Moment of Inertia

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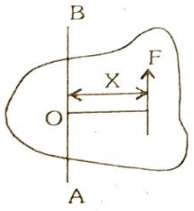

Fig (M)

We have measured the moment of a force about a point as the product of its magnitude and the perpendicular distance between the line of action of the force and the point about which the force causing rotation.

Referring the figure M, the moment of the force F about O, an AB axis,

$$M_o = F \times x$$

This moment is also called as the first moment of the force about O.

Let is be MO_1

If this moment is again multiplied by X, then we get the moment of moment of the force, or second moment of the force. Let it be MO_2. The second moment of the force, MO_2, is also called as the Moment of inertia.

Moment of inertia MO_2 = First moment x Distance

$= MO_1 XX$

$= (FXX) X X$

$= FX^2$

The moment of inertia is briefly written as M.I. and denoted by the symbol I.

Moment of Inertia of plane figures:

Moment of inertia of a plane figure is generally called as area moment of inertia. In SI system of units, units of area moment of inertia are mm^4, cm^4. The moment of inertia is denoted by I and causes with it the symbol of the axes about which it is calculated. Thus the moment of inertia about an axis AB is denoted by I_{AB}. The moment of inertia about centroidal axes is denoted by I_{xx} and I_{yy}.

The moment of inertia of simple and composite plane figure are determined separately. The M.I. of simple plane figures are determined by the method of integration and the M.I. of composite plane figures are determined by applying the theorems of moment of inertia.

Parallel axis theorem:

It states that "the moment of inertia of a lamina about any axis in the plane of lamina is equal to the sum of the moment of inertia about a parallel centroidal axis in the plane of lamina and the product of the area of the lamina and square of the distance between the two axes.

Perpendicular Axis theorem:

It states that "if I_{ox} and I_{oy} be the moment of inertia of a lamina about two mutually perpendicular axes and OX and OY in the plane of the lamina and I_{oz} be the moment of inertia of the lamina about an axis normal to the lamina and passing through the point of intersection of the axes OX and OY, then

$$I_{oz} = I_{ox} + I_{oy}$$

M.I. of common simple plane figures.

S.No	Name	I_{xx}	I_{yy}
1.	Rectangle	$\frac{bh^3}{12}$	$\frac{hb^3}{12}$
2.	Hollow Rectangle	$\frac{1}{12}(BH^3 - bh^3)$	$\frac{1}{12}(HB^3 - hb^3)$
3.	Square	$\frac{a^4}{12}$	$\frac{a^4}{12}$
4.	Hollow Square	$\frac{1}{12}(A^4 - a^4)$	$\frac{1}{12}(A^4 - a^4)$
5.	Triangle	$\frac{bh^3}{36}$	$\frac{hb^3}{48}$
6.	Circle	$\frac{\pi d^4}{64}$	$\frac{\pi d^4}{64}$
7.	Hollow Circle	$\frac{\pi}{64}(D^4 - d^4)$	$\frac{\pi}{64}(D^4 - d^4)$
8.	Semi Circle	$0.0068d^4$	$\frac{\pi d^4}{128}$
9.	Quadrant	$0.055r^4$	$0.55r^4$

Polar moment of inertia:

The area moment of inertia (ie. M.I. of plane figures) for an area relative to an axis perpendicular to the plane of the area is called the polar moment of inertia. It is denoted by I_P.

Mathematically, $Ip = I_{xx} + I_{yy}$

Where I_{xx} and I_{yy} are the M.I. of the plane figure about its centroidal axes.

In general, the polar moment of inertia is equal to the sum of area of moments of inertia about any two mutually perpendicular axes in its plane and intersecting on the polar axis.

Radius of Gyration:

Radius of gyration (also called as radius of rotation) about an axis is defined as the distance from that axis at which all the elemental parts of the lamina would have to be placed, such that th moment of inertia about the axis is same. We known area moment of inertia,

$I = \text{area} \times \text{distance}^2$

i.e $I = Ar2$ or $r = \sqrt{\frac{1}{A}}$

where 'r' is the radius of gyration and A is the area of the section.

Hence, radius of gyration about XX axis,

$r_{xx} = \sqrt{\frac{I_{xx}}{A}}$ and

about yy axis

$r_{yy} = \sqrt{\frac{I_{yy}}{A}}$

Radius of Gyration – Standard results.

1. Rectangular section

$r_{xx} = \frac{h}{\sqrt{12}}$

$r_{yy} = \frac{b}{\sqrt{12}}$

2. Circular section

$r_{xx} = r_{yy} = \frac{d}{4}$

3. Triangular section

$r_{xx} = \frac{h}{\sqrt{18}}$

$r_{yy} = \frac{6}{\sqrt{24}}$

4. Square section

$r_{xx} = r_{yy} = \frac{a}{\sqrt{12}}$

Unit of radius of gyration is mm.

The radius of gyration of a composite plane area about an axis is not equal to the sum of the radii of gyration of the individual components about the same axis.

Principal Moment of Inertia.

The perpendicular axes about which product of inertia is zero are called "Principal axes" and the moments of inertia with respect to these axes are called as principal moments of inertia.

Among two values of moments of inertia about the set of principal axes. One will be maximum and the other will be minimum. The maximum moment of inertia is known as Major principal moment of inertia and the minimum moment of inertia is known as Minor principal moment of inertia.

Mass moment of inertia:

The moment of inertia of solid figure is generally referred as "Mass moment of inertia". It is denoted by the symbol $(I)_{mass}$ or Simply I_m.

Kinematics of particles:

Kinematics is the study of motion of a moving body without considering the forces which cause the motion and kinetics is the study of motion of a moving body with also considering the external forces which cause the motion.

Types of plane motion:

1. Rectilinear motion
2. Curvilinear motion

The motion of a particle along a straight line is known as rectilinear motion (or straight line motion).

Example :

A car moving on a straight road.
A stone falling vertically downward.
A ball thrown vertically upward, etc.

The motion of a particle along a curved path is known as curvilinear motion.

Characteristics of kinematics;

The study of kinematics is concerned with the relations among the displacement, velocity, acceleration and time for a moving particle.