Applied Probability

1. Probability $= \frac{\text{Number of favourable cases}}{\text{Number of possible cases}}$
   $= \frac{n(A)}{n(S)}$
2. $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
3. $P(\bar{A}) = 1 - P(A)$
4. $P(\phi) = 0$; $P(S) = 1$
5. $P(\bar{A} \cap B) = P(B) - P(A \cap B)$
6. A and B are independent events then, $P(A \cap B) = P(A).\ P(B)$
7. Conditional probability:
   The conditional probability of A, when the event B has already defined is
   $P\left(\frac{A}{B}\right) = \frac{P(A \cap B)}{P(B)}$, $P(B) \neq 0$
   Similarly,
   $P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)}$
8. $P(A \cap B) = P(A)\ P\left(\frac{B}{A}\right)$
   $P(A \cap B) = P(B).\ P\left(\frac{A}{B}\right)$
9. Bay's Theorem:
   Let $A_1, A_2, \ldots\ldots A_n$ be n mutually exclusiveand exhausitive events. Let B be an independent event such that $B \subset \bigcup_{i=1}^{n} A_i$ is the conditionalprobability of B given that A_i has already occurred, then
   $P\left(\frac{A_i}{B}\right) = \frac{P(A_i) P\left(\frac{B}{A_i}\right)}{\sum_{i=1}^{n} P(A_i) P\left(\frac{B}{A_i}\right)}$

Probability Mass Function:

Let X be a one - dimensional discrete random variable takes the values $x_1, x_2, \ldots\ldots$

If i) $P(x_i) \geq 0$

ii) $\sum_{i=1}^{\infty} P(x_i) = 1$

Then, the function P is called the probability mass function.

Probability densitieve function:

If 1) $f(x) \geq 0$, $-\infty < x < 0$

2) $\int_{-\infty}^{\infty} f(x)\, dx = 1$

Then, f(x) is called the probability density function

Cumulative Distributive Function:

$F(x) = P(X \leq x) = \int_{-\infty}^{x} f(x)\, dx$

In case of discrete variables,

$F(x_1) = P(X \leq x_i) = \sum_{n=1}^{i} P(x_n)$

Binomial Distribution:

1. $P(X = r) = P(r)$
$= {}^{n}C_{r} p^{r} q^{n-r}$
$(r = 0, 1, 2, \ldots. n)$

2. i) Mean of binomial distribution = np
ii) Variance = npq
iii) Standard deviation S.D. $= \sqrt{npq}$
iv) $p + q = 1$

Poisson Distribution

$P(X = x) = \frac{e^{-\lambda} \lambda^{x}}{x!}$, $x = 0, 1, 2, \ldots\ldots$

$= 0$ otherwise

Remarks:

i) Poisson distribution is a limiting case of binomial distribution under the conditions, $n \to \infty$ and $p \to 0$
ii) $np = \lambda$
iii) Mean $= \lambda$
iv) Variance $= \lambda$
v) S.D. $= \lambda$

Normal Distribution:

$f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2}$ $-\infty < x < \infty$

Standard Normal Distribution:

$\phi(z) = \frac{1}{\sqrt{2\pi}} e^{-\frac{1}{2} z^2}$

Where $z = \frac{x-\mu}{\sigma}$

Properties:

i) Mean = median = mode

ii) The normal curve is perfectly symmetrical about the mean. This means that if we fold the curve along the vertical line at μ, the two halves of the curve will coincide.

iii) Maximum ordinate is at x = μ.

Its value is $\frac{1}{\sigma\sqrt{2\pi}}$