)}{(x_2-x_0)(x_2-x_1)}y_2$$

$$= \frac{(x-1)(x-3)}{-1\times-3} \times 1 + \frac{(x-0)(x-3)}{1\times-2} \times 2 +$$

$$+ \frac{(x-0)(x-1)}{3\times2} \times 10$$

$$\frac{x^2-4x+3}{3} - (x^2-3x) + \frac{5x^2-5x}{3}$$

$$= \frac{x^2-4x+3-3x^2+9x+5x^2-5x}{3}$$

$$= \frac{3x^2+3}{3} = x^2+1$$

26. (3)

$$\frac{d^2y}{dx^2} = \frac{1}{h^2}[\nabla^2 y_i + (r+1)\nabla^3 y_i + \cdots]$$

$= \frac{1}{h^2}[\nabla^2 y_1]$ approximately

$$= \frac{1}{h^2}[\nabla y_{i+1} - \nabla y_i]$$

$$= \frac{1}{h^2}[y_{i+2} - y_{i+1} - (y_{i+1} - y_i)]$$

$$= \frac{1}{h^2}[y_{i+2} - 2y_{i+1} + y_i]$$

which is also equal to $\frac{d^2y}{dx^2} = \frac{1}{h^2}(y_{i+1} - 2y_i + yi-1$

27. (2)

$\int_a^{a+2h} f(x)dx = \frac{h}{3}[f(a) + 4f(a+h) + f(a+2h)]$ is called

Simpson's $\frac{1}{3}$ rule.

28. (3)

Since only 3 values of x are given. Assume

$y = f(x) = a_0 + a_1x + a_2x(x-l)$

put x=0, y=1

$\Longrightarrow 1 = a_0 + a_1(0) + a_2(0)$

$\Longrightarrow a_0 = 1$

put x - 1, y=0

$\Longrightarrow 0 = a_0 + a_1 + a_2(0)$

$\Longrightarrow 0 = a_0 + a_1$

$\Longrightarrow 0 = l + a_1 \Longrightarrow \therefore a_{1=-1}$

put x=3, y= 10

$\therefore 10 = a_0 + 3a_1 + 6a_2$

$\therefore f(x) = l - x + 2x(x-l)$

$= l - x + 2x^2 - 2x$

$= 2x^2 - 3x + 1$

29. (3)

Paricular case of R.K. formula of second order is Euler's modified formula.

30. (2)

Simpson's one-third rule is

$$\int_{x_0}^{x_0+nh} ydx$$

$$= \frac{h}{3}[(y_0 + y_n) + 4(y_1 + y_2 + \cdots y_{n-1}) + 2(y_2 + y_4 + \cdots y_{n-2})]$$

In this case we reglect all differences above the second.

So y will be a polynomial of second degree only, i.e., $y = ax^2+bx+c$

31. (3)

The error in R.K. method of fourth order to solve an ordinary differential equation is of $0(h^4)$.

32. (1)

Required degree = 2

33. (3)

Formula

$(1 + \Delta)(1-\nabla) = 1$

34. (3)

Newton – Rapson formula

$$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

35. (4)

X	$y = \frac{1}{1+x^2}$	
0	1	y_0
0.2	0.9615	y_1
0.4	0.8621	y_2
0.6	0.7353	y_3
0.8	0.6098	y_4
1	0.5	y_5

By Trapezoidal Rule,

$$\int_0^1 \frac{dx}{1+x^2}$$

$= \frac{h}{2}[(y_0 + y_5) + 2(y_1 + y_2 + y_3 + y_4)]$

$= \frac{0.2}{2}[1.5+2(3.1687)]$

0.1×7.8374

$= 0.78374$

36. (1)

Simpsons $\frac{1}{3}$ rule of Integration is exact for allpolynomial of degree not exceeding 2.

37. (3)

Particular case of Runge Kutta formula of second order is Euler's modified formula.

38. (2)

Newton Raphson formula of finding the root of f(x)=0 is

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

39. (1)

$h = \frac{2-0}{2} = 1$

$x_0 = 0$

$y_0 = \frac{1}{1+x_0} - \frac{1}{1+0} - 1$

$x_1 - 1$

$y_1 - \frac{1}{1+1} - \frac{1}{2} = 0.5$

$x_2 - 2$

$y_2 - \frac{1}{1+2} - \frac{1}{3} - 0.3333$

∴ By Trapezoidal rule

$\int_0^2 \frac{dx}{1+x} = \frac{h}{2}[y_0 + 2y_1 + y_2]$

$= \frac{1}{2}[1 + 2(0.5)+0.3333]$

$= 1.1666$

40. (3)

Any number of intervals.

41. (1)

h = interval length

= 0.3

By Simpsons $\frac{1}{3}$ rule

$k = \frac{h}{3} = \frac{0.3}{3}$

= 0.1

42. (1)

Let $x = \sqrt{a}$

$\Longrightarrow x^2 - a = 0$

Let $f(x) = x^2 - a$

then$f'(x) = 2x$

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$= x_n - \left(\frac{x_n^2 - a}{2x_n}\right)$

$= \frac{1}{2}\left[\frac{2x_n^2 - x_n^2 + a}{x_n}\right]$

$= \frac{1}{2}\left[x_n + \frac{a}{x_n}\right]$

43. (1)

Newton Raphson Method

$x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

$f(x) = x \sin x + \cos x$

$f'(x) = x \cos x + \sin x - \sin x$

$= x \cos x$

∴By Newton Raphson method

$x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n}$

44. (1)

Since only 4 values of x are given assume f(x) to be a polynomial of degree 3

Let $f(x) = a_0 + a_1x + a_2x(x-1)$

$+ a_3x(x-1)(x-2)$

put $x = -1$, $f(x) = 1$

$\Rightarrow 1 = a_0 - a_1 + 2a_2 - 6a_3$ .... (1)

$x = 0$ then $f(x) = 1$

$\therefore 1 = a_0 + a_1 0 + a_2 0 + a_3 0$

$\Rightarrow a_0 = 1$ .... (2)

put $x = 0$ then $f(x) = l$

$\therefore 1 = a_0 + a_1$

$\Rightarrow a_i = 0$ .... (3)

put $x = 2$

then $-5 = a_0 + 2a_1 + 2_2a$

$\therefore l + 0 + 2a_2 = -5$

$2a_2 = -6$

$a_2 = -3$ .... (4)

put the values of a_0, a_1, a_2 in (1)

$l - 0 - 6 - 6a_3 = 1$

$6a_3 = -6$

$a_3 = -1$

$\therefore f(x) = 1 + 0x + (-3)x(x-1) + x(x-1)(x-2)$

$= 1 - 3x^2 + 3x - x(x^2 - 3x + 2)$

$= 1 - 3x^2 + 3x - x^3 + 3x^2 - 2x$

$= 1 + x - x^3$

$\therefore f(x) = 1 + x - x^3$

45. (2)

$x_0 = 1$; $y_0 = 1$

$x_1 = 1.5$; $y_1 = 2.875$

$x_2 = 2$; $y_2 = 7$

$x_3 = 2.4$; $y_3 = 14.125$

$x_4 = 3$; $y_4 = 25$

Simpson's $\frac{1}{3}$ formula

$= \frac{h}{3}[y_0 + y_n + 4(y_1 + y_3 + \ldots) + 2(y_2 + y_4 + \ldots)]$

$= \frac{0.5}{3}[1 + 25 + 4(2.875 + 14.125) + 2(7)]$

$= 18.00$

46. (1)

If a function f(x) is continuous between a and b and f(a) and f(b) are of opposite sign then there exists atleast one root between a and b.

$f(x) = x^3 - 4x - 9$

$f(2) = 8 - 8 - 9$

$= -9 = -ve$

$f(3) = 27 - 12 - 9$

$= 6 = +ve$

Hence, the root lies between 2 and 3

By first approximation

$x_0 = \frac{3+2}{2} = 2.5$

Again $f(2.5) = 15.625 - 10 - 9 = -ve$

Hence, the root lies between 2.5 and 3

Second approximation

$$\frac{2.5 + 3}{2} = 2.75$$

47. (3)

Since only 3 values of x are given assume f(x) be a polynomial of degree 2

Let $f(x) = a_0 + a_1x + a_2x(x-l)$

when $x=0$ $f(x) = 1$

$\Rightarrow 1 = a_0 + a_1 0 + a_2 0$

$\Rightarrow a_0 = 1$

when $x=l$; $f(x) = 0$

$\Rightarrow 0 = a_0 + a_1$

$\Rightarrow a_1 = -1$

when $x= 3$, $f(x)=10$

$\Rightarrow 10 = a_0 + 3a_1 + 6a_2$

$\Rightarrow a_2 = 2$

Now$f(x) = l - x + 2x(x-l)$

$= l - x + 2x^2 - 2x$

$= 2x^2 - 3x + 1$

∴ Required polynomial

$= 2x^2 - 3x + 1$

48. (2)

Required degree = 2

49. (2)

$y_{n+1,p} = y_{n-3} + \frac{4h}{3}[2y_{n-2} - y_{n-1} + 2y_n]$

put n = 3

$y_{4,p} = y_0 + \frac{4h}{3}[2y_1 - y_2 + 2y_3]$

$= 1 + \frac{4}{3}(0.1)(2 \times 0.9 - 0.8 + 2 \times 0.7)$

$= 1 + \frac{0.4}{3}(1.8 - 0.8 + 1.4)$

$= 1 + \frac{0.4}{3}(2.4)$

$= 1 + 0.32.$

$= 1.32 \approx \frac{33}{25}$

50. (1)

$x = \sqrt[n]{M}$

$x^n = M$

Let $f(x) = x^n - M$

$f'(x) = nx^{n-1}$

By Newtons formula

$$x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$$

$= x_k - \frac{(x_k^n - M)}{nx_k^{n-1}}$

$= \frac{nx_k x_k^{n-1} - x_k^n + M}{nx_k^{n-1}}$

$= \frac{x_k^n(n-1) + M}{nx_k^{n-1}}$

51. (4) h=1/2

X	$y = \frac{1}{1+x^2}$
0	1
$\frac{1}{2}$	$\frac{4}{5}$
1	$\frac{1}{2}$

$\int_0^1 \frac{1}{1+x^2} dx = \frac{h}{3}[y_0 + y_2 + 4y_1]$

$= \frac{1}{6}\left[1 + \frac{1}{2} + \frac{16}{5}\right]$

$= \frac{1}{6}\left[\frac{10+5+32}{10}\right]$

$= \frac{1}{6} \times \frac{47}{10}$

$= \frac{47}{60}$

52. (3)

Euler's modified formula is the particular case of Runge - Kutta formula.