Numerical Methods

1. Newton's formula to find a root of f(x) = 0
   $X_{n+1} = X_n - \frac{f(X_n)}{f(X_n)}$
2. Condition for convergence in Newton – Raphson method
   $$|f(x)"f^n(x)| < |f'(x)|^2$$
3. Order of convergence in Newton Raphso method = 2.
4. In Newton - Raphson method the error at any stage is proportional to the square of the error in the previous stage.
5. Newton's method is useful where the graph of the function when it crosses the x-axis is nearly vertical.
6. Newton - Raphson method is convergent - Biquadratically.
7. **Lagrange's Interplation Formula:**
   Let y = f(x) be a function which takes the values
   $y_0, y_1, y_2, \ldots\ldots y_n$ corresponding to
   $x_0, x_1 \ldots\ldots x_n$ then,
   $$y = f(x) = \frac{(x-x_1)(x-x_2)\ldots(x-x_n)}{(x_0-x_1)(x_0-x_1)\ldots(x_0-x_n)}y_0$$
   $$\frac{(x-x_0)(x-x_2)\ldots(x-x_n)}{(x_1-x_0)(x_1-x_2)\ldots(x_1-x_n)}y_1$$
   $+ \ldots\ldots\ldots$
   $+ \ldots\ldots\ldots$
   $$+ \frac{(x-x_0)(x-x_1)\ldots(x-x_{n-1})}{(x_n-x_0)(x_n-x_1)\ldots(x_n-x_{n-1})}y_n$$

Forward difference formula to compute Numerical differentiation:

$$f'(x_0) = \frac{1}{h}\left[\Delta y_0 + \frac{1}{2}\Delta^2 y_0 + \frac{1}{3}\Delta^3 y_0 + \frac{1}{4}\Delta^4 y_0 + \cdots\right]$$

$$f''(x_0) = \frac{1}{h^2}\left[\Delta^2 y_0 - \Delta^3 y_0 + \frac{11}{12}\Delta^4 y_0 \ldots\ldots\right]$$

$$f'''(x_0) = \frac{1}{h^3}\left[\Delta^3 y_0 - \frac{3}{2}\Delta^4 y_0 + \cdots\right]$$

Backward difference formula to compute derivatives:

$$f'(x_0) = \frac{1}{h}\left[\nabla y_0 - \frac{1}{2}\nabla^2 y_0 + \frac{1}{3}\nabla^3 y_0 + \frac{1}{4}\nabla^4 y_0 + \cdots\right]$$

$$f''(x_0) = \frac{1}{h^2}\left[\nabla^2 y_0 + \nabla^3 y_0 + \frac{11}{12}\Delta^4 y_0 + \cdots\ldots\right]$$

$$f'''(x_0) = \frac{1}{h^3}\left[\nabla^3 y_0 + \frac{3}{2}\nabla^4 y_0 + \cdots\right]$$

Numerical Integration:

Trapezoidal Rule:

$$\int_{x_0}^{x_0+nh} y(x)dx = \frac{h}{2}[(y_0 + y_n) + 2(y_1 + y_2 + \cdots) + y_{n-1}]$$

Simpson's $\frac{1}{3}$ rule:

$$\int_{x_0}^{x_0+nh} y(x)dx = \frac{h}{3}[y_0 + 4(y_1 + y_3 + \cdots)2(y_2 + y_4 + \cdots) + y_n]$$

Simpson's $\frac{3}{8}$ rule:

$$\int_{x_0}^{x_0+nh} y(x)dx = \frac{3h}{8}[(y_0 + y_n) + 3(y_1 + y_2 + y_4 + y_5 + \cdots y_{n-1}) + 2(y_3 + y_6 + \cdots y_{n-3})]$$

Errors:

i) Error in the trapezoidal rule is of order h^2.

ii) Error in the simpson's $\frac{1}{3}$ rule is of order h^4.

Properties:

i) Simpson's $\frac{1}{3}$ rule is a closed formula.

ii) In Trapezoidal rule, if y is a linear function of x then the result is least accurate. If y is a polynomial of degree 2, then simpson's $\frac{1}{3}$ rule is more accurate.

iii) Simpson's $\frac{1}{3}$ rule approximates the area of two adjacent strips by the area under a quadratic parabola.