Ohm's and Kirchhoff's Law Study Material

Ohm's Law:

When the temperature remains constant, current flowing through a circuit is directly proportional to potential difference across the conductor.

$E \alpha I$

or $E = IR$

where R is the constant of proportionality becomes the resistance

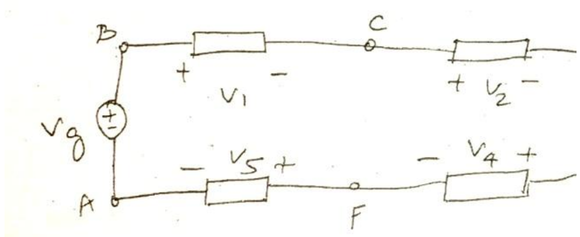
E is in volts

I is in ampers

KIRCHOFF's LAWS:

i) Kirchoff's Voltage Law:

The algebraic sum of all branch voltages around any dosed loop at a network is zero at all instant of time.



According to Kirchoff's voltage law in the above circuit.

$V_g - V_1 - V_2 - V_3 - V_4 - V_5 = 0$

Mathematically KVL can be written as

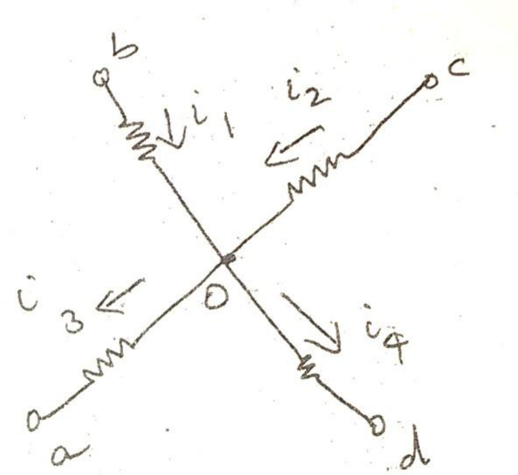
$\int V = 0$

i.e.

Around any closed loop, at any instant of time, the sum of voltage drops must be equal to the sum of voltage rises.

Kirchoff's Current Law (KCL):

"Sum of the currents entering a node is same as the sum of the currents leaving the node".



According to KCL, at node 0,

We have,

$i_1 + i_2 - (i_3 + i_4) = 0$

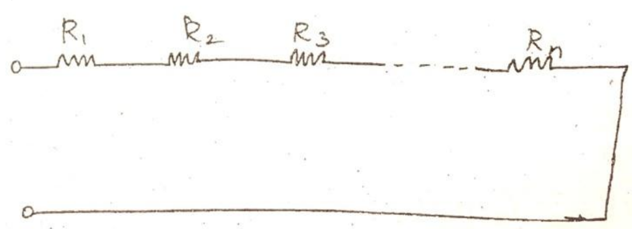
or $i_1 + i_2 = i_3 + i_4$

Mathematically

$\sum_i = 0$

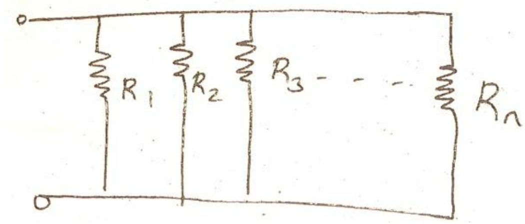
Series and Parallel networks:

i) Resistance in series



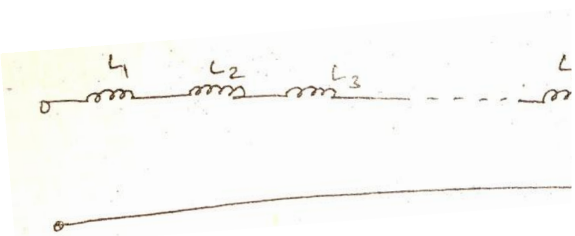
$R_{eq} = R_1 + R_2 + R_3 \ldots R_n$

Resistance in parallel:



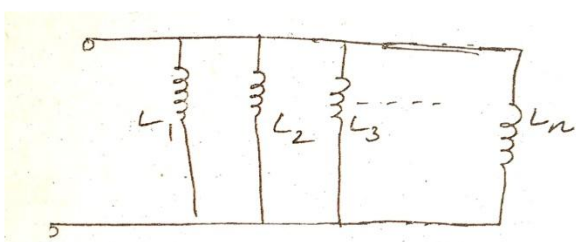
$$\frac{1}{R_{eq}} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \cdots + \frac{1}{R_n}$$

Inductance in series:



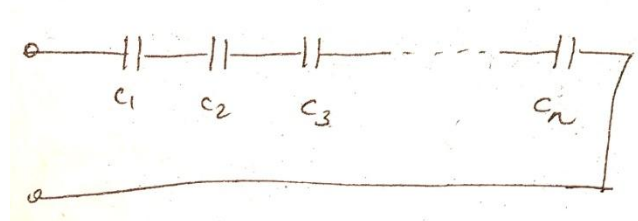
$L_{eq} = L_1 + L_2 + L_3 \ldots + L_n$

Inductance in Parallel:



$$\frac{1}{L_{eq}} = \frac{1}{L_1} + \frac{1}{L_2} + \frac{1}{L_3} + \cdots + \frac{1}{L_n}$$

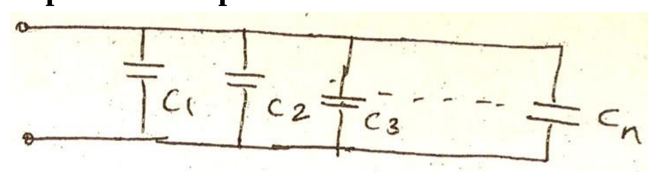
Capacitance In series:



$$\frac{1}{L_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

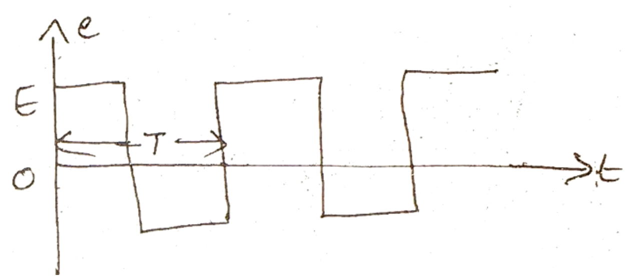
$$\frac{1}{C_{eq}} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3} + \cdots + \frac{1}{C_n}$$

Capacitance in parallel:



Alternating quantities:

AC source gives a voltage that varies with time, voltage changes not only in magnitude but also in direction (or) polarity.



• The time taken to complete one cycle (or the time interval, T after which the waveform repeats itself) is called the time period of the quantity.

• The number of cycles occurring per second is called frequency, f.

$f = \frac{1}{T}$ Hz

• The maximum value, positive or negative of the alternating quantity is called its amplitude.

• Average value = $\frac{\text{Area under the curve over one complete cycle}}{\text{Base (Time period)}}$

Effective value (or) RMS value:

RMS value = $\sqrt{\frac{i_1^2+i_2^2+\cdots+i_N^2}{N}}$

Form factor and crest factor:

Form factor = $\frac{\text{RMS value}}{\text{Average value}}$

Crest (peak) factor = $\frac{\text{Maximum value}}{\text{RMS value}}$

In AC circuits apart from resistance, we have two more parameters.

1. Inductance 2. Capacitance

Resistance:

When a sinusoidal current

$i = I_n \sin wt$ is applied to it the voltage across the terminals of the resistor is given by Ohms law.

$e = íR$

Hence $e = I_m R \sin wt = E_m \sin wt$

The maximum value $E_m = I_m R$ and then the rms value of the voltage

$E = IR$

The power dissipated in the resistor is obtained by finding the average of the instantaneous power.

$p = e.i = E_m \sin wt. I_m.\sin wt$

Average power,

$p = \frac{E_m I_m}{2}\int_0^{2\pi} \sin^2 \theta d\theta$

where θ = wt

$= \frac{E_m I_m}{2}$

$= \frac{E_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}} = EI$

$\therefore p = I^2 R = \frac{E^2}{R}$

Inductance:

When a time varying current passes through a circuit varying flux is produced. Because of this change of flux, voltage is induced in the circuit proportional to the time rate of change of flux or current.

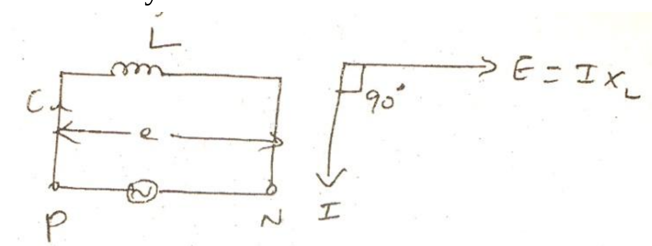
Emf Induced $\alpha \frac{di}{dt}$

$e\alpha \frac{di}{dt}$

$e = L\frac{di}{dt}$

where L, the constant of proportionality has come to be called self inductance of the circuit. The self inductance is the property of a coil by which it opposes any change of current.

Unit: Henry



$i = I_m \sin wt$

$e = L\frac{di}{dt} = I_m wL \cos wt$

$= I_m X_L \sin\left(w + \frac{\pi}{2}\right)$

where $X_L = wL$

$p_{qv} = \frac{1}{2\pi}\int_0^{2\pi} E_m\, I_m \sin\theta \cos\theta\, d\theta$

$= \frac{E_m I_m}{2\pi}\int_0^{2\pi} \frac{\sin 2\theta}{2} d\theta$

$= 0$

Energy stored in the inductance

$= \int_0^{I_0} Li\, di$

$= \frac{1}{2} L I_m^2$

$= \frac{1}{2} L \left(\sqrt{2} l\right)^2$

$= \frac{1}{2} L\, 2I^2 = LI^2$

where $I_m = \sqrt{2}l$

The coil windings in electric motors, transformers and similar devices have inductances.

In AC circuits apart from resistance, we have two more parameters.

Capacitance:

A capacitor is a circuit element that, like the inductor, stores energy during periods of time and returns the energy during others.

In the capacitor, storage takes place in an electric field unlike the inductance where storage is in a magnetic field.

A capacitor is formed by two parallel plates separated by an insulating medium. The emf across capacitor is proportional to the change in it, i.e.

$e \propto or\ e = \frac{q}{c}$

$q = \int i\, dt$

$\therefore e = \frac{1}{c} \int i\, dt$

Farad is the unit of capacitance. Usually μF (micro farad) is used

1 μF = 10^{-6} F

1 μF = 10^{-12} f

(pico farad)

$e = \frac{1}{c} \int idt = \frac{1}{c} \int I_m \sin wt\, dt$

$= \frac{I_m}{wC} \cos wt$

$= I_M X_c \sin\left(wt - \frac{\pi}{2}\right) (\because w = 2\pi f)$

Total work done (energy stored) in giving a voltage upto E_{max}

$= C \int_0^{E_{max}} C\, de$

$= \frac{1}{2} C E_{max}^2 = CE^2$

E = rms value of the voltage