Force, Moments of a Force, Couple

Force

Principal of transmissibility of Forces:

If a force acts at any point on a rigid body it may also be considered to act at any other point on its line of action.

Parallelogram Law of Forces:

It stated that "it two forces acting simultaneously at a point be represented in magnitude and direction by the two adjacent sides of parallogram, then the resultant of these two forces is represented in magnitude and direction by the diagonal ofd that parallogram originating from that point ".

Mathematically, R=$\sqrt{P^2 + Q^2} + (2PQ\cos\theta)$

Triangle law of forces:

It stated that "If two forces acting at a point are represented by the two sides of a triangle taken in order, then their resultant force is represented by the third side taken in opposite order".

System of forces:

Force:

Force is an agent which changes or tends to change the state of rest or of uniform motion of a body upon which it acts. A Force represents the action of one body on another. Force is a vector quantity (i.e, Force has got both magnitude and direction)

Characteristics of a force:

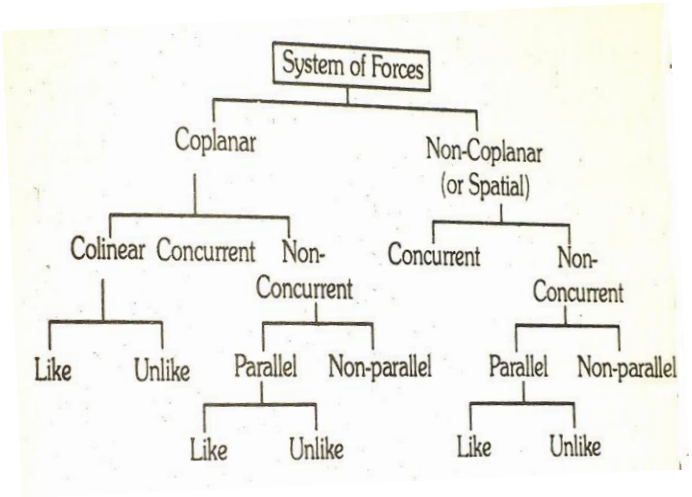
A Force is characterized by

1. Magnitude
2. Line of action
3. Direction

The magnitude of a force is denoted by a certain number of units.

System of Forces:

A body with two or more forces acting simultaneously on it constitutes a system of forces. Force system is classified into subdivisions as shown in the line diagram given below.



Coplanar Forces:

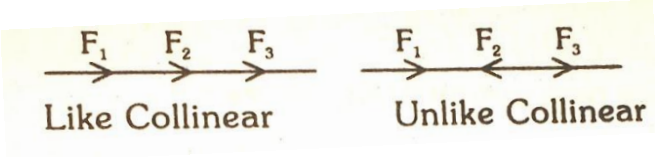
In Coplanar force system, the forces do not act in one plane. This system is also called as "Forces in Plane".

Non – Coplanar Force:

In Non – Coplanar force system, the forces do not act in one plane. This system is also called as "Forces in Space".

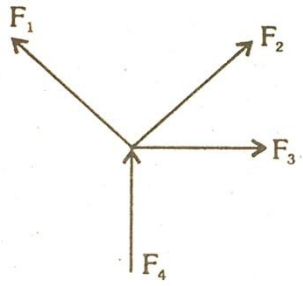
Collinear Forces:

The Forces which acts on a common line of action are called collinear forces. If they act in same direction, they are called the collinear and it they act in opposite direction, they are called "unlike collinear".



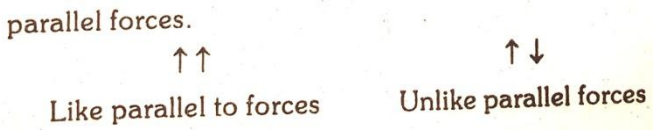
Concurrent Forces:

In concurrent force system, forces intersect at a common point.



Parallel Forces:

In parallel force system line of action of **forces** are parallel to each other. Parallel forces acting in same direction are called like parallel forces and the parallel forces acting in opposite direction are called unlike parallel forces.



Like Collinear Coplanar Forces:

Forces acting in same direction lie on a common line of action and acts in a single plane.

Unlike Collinear Coplanar Forces:

Forces acting in opposite direction, lies on a common line of action and acts in a single plane.

Coplanar Concurrent Forces:

Forces intersect at a common point and lies in a single plane.

Coplanar Noncurrent Forces:

Forces which do not intersect at a common point, but acts in one plane. They may be either parallel or non-parallel.

Non- Coplanar Concurrent forces:

Forces intersect at one point, but their lines of action do not lie on the same plane.

Non Coplanar non –concurrent forces:

Forces do not intersect at one point and also their line of action does not lie on the same plane.

Resultant Force:

If a number of forces acting on a particle simultaneously are replaced by a single force, which could produce the same effect as produced by the given forces, that single force is

called Resultant force. It is an equivalent force of all the given forces.

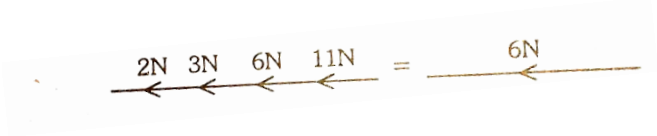
Resultant force of all the force systems can be determined by two methods.

1. Analytical method
2. Graphical method.

Q. Find the resultant force of the collinear forces shown below:

Resultant Force R = 8 + 10 + 12 = 30N

Q. Find the resultant force of the collinear forces shown below.:



Magnitude of Resultant force

R = 2 – 3 + 6 – 11 = -6N

As R is negative measure, Resultant force acts in the negative directions. i.e. towards left.

Resultant force of concurrent forces:

In order to find the resultant force of concurrent forces, we shall take.

i) Resultant force of two concurrent forces and

ii) Resultant forces of more than two concurrent forces.

Resultant force of two concurrent forces:

The analytical method of finding out the resultant of two concurrent forces can be developed from the parallelogram law of forces.

$R = \sqrt{P^2 + Q^2 + 2PQ\ cos\ \theta}$

Q. Two concurrent forces of 12N and 18N are acting at an angle of 60°. Find the resultant force.

Let p = 12N and

Q = 18N and

θ = 60°

Resultant force,

$R = \sqrt{P^2 + Q^2 + 2PQ\ cos\ \theta}$

$R = \sqrt{12^2 + 18^2 + (2\ x\ 12\ x\ 18\ x \cos 60^0)}$

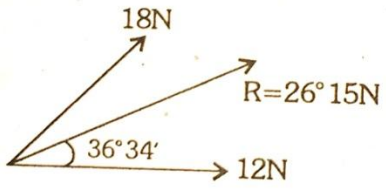
$= \sqrt{684}$ = 26.15N

Inclination of Resultant force with the force P,

$\tan \alpha = \frac{Q\ sin\theta}{P + Q \cos\ \theta}$

$= \frac{18 sin\, 60^0}{12 + (18 \cos\ 60^0)} = 0.742$

$\alpha = \tan^{-1}(0.742) = 36° 34'$



Q. Two concurrent forces acts at an angle of 30°. The resultant force is 15N and one of the forces is 10N. Find the other force.

Let R =15N. P = 10N, θ =30°, Q = ?

$R^2 = P^2 + Q^2 + 2PQ \cos \theta$

$15^2 = 10^2 + Q^2 + (2 \times 10 \times Q \cos 30°)$

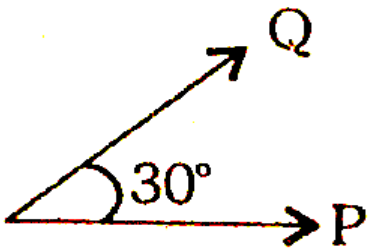
$15^2 = 100 + Q^2 + 17.32\ Q$

or $Q^2 + 17.32\ Q - 125 = 0$

$Q = \frac{-17.32 \pm \sqrt{(17.32)^2 - (4\ x\ 1\ x - 125)}}{2\ x\ 1}$

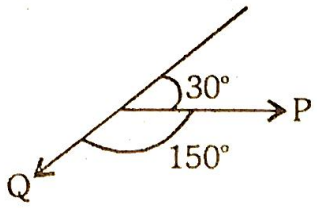
$= \frac{-17.32 \pm 28.28}{2}$

= -22.8N or 5.48N



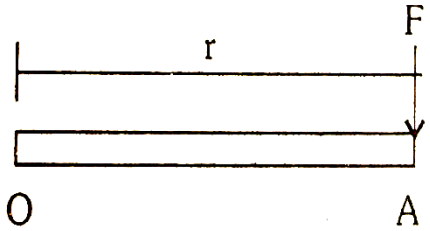
Reject – 22.8N force, because, if the force is reversed the included angle will be 150° not 30°.

Hence Q = 5.48N



Moment of a Force:

Moment of a force about a point is defined as the product of the force and the perpendicular distance of the line of action of the force from that point.



In the above figure, F is a downward force applied at 'A' and r is the perpendicular distance of the line of action of the force from the point O.

The moment (μ) of the force F about 'O' is given by

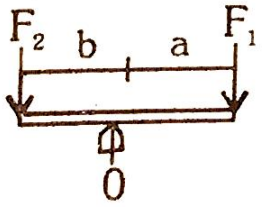
μ_o = F x r (i.e. Force x Perpendicular distance)

Downward force applied at A_1 will have a tendency to rotate OA about the point 'O'. Hence moment may also be defined as the turning effect produced by a force.

Moment is classified into two types.

) Clockwise moment

ii) Anticlockwise moment



In the above figure, downward force F_1 acting on the right hand side of the fulcrum 'O' at a distance of 'a' produces clockwise moment about 'O'.

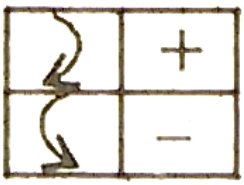
∴ Moment of F_1 about 'O' = F_1 x a (clockwise)

But downward force F_2 applied on the left hand side of the fulcrum 'O', at distance of b, produces anticlockwise moment about 'O'.

∴ Moment of F_2 about 'O' = F_2 x a (clockwise)

Sign convention:

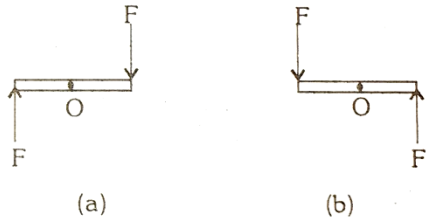
Positive sign for clockwise moment, negative sign for anticlockwise moment.



Unit of moment:

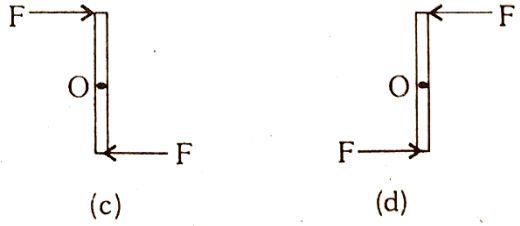
In S.I. system, unit of moment is Newton – metre (Nm) Force is measured in Newton and the distance is measured in metre.

Moment of vertical forces:



About a point (at 'O') right hand side downward force and left hand side upward force produces clockwise moment, shown in fig. (a). Similarly about a point (at 'O'), right hand side upward force and left hand side downward force produces anticlockwise moment, shown in fig. (b).

Moment of Horizontal Forces :

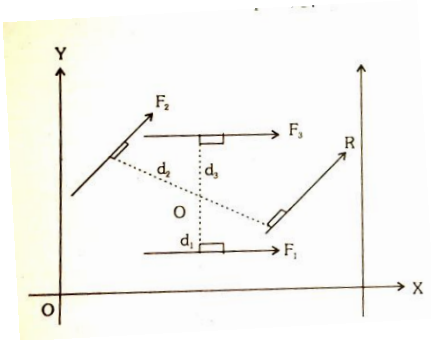


About a point (at 'O' in fig). upward right hand side force and downward left hand side produces clockwise moment (fig. c.). Similarly about a point, upward left hand side force and downward right hand side force produces anticlockwise moment.

Varignon's theorem :

The algebraic sum of the moments of any number of forces about any point in their plane is equal to the moment of their resultant about the same point. Varginon's theorem is also known as theorem of moments.

Consider a rigid body subjected to three coplanar forces F_1, F_2 and F_3 as shown in figure, at perpendicular distances d_1, d_2 and d_3 from a point O.



Let the resultant force R is at a distance 'd' from O.

From Varignon's theorem,

Sum of the moments of the forces F_1, F_2, and F_3 about 'O' is equal to the moment of resultant force R_1 about the same point O'.

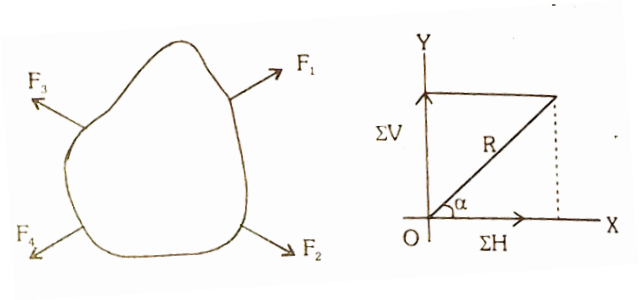
i.e. $F_1d_1 + F_2d_2 + F_3d_3 = R.d.$

Sum of the moment of all the forces about a point = moment of their resultant force about the same point.

Varignon's theorem is used in locating the resultant force.

Resultant force of Non – Concurrent & Non – Parallel forces :

The magnitude and direction of resultant force can be determined by analytical method as same for concurrent force system, But, location of the resultant force of non-concurrent and nonparallel force system is determined by the concept of moment and Varignon's principle.



$R = \sqrt{(\sum H)^2 + (\sum V)^2}$ *and*

Direction of resultant force, $a = \tan^{-1}\left[\frac{\sum V}{\sum H}\right]$

$\therefore \sum M = R \times X$

Where

$\sum m$ = algebraic sum of moments of given forces about a particular point.

R = Resultant force

x = the perpendicular distance of the line of action of Resultant force from the reference point, about which algebraic sum of moment of given forces is determined.

Couple:

Two forces F and –F having the same magnitude, parallel lines of action and opposite sense are said to form a couple.

Sum of the moment of couple forces about any point is same in magnitude and nature.

Moment of a couple = Force x arm of the couple.

M = F x a

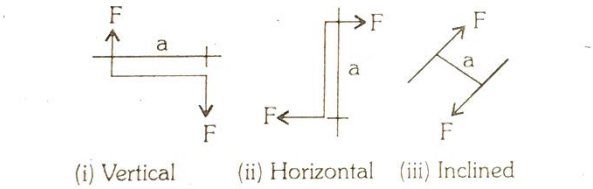
Difference between Moment and Couple :

The couple is a pure turning effect which may be moved anywhere in its own plane or into a parallel plane without change of its effect on the body, but the moment of a force must include a description of the reference axis about which the moment is taken.

Types of couple :

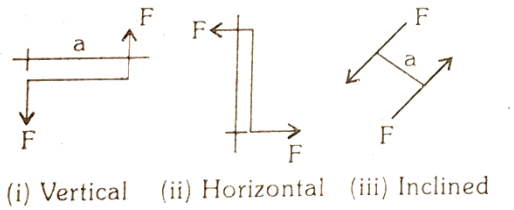
1. Clockwise couple
2. Anticlockwise couple

Clockwise couple acting on a body will have a tendency to rotate the body in clockwise direction, similarly anticlockwise couple will have a tendency to rotate a body in anticlockwise direction.



(i) Vertical (ii) Horizontal (iii) Inclined

Clockwise Couple



(i) Vertical (ii) Horizontal (iii) Inclined

Anticlockwise Couple