Previous Year Solved Questions

1. **The inverse Laplace transform of $\frac{1}{S^2+4S+13}$ is**

   1) $\frac{1}{2}e^{-t}\sin 2t$ 2) $\frac{1}{3}e^{-2t}\sin 3t$

   3) $\frac{1}{3}e^{-2}\cos 3t$ 4) $\frac{1}{2}e^{-t}\cos 2t$

2. **If satisfies $y''+3y'+2y=e^{-t}$ with $y(0)=0 = y'(0)$ then the Laplace transform of y is**

   1) $\frac{1}{(S+1)(S+2)}$ 2) $\frac{1}{(S+1)(S+2)^2}$

   3) $\frac{1}{(S+1)^2(S+2)}$ 4) $\frac{1}{(S-1)^2(S-2)}$

3. **If $L\{f(t)\} = \frac{S^2}{(S^2+4)^2}$, then f(t) =**

   1) $\frac{1}{4}\sin 2t + \frac{1}{2}\cos 2t$ 2) $\frac{1}{4}\sin 2t + t\cos 2t$

   3) $\frac{1}{2}\sin 2t + \frac{1}{4}\cos 2t$ 4) $\frac{1}{4}\cos 2t + t\sin t$

4. **Fourier sine transform of $e^{-|x|}$ is**

   1) $\frac{2S}{1-S^2}$ 2) $\frac{1}{S^2+1}$

   3) $\frac{S}{1+S^2}$ 4) $\frac{1}{S^2-1}$

5. **If $F_C(S)$ is the Fourier cosine transform of f(x) and $F_C(ax) = kF_C\left(\frac{S}{a}\right)$ then the value of k is**

   1) $\frac{1}{a}$ 2) a

   3) $\frac{1}{a^2}$ 4) $\frac{-1}{a}$

6. **The inverse Z-transform of $\frac{z}{(z+1)^2}$ is**

   1) $n(-1)^n$ 2) $n(-1)^{n-1}$

   3) $(n-1)(-1)^n$ 4) $n(-1)^{n+1}$

7. **The value of the integration $\int_0^\infty e^{-2t}t\sin t\,dt$ is**

   1) $\frac{1}{25}$ 2) $\frac{2}{25}$

   3) $\frac{3}{25}$ 4) $\frac{4}{25}$

8. **The Fourier sine transform of $xe^{-x^2/2}$ is**

   1) $e^{-s^2/2}$ 2) $Se^{-s^2/2}$

   3) $e^{-s/2}$ 4) $Se^{-s/2}$

9. **The Z-transform of $\{na^n\}$ is**

   1) $\frac{z}{(z-a)^2}$ 2) $\frac{a^2z}{(z-a)^2}$

   3) $\frac{az}{(z-a)^2}$ 4) $\frac{az^2}{(z-a)^2}$

DETAILED SOLUTIONS

1. **(2)**

   Formula :

   $L^{-1}\left(\frac{1}{(S-a)^2+b^2}\right) = \frac{1}{b}e^{at}\sin bt$

   $L^{-1}\left(\frac{1}{S^2+4S+13}\right) = L^{-1}\left(\frac{1}{(S+2)^2+9}\right)$

   $= \frac{1}{3}e^{-2t}\sin 3t$

2. **(3)**

   $L(y''+3y'+2y) = L(e^{-t})$

   $\Rightarrow (S^2L(y)-S\,y(0)-y'(0)]+3[sL(y)-y(0)]+2L(y)$

   $= \frac{1}{S+1}$

   Substituting $y(0) = y'(0) = 0$

   $L(y)[S^2+3S+2] = \frac{1}{S+1}$

   $\Rightarrow L(y) = \frac{1}{(S^2+3S+2)(S+1)}$

   $= \frac{1}{(S+1)(S+2)(S+1)}$

   $= \frac{1}{(S+1)^2(S+2)}$

3. **(1)**

   Formula:

   $L^{-1}\left(\frac{S^2}{(S^2+a^2)^2}\right) = \frac{1}{2a}[\sin at + at\cos at]$

   $\therefore f(t) = L^{-1}\left(\frac{S^2}{(S^2+4)^2}\right)$

   $= L^{-1}\left(\frac{S^2}{(S^2+2^2)^2}\right)$

   $= \frac{1}{2\times 2}(\sin 2t + 2t\cos 2t)$

$= \frac{1}{4}\sin 2t + \frac{t}{2}\cos 2t$

4. **(3)**

Fourier sine transform of $e^{-|x|} = \frac{S}{1+S^2}$

5. **(1)**

Formula:

$$F_c\{f(ax)\} = \frac{1}{a} F_c\left(\frac{S}{a}\right)$$

$$\therefore F_c(ax) = k\, F_c\left(\frac{S}{a}\right)$$

$$\Rightarrow k = \frac{1}{a}$$

6. **(4)**

Formula:

$$Z^{-1}\left(\frac{az}{(z-a)^2}\right) = na^n$$

$$Z^{-1}\left(\frac{z}{(z+1)^2}\right) = (-1)\, Z^{-1}\left(\frac{(-1)z}{(z-(-1))^2}\right)$$

$$= (-1)n\,(-1)^n$$

$$= n(-1)^{n+1}$$

7. **(4)**

By the definition of Laplace transform

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\, dt$$

Consider $\int_0^\infty e^{-st} t \sin t\, dt$

$= L(t \sin t)$

$= \frac{-d}{ds}(L(\sin t))$

$= \frac{-d}{ds}\left(\frac{1}{s^2+1}\right)$

$= -\left[\frac{(s^2+1).0-1*2s}{(s^2+1)^2}\right]$

$= \frac{2s}{(s^2+1)^2}$

In $\int_0^\infty e^{-2t} t \sin t\, dt$, $s = 2$

$\therefore \int_0^\infty e^{-2t} t \sin t\, dt = \frac{2*2}{(2^2+1)^2} = \frac{4}{25}$

8. **(1)**

Sine transform of $xe^{-x^2/2}$ is $e^{-x^2/2}$

9. **(3)**

We know that

$$z(a^n) = \frac{z}{z-a}$$

and

$$Zn(f(n)) = -z\frac{d}{dz}(Z\, f(n))$$

$$\therefore z(n\, a^n) = -z\frac{d}{dz}(Z\,(a^n))$$

$$= -z\frac{d}{dz}\left(\frac{z}{z-a}\right)$$

$$= -z\left(\frac{(z-a).1-z.1}{(z-a)^2}\right) = \frac{az}{(z-a)^2}$$

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