Previous Year Solved Questions

1. The mapping $w = z^2 - 2z - 3$ is
   1) conformal everywhere
   2) not conformal at z=-1 and z = -3
   3) conformal with |z| = 1
   4) not conformal at z = l

2. The residue at z = 0 of $\frac{1+e^2}{z\cos z \ + \sin z}$ is
   1) 0 2) 1
   3) 4 4) -1

3. The value of $\int_c \frac{3z^2+7z+1}{z+1} dz$ where C is |z| =0.5 is
   1) 0 2) $\frac{\pi i}{2}$
   3) πi 4) 2πi

4. Singularity of ze^{1/x^2} at z = 0 is of the type
   1) isolated singularity
   2) removable singularity
   3) essential singularity
   4) isolated and removable singularities

5. The analytic function which maps an angularregion $0 \le \theta \le \frac{\pi}{4}$ on to the upper half plane is
   1) 4z 2) z^4
   3) 2θ 4) z^2

6. Let f(z) = u + iv be an analytic function which of the following statements are correct?
   a) both u and v satisfy Laplace equation
   b) Family of curves u=c, and v = C2 cut orthogonally
   c) $u_x = -v_y$ and $u_y = v_x$
   d) u-iv is also an analytic function
   1) (a) and (b) only
   2) (a), (b) and (c) only
   3) (a), (b) and (d) only
   4) (b), (c) and (d) only

7. The image of the straight line 2x+3y + 5 = 0under the transformation w= $\frac{1}{z}$ is a
   1) straight line 2) parabola
   3) ellipse 4) circle

8. The value of the Integral $\int_c \frac{z^2+5}{(z-1)^2} dz$, wherez is a complex number and C is the circle | z-1| = 1, is
   1) 12πi 2) 6πi
   3) 4πi 4) 2πi

9. The residue of the functionf(z) =$\frac{z^3+5z+7}{(z-2)(z+3)^3}$at the simple pole is
   1) 1 2) 5
   3) 25 4) 30

10. C-R equation for a functionW=P(r, θ) + iQ(r,θ) to be analytic, in Polarform are
    1)$\frac{\partial P}{\partial r} = \frac{1}{r}\frac{\partial Q}{\partial \theta}; \frac{\partial Q}{\partial r} = -\frac{1}{r}\frac{\partial P}{\partial \theta}$
    2)$\frac{\partial Q}{\partial \theta} = \frac{1}{r}\frac{\partial P}{\partial r}; \frac{\partial P}{\partial \theta} = \frac{1}{r}\frac{\partial Q}{\partial r}$
    3) $\frac{\partial P}{\partial r} = -\frac{1}{r}\frac{\partial Q}{\partial \theta}; \frac{\partial Q}{\partial r} = \frac{1}{r}\frac{\partial P}{\partial \theta}$
    4) $\frac{\partial P}{\partial \theta} = \frac{1}{r}\frac{\partial Q}{\partial r}; \frac{\partial Q}{\partial \theta} = -\frac{1}{r}\frac{\partial P}{\partial \theta}$

11. If f(z) + u+iv le an analytic function and u and v are harmonic, then u and v will satisfy
    1) one dimensional wave equation
    2) one dimensional wave equation
    3) Laplace equation
    4) Poisson equation

12. In the analytic function f(z)=u+iv the curves $u(x, y) = C_1$ and $V(x, y) = C_2$ are orthogonal If the product of the slopes m_1 and m_2 are
    1) $m_1 \ m_2=0$ 2) $m_1 \ m_2=-\pi$
    3) $m_1 \ m_2=-\frac{\pi}{2}$- 4) $m_1 \ m_2=-1$

13. If the Imaginary part of the analytic function $f(z)=u + iv$ is constant, then
1) u is not a constant
2) f(z) is not a complex constant
3) f(z) is equal to zero
4) u is a constant

14. If $f(z) = P(r, \theta) + iQ(r, \theta)$ is analytic, then $f'(z)$ is equal to
1) $e^{i\theta}\left[\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial \theta}\right]$ 2) $e^{-i\theta}\left[\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial \theta}\right]$
3) $e^{-i\theta}\left[\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial r}\right]$ 4) $e^{i\theta}\left[\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial r}\right]$

15. The necessary and sufficient conditions for the function $f(x) = u(r, \theta) + i(r, \theta)$ to be analytic is
1) $u_r = rv_0$ and $u_0 = \frac{-1}{r}v_r$
2) $u_r = \frac{1}{r}v_\theta$ and $u_\theta = -rv_r$
3) $u_\theta = \frac{1}{r}v_\theta$ and $u_\theta = rv_r$
4) $u_r = rv_0$ and $u_0 = \frac{1}{r}v_r$

16. The function $f(z) = z + 2\bar{z}$ is
1) analytic in the upper half plane
2) analytic inside the unit circle
3) analytic everywhere in complex plane
4) not analytic anywhere In the complex plane

17. In the two dimensional fluid flow, if the stream function is $\psi = \frac{-y}{x^2+y^2}$, then the velocity potential ϕ is
1) $\phi = \frac{x}{x^2+y^2}$
2) $\phi = \frac{y}{x^2+y^2}$
3) $\phi = \frac{-x}{x^2+y^2}$
4) $\phi = \frac{-y}{x^2+y^2}$

18. The value of the Integral $\int \frac{2z+1}{z^2+2}dz$, where C is $|z| = \frac{1}{2}$ is
1) $2\pi i$ 2) $3\pi i$
3) $5\pi i$ 4) 0

19. The residue of $z\cos\frac{1}{z}$ at $z = 0$ is
1) 1 2) -1
3) $\frac{1}{2}$ 4) $\frac{-1}{2}$

20. The analytic function, whose real part $\left(\frac{x}{x^2+y^2}\right)$ is given by
1) z^2 2) $\bar{z}$
3) $\frac{1}{z}$ 4) $z + \frac{1}{z}$

21. Cauchy - Riemann conditions for $f(z)=u(r, \theta) + iV(r, \theta)$ is
1) $u_r = V_0; u_\theta = -V_r$
2) $ru_r = v_\theta; rv_r = u_0$
3) $ru_\theta = V_r; rv_\theta = -u_r$
4) $ru_r = v_\theta; u_\theta = -rV_r$

22. Choose the correct answer. If $w = f(z)$ is analytic, then
1) $\frac{\partial w}{\partial z} = i\frac{\partial w}{\partial z}$
2) $\frac{\partial w}{\partial z} = \frac{\partial w}{\partial y}$
3) $\frac{\partial^2 w}{\partial z\,\partial \bar{z}} \neq 0$
4) $\frac{\partial w}{\partial \bar{z}} = 0$

23. The value of m such that $2x-x^2+my^2$ may be harmonic is
1) 3 2) 1
3) 2 4) 4

24. Image of circle $|z| = 2$ by the transformation $w=z+2+3i$, is
1) $|w+(2+3i)| = 2$ 2) $|w-(2+3i)| = 3$
3) $|w-(2+3i)| = 2$ 4) $|w| = 1$

25. Value of integral $\int_c (z+1)dz$, where C is asquare with vertices 0, 1, 1 + i and i is equal
1) zero 2) 1
3) -1 4) $2\pi i$

26. Residue of $\frac{z^2}{z^2+a^2}$ at z=al is

1) $\frac{1}{2a}$ 2) i a

3) $\frac{ia}{2}$ 4)2ai

27. The conditions CR equations f(z) = u + iv to be analytic are

1) $U_x = -V_y, V_x=U_y$

2) $U_x=V_y, V_x = -U_y$

3) $U_x = V_x, U_y = -V_y$

4) $U_x = U_y, V_x = -V_y$

28. If the real part of the analytic function f(z)=u+iv is constant then

1) v is not constant

2) f(z) is not a complex constant

3) v is a constant

4) f(z) is equal to zero

29. The function $z\bar{z}$ is

1) analytic at (1, 1) 2) no where analytic

3) analytic at (-1, -1) 4) analytic in (2, 2)

30. If f(z) = u + iv is analytic then f'(z) =

1) u_x+iv_y 2) u_y+iv_x

3) u_x+iv_x 4) u_x+iv_x

31. If f(z) = u+iv is an analytic function and v=xy then f(z) is equal to

1) $\frac{-z^2}{2}$ 2) z^2+z

3) $\frac{z^2}{2}$ 4) z^2-z

32. If w=P(r, θ) + iQ(r, θ) be a analytic function then

1) $\frac{\partial P}{\partial r} = \frac{\partial Q}{\partial \theta}$ 2) $\frac{\partial P}{\partial \theta} = \frac{\partial Q}{\partial r}$

3) $\frac{\partial P}{\partial r} - \frac{1}{r}\frac{\partial Q}{\partial \theta}$ 4) $\frac{\partial P}{\partial r} = \frac{-1}{r}\frac{\partial Q}{\partial \theta}$

33. If w = u(x,y) + iv(x,y) be an analytic function is equal to

1) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ 2) $\frac{\partial u}{\partial x} - i\frac{\partial v}{\partial y}$

3) $-\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$ 4) $\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

34. If f(z) = u + iv is an analytic function, then

1) only u is harmonic function

2) only v is harmonic function

3) both u and v are harmonic function

4) both u and v are not harmonic function

35. The value of $\int_C \frac{z^2}{x+3} dz$ where C is the circle |z| = 2 is

1) 0 2) 4

3) -4 4) $-\frac{4}{5}$

36. The residue of the function $e^{1/z}$ at the singular point z = 0 is

1) 0 2) -1

3) 1 4) ∞

37. Which one of the following is not an analytic function?

1) e^z 2) sin z

3) cos z 4) $|z|^2$

38. If c is a real constant an analytic function whose real part xy is

1) $\frac{-iz^2}{2} + ic$ 2) $\frac{z^2}{2} + c$

3) $\frac{-z^2}{2} + ic$ 4) $\frac{-iz^2}{2} + ic$

39. The bilinear transformation which maps z = 1, i, ∞ into W = i, 1, o is

1) $\frac{1}{x}$ 2) $\frac{i}{z}$

3) iz 4) $\frac{1}{zi}$

40. If w = u(x,y) + iv(x, y) is an analytic function of z = x + iy, then

1) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = \frac{-\partial v}{\partial x}$

2) $\frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

3) $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x}$

4) $\frac{\partial u}{\partial x} = \frac{-\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

41. Invariant points of a bilinear transformation w = $-\frac{1+iz}{1-iz}$ is

1) $\frac{-1}{2}\{\sqrt{1+i} \pm 6i\}$ 2) $\frac{-1}{2}((1+i) \pm \sqrt{6i})$

3) $\frac{1}{2}((1+i) \pm \sqrt{3+i})$ 4) $\frac{-1}{2}(6i \pm \sqrt{3+i})$

42. The critical points of the conformal transformation $w = z + \frac{1}{z}$ are

1) 1 and 0 2) -1 and 0

3) 1 and -1 4) 1 and 1

43. For the function $f(z) = \frac{e^{\frac{1}{z-1}}}{(z-2)^2}$, the point z = 1 is

1) a simple pole

2) a multiple pole

3) a removable singularity

4) an essential singularity

44. The value of the integral $\int_{|z|=3} \frac{z^3+1}{(z-1)(z-2)} dz$ is

1) πi 2) $2\pi i$

3) $6\pi i$ 4) $3\pi i$

45. Which of the following is a harmonic function?

1) $e^x x$ 2) $e^x \sin y$

3) $x^2 + y^2$ 4) $\sin x \cos y$

46. If W = u + iv is an analytic function of z = x + iy then which of the following is not true?

1) $\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$ 2) $\frac{dw}{dz} = \frac{\partial w}{\partial x}$

3) $\frac{dw}{dz} = -i\frac{\partial w}{\partial y}$ 4) $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$

47. The bilinear transformation which maps z = 1, i, ∞ into i, 1, 0 is

1) $\frac{1}{z}$ 2) $\frac{i}{z}$

3) iz 4) $\frac{1}{zi}$

48. The critical points of the transformation $w = (z-1)(z-2)$

1) z = l 2) z=2

3) $z = \frac{3}{2}$ 4) $z = \frac{4}{5}$

49. If f(z) = u +iv is an analytic function of z such that $u = x^2 - y^2$, then the value of v is

1) xy 2) 2xy

3) -xy 4) -2xy

50. If w = f(z) = u + iv is an analytic function of z = x + iy then which of the following is/are true?

1) $\frac{dw}{dz} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial y}$ 2) $f'(z) = \frac{\partial v}{\partial y} + i\frac{\partial v}{\partial x}$

3) $\frac{\partial w}{\partial y} = i\frac{\partial w}{\partial x}$ 4) $\frac{\partial w}{\partial x} = -i\frac{\partial w}{\partial y}$

1) (a) only 2) (d) only

3) (b) and (d) only 4) (b) and (c) only

51. If c is a real constant on analytic function whose real part is xy is

1) $\frac{-iz^2}{2} + ic$ 2) $\frac{z^2}{2} + c$

3) $\frac{z^2}{2} + ic$ 4) $\frac{iz^2}{2} + c$

52. The bilinear transformation which maps z = 0,1, ∞ onto w = -l, -i, 1 respectively is given by

1) $w = \frac{z}{z-i}$ 2) $w = \frac{z}{z+i}$

3 $w = \frac{z-1}{z+i}$ 4) $w = \frac{z+i}{z-i}$

53. $\int_{|x-1|-1} \frac{z^2+1}{z^2-1} dz =$

1) 0 2) 2π

3) $2\pi i$ 4) $-2\pi i$

54. In the Laurent series for $f(z) = \frac{1}{z-4}$ at centred at z = 1, the coefficient of $(z-1)^{-2}$ is

1) 3 2) -3

3) 9 4) -9

55. $f(z) = e^x \cos y + ie^x \sin y$ is analytic

1) at the origin only

2) nowhere In the complex plane

3) everywhere In the complex plane

4) at all points on the real axis only

56. Given that P+Q and Q-P can be the real and imaginary parts of an analytic function

respectively, which of the following is an analytic function?

1) (P+Q) + i(P-Q) 2) (P-Q) + i(P+Q)
3) P-iQ 4) p + iQ

57. The bilinear transformation $w = \frac{az+b}{cz+d}$ is
1) not conformal anywhere
2) conformal everywhere except at one point
3) conformal everywhere
4) analytic everywhere

58. If C is the curve $16x^2 + y^2 = 1$ the value of $\int_c \frac{dz}{z^2+9}$ is
1) $\frac{\pi}{3}$ 2) $\frac{-\pi}{3}$
3) 0 4) $\frac{1}{3}\tan^{-1}\left(\frac{2\pi}{3}\right)$

59. For the function $e^{1/z}$ the point z = 0 is
1) a simple pole
2) a pole of infinite order
3) an essential singularity
4) a regular point

60. Which of the following is not an analytic function?
1) e^z 2) sin z
3) coshz 4) $|z|^2$

61. If w = u(x, y) + iv (x, y) is analytic, then $\left|\frac{dw}{dz}\right|^2 =$
1) $\frac{\partial(u,v)}{\partial(x,y)}$ 2) $\frac{\partial(x,y)}{\partial(x,y)}$
3) $\frac{\partial(v,u)}{\partial(x,y)}$ 4) $\frac{\partial(y,x)}{\partial(u,v)}$

62. Given that $u = e^x \sin y$ as the real part of an analytic function, the corresponding imaginary part is
1) $e^y\sin x$
2) $-e^x\cos y$ + ic where c is a real constant
3) $-e^x\cos y+c^2$ where c is a real constant
4) $e^x\cos (xy)$

63. The transformation $w = z^2$ transforms the line y= 1 into
1) a circle, centre (0, 1) and radius 1
2) a straight line $\frac{y}{x}=1$
3) a hyperbola with vertex (1, 0)
4) a parabola with vertex at (-1, 0)

64. The value of the integral $\int_c \frac{e^z dz}{z(z-\pi)}$ where c is
the circle $|z-1|=2$ is
1) 0 2) 4i
3) -2i 4) 2i

DETAILED SOLUTIONS

1. (4)
The mapping f(z) is analytic and f'(z)≠ 0 then the mapping w = f(z) is conformal.
$f(z) = z^2 - 2z - 3$
$f'(z) = 2z - 2$
$f'(z) = 0$
$2z-2 = 0$
$2(z-1) = 0$
$z = 1$
$\therefore f'(1) = 0$
This implies w = f(z) is not conformal at z=1.

2. (2)
If $f(z) = \frac{\phi(z)}{\psi(z)}$
and $\psi(a) = 0 \phi(a) \neq 0$
then
Residue (at z = a) $\frac{\phi(a)}{\psi'(a)}$
consider $\phi(a) = \frac{1+e^z}{z\cos z+\sin z}$
$\phi(z) = 1 + e^z$
$\phi(z) = =$ zcos+sinz
$\phi(0) = 1 + e^0 = 2$
$\psi(0) = 0 + 0 = 0$
Now $\psi'(z)$ = cos z-z sin z+cos z
= 2cos z-z sin z
Residue (at z = 0)

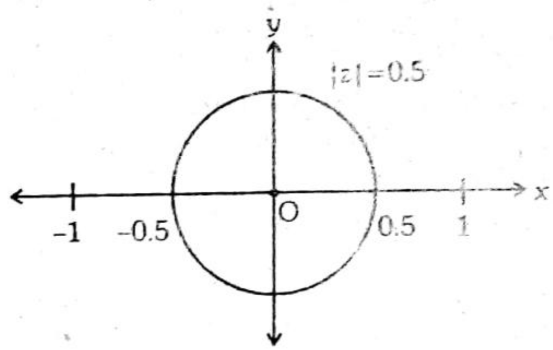
$= \frac{\phi(0)}{\psi'(0)}$

$= \frac{1+e^0}{2\cos 0 - 0\sin 0}$

$= \frac{2}{2}$

$= 1$

3. (1)



Cauchy's theorem:

If a function f(z) is analytic at all points inside
and on a closed contour C, then

$\int_c f(z)\,dz = 0$

let $f(z) = \frac{3z^2+7z+1}{z+1}$

f(z) is not analytic at z = -l

but z = -l lies outside the circle |z| = 0.5

∴ f(z) is analytic inside and on |z| = 0.5

∴ By Cauchy's theorem $\int_c f(z)dz = 0$

4. (3)

$f(z) = ze^{x^{\frac{1}{2}}}$

$= z\left[1 + \frac{\left(\frac{1}{z^2}\right)}{1!} + \frac{\left(\frac{1}{z^2}\right)^2}{2!} + \frac{\left(\frac{1}{z^2}\right)^3}{3!} + \cdots\right]$

$= z + \frac{1}{z} + \frac{1}{2z^3} + \frac{1}{6z^5} \ldots$

The principal part powers of $\left(\frac{1}{z-0}\right)$contains infinite number of levins. Therefore z = 0 is an essential singularity.

5. (2)

Required analytic function is

$f(z) = z^4$

6. (1)

Statements (a) and (b are true

C-R equation is

$u_x = v_y$; $u_y = -v_x$

∴ Statement (c) is not correct

Given f(z) = u + iv is an analytic function

∴ $u_x = v_y$; $u_y = - v_x$ ...(1)

Consider $\overline{f(z)}$= u - iv

= P + iQ

where P = U ; Q = -V

Now $P_x = u_x$; $Q_x = -v_x$

$P_y = u_y$; $Q_y = -v_{ya}$

by (1)

$P_x \neq Q_y$ and

$P_y \neq -Q_x$

∴$\overline{f(z)}$ does not satisfy C-R equation

∴$\overline{f(z)}$= u-iv is not analytic

∴ Statement (d) is not true

7. (4)

Let z = x + iy

and w = u + iv

$w = \frac{1}{z}$

$\Rightarrow z = \frac{1}{w}$

$\Rightarrow x + iy = \frac{1}{u+iv}$

$= \frac{1}{u+iv} \times \frac{u-iv}{u-iv}$

$= \frac{u-iv}{u^2+v^2}$

$= \frac{u}{u^2+v^2} - i\frac{v}{u^2+v^2}$

$\therefore x = \frac{u}{u^2+v^2}; v = \frac{-v}{u^2+v^2}$

Given straight line is

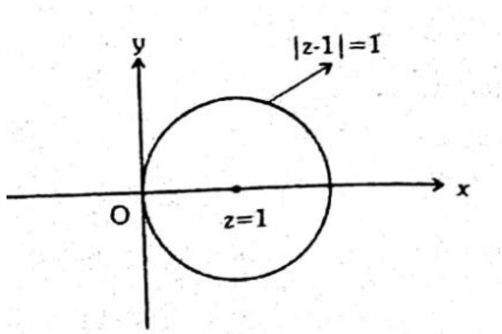
2x + 3y + 5 = 0

$\Rightarrow 2\left(\frac{u}{u^2+v^2}\right) + 3\left(\frac{-v}{u^2+v^2}\right) + 5 = 0$

$\Rightarrow 5(u^2 + v^2)$ 2u-3v = 0

∴ Image is a circle.

8. (3)



By Cauchy's integral formula,
if $z_0 \epsilon$ C then

$f'(z_0) = \frac{1}{2\pi i}\int_c \frac{f(z)dz}{(z-z_0)^2}$

consider $\int_c \frac{(z^2+5)dz}{(z-1)^2}$

$f(z) = z^2+5$

$f'(z) = 2z$

$f'(l) = 2(1)=2$

By (1)

$f'(1) = \frac{1}{2\pi i}\int_c \frac{(z^2+5)dz}{(z-1)^2}$

$\Rightarrow \int_c \frac{(z^2+5)dz}{(z-1)^2} = 2\pi i\ f'(1)$

$= 2\pi i \times 2$

$= 4\pi i$

9. (1)

Solution:

Residue of f(z) at a simple pole z = a

is $\lim_{x \to 0} (z-a)\ f(z)$

For $f(z) = \frac{z^3+5z-7}{(z-2)(z+3)^2}$

z = 2 is a simple pote and

z = -3 is a pole of order 2

∴ Residue at the simple pole z = 2 is

$\lim_{x \to 2} (z-2)\frac{z^3+5z-7}{(z-2)(z+3)^2}$

$= \lim_{x \to 2} \frac{z^3+5z+7}{(z+3)^2}$

$= \frac{8+10+7}{25} = \frac{25}{25}$

$= 1$

10. (1)

If $w = P(r, \theta) + iQ(r, \theta)$

is analytic, then polar form of C-R equations are

$$\frac{\partial P}{\partial r} = \frac{1}{r}\frac{\partial Q}{\partial \theta}$$

$$\frac{\partial Q}{\partial r} = \frac{-1}{r}\frac{\partial P}{\partial \theta}$$

11. (3)

f(z) = u + iv is an analytic function, then u and v satisfy Laplace equation.

12. (4)

$u(x, y) = C_1$ and $v(x, y) = c_2$

are orthogonal if prouduct of their slopes = -1

i.e., $m_1m_2 = -1$

13. (4)

f(z) = u + iv

Given imaginary part = constant

⇒ v = constant

$\Rightarrow \frac{\partial v}{\partial x} = 0; \frac{\partial v}{\partial y} = 0$

by C-R equations

$$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y} = 0$$

and $\frac{\partial u}{\partial y} = \frac{\partial v}{\partial x} = 0$

$\Rightarrow \frac{\partial u}{\partial x} = 0$

and $\frac{\partial u}{\partial v} = 0$

⇒ u is constant.

14. (3)

$f'(z) = e^{-i\theta}\left[\frac{\partial P}{\partial r} + i\frac{\partial Q}{\partial r}\right]$

15. (2)

The necessary and sufficient conditions for the function

$f(z) = u(r, \theta) + i\ v(r, \theta)$ to be analytic is

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$$

and $\frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$

$\Rightarrow u_r = \frac{1}{r} v_\theta$

and $v_r = \frac{-1}{r} v_\theta$

i.e. $u_\theta = -r v_r$

16. (4)

Let f(z) = u + iv

$f(z) = z + 2\bar{z}$

= (x + iy) + 2(x-iy)

= 3x-iy

$\Rightarrow$ u = 3x; v = -y

$u_x = 3$; $u_y = 0$

$v_x = 0$; $v_y = -l$

Clearly $u_x \neq v_y$

So C.R. equations are not satisfied.

This implies f(z) is not analytic everywhere in the complex plane.

17. (1)

The complex potential

$f(z) = \phi + i\psi$

Given $\psi = \frac{-y}{x^2+y^2}$

$$\psi_x = \frac{(x^2+y^2)0 + y.2x}{(x^2+y^2)^2}$$

$= \frac{2yx}{(x^2+y^2)^2}$

ψ_x (z, 0) = 0

$\psi_y = \frac{(x^2+y^2)(-1)+2y^2}{(x^2+y^2)^2}$

$= \frac{-x^2-y^2+2y^2}{(x^2+y^2)^2}$

$= \frac{y^2-x^2}{(x^2+y^2)^2}$

$\psi_y (z, 0) = \frac{z^2}{z^4} = \frac{-1}{z^2}$

∴ By Miline – Thomdon method

$f(z) = \int[\psi_y(z,0) + i\psi_x(z,0)]\, dz$

$= \int\left(\frac{-1}{z^2} + i0\right) dz$

$= -\int\frac{dz}{z^2} = -\left(\frac{-1}{z}\right)$

$= \frac{1}{z}$

$= \frac{1}{x+iy}$

$= \frac{1}{x+iy} \times \frac{x-iy}{x-iy}$

$= \frac{x-iy}{x^2+y^2}$

$\Rightarrow f(z) = \left(\frac{x}{x^2+y^2}\right) + i\left(\frac{-y}{x^2+y^2}\right)$

∴ The velocity potential

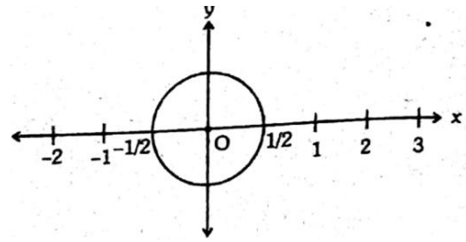
$\phi = \frac{x}{x^2+y^2}$

18. (1)

Given circle $|z| = \frac{1}{2} \Rightarrow \sqrt{x^2+y^2} = \frac{1}{2}$

$$\Rightarrow x^2 + y^2 = \left(\frac{1}{2}\right)^2$$

This is a circle with centre at origin and radius $= \frac{1}{2}$



Consider $\int_c \frac{2z+1}{z^2+z}\, dz$

Equating the denominator to zero

$z^2 + z = 0$

$\Rightarrow z(z+1) = 0$

$\Rightarrow z = 0, -1$

The point z = 0 lies inside the circle.

Let $f(z) = \frac{2z+1}{z^2+z}$

Residue of f(z) at z = 0

$= \lim_{x \to 0} (z-0)\, f(z)$

$= \lim_{x \to 0} z.\frac{2z+1}{z(z+1)}$

$= \lim_{z \to 0} \frac{2z+1}{z+1}$

= 1

By Cauchy's residue theorem

$\int_c f(z)\,dz = 2\pi i$ [sum of residues of points inside c]

z = 0 is the only point inside c

$\therefore \int_c \frac{2z+1}{z^2+z}\,dz = 2\pi i$ (Residue at z = 0)

$= 2\pi i \times 1 = 2\pi i$

19. (4)

$\cos\left(\frac{1}{z}\right) = 1 - \frac{\left(\frac{1}{z}\right)^2}{2!} + \frac{\left(\frac{1}{z}\right)^4}{4!} - \ldots.$

$= 1 - \frac{1}{2z^2} + \frac{1}{24z^4} - \cdots$

$z \text{ coscos}\left(\frac{1}{z}\right) = z - \frac{1}{2z} + \frac{1}{24z^2} - \ldots.$

Residue of z = 0

$=$ coefficient of $\frac{1}{z-0} = \frac{1}{z}$

$= \frac{-1}{2}$

20. (3)

clearly $\frac{1}{z} = \frac{1}{x+iy}$

$= \frac{1}{x+iy} \times \frac{x-iy}{x-iy}$

$= \frac{x-iy}{x^2+y^2}$

$= \frac{x}{x^2+y^2} - i\frac{y}{x^2+y^2}$

$\therefore \text{Re}\left(\frac{1}{z}\right) = \frac{x}{x^2+y^2}$

$= \frac{1}{z}$

Method : 2

$u = \frac{x}{x^2+y^2}$

$$\frac{\partial u}{\partial x} = \frac{(x^2+y^2) - x.2x}{(x^2+y^2)^2}$$

$= \frac{y^2-x^2}{(x^2+y^2)^2}$

$\frac{\partial u}{\partial x}(z, 0) = \frac{0^2-z^2}{(z^2+0^2)^2} = \frac{-z^2}{z^4} = \frac{-1}{z^2}$

Now,

$\frac{\partial u}{\partial y} = \frac{0-x.2y}{(z^2+0^2)^2} = \frac{-2xy}{(x^2+y^2)^2}$

$\frac{\partial u}{\partial y}(z, 0) = \frac{-2.z.0}{(z^2+0^2)^2}$

$= 0$

By Milne - Thompson method

$f(z) = \int\left[\frac{\partial u}{\partial x}(z, 0) - i\frac{\partial u}{\partial y}(z, 0)\right] dz$

$= \int\left(\frac{-1}{z^2} - i\,0\right) dz$

$= -\int \frac{1}{z^2}\,dz = \frac{1}{z}$

$\therefore f(z) = \frac{1}{z}$

21. (4)

Cauchy - Riemann equations of

$f(z) = u(r, \theta) + iv\,(r, 0)$

is $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta}$

and $\frac{\partial v}{\partial r} = \frac{1}{r}\frac{\partial u}{\partial \theta}$

$\Rightarrow ru_r = v_0$

and $rv_r = -u_\theta$

22. (4)

w = f(z) is analytic then it is independent of $\bar{z}$

$\therefore \frac{\partial w}{\partial \bar{z}} = 0$

23. (2)

Let $u = 2x - x^2 + my^2$

$$\frac{\partial u}{\partial x} - 2 - 2x$$

$\frac{\partial^2 u}{\partial x^2} = -2$

$$\frac{\partial u}{\partial y} = 2my$$

$\frac{\partial^2 u}{\partial y^2} = 2m$

Given u is harmonic

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0$$

$\Rightarrow -2 + 2m = 0$

$\Rightarrow m = 1$

24. (3)

w = z + 2 + 3i

z = w – 2 - 3i

|z|= 2

$\Rightarrow |w - 2 - 3i| = 2$

$\Rightarrow |w-(2+3l)| = 2$

25. (1)

By Cauchy - Goursat theorem if f is analytic function in the region enclosed by the curve C, then

$\int_c f(z)dz = 0$

f(z) = z + 1 is analytic in every region

$\therefore$ f(z) = z + 1 is analytic inside C

$\therefore \int_c (z+1)dz = 0$

26. (3)

Result: Residue of f(z) at the pole z = a is

$\lim_{x \to 0} (z-a) f(z)$

Let $f(z) = \frac{z^2}{z^2+a^2}$

$= \frac{z^2}{(z+ai)(z-ai)}$

Residue at z = ai

$= \lim_{z \to ai} (z - ai)f(z)$

$= \lim_{z \to ai} (z - ai) \frac{z^2}{(z+ai)(z-ai)}$

$= \lim_{z \to ai} \frac{z^2}{(z+ai)}$

$= \frac{(ai)^2}{ai+ai}$

$= \frac{(ai)^2}{2ai}$

$= \frac{ai}{2}$

27. (2)

CR equations are

$u_x = v_y$; $u_y = -v_x$

i.e. $u_x = v_y$, $v_x = -u_y$

28. (3)

By CR equations

$u_x = v_y$; $u_y = -v_x$

$\therefore v_m$

since u is constant

$\Rightarrow u_x = 0$ and $u_y = 0$

By CR equations

$0 = u_x = v_y$

$\Rightarrow v_y = 0$ ....(1)

Also

$0 = u_y = -v_x$

$\Rightarrow v_x = 0$ ....(2)

By (1) and (2)

v is a constant.

29. (2)

Let z = x + iy

then $\overline{z}$ = x - iy

$f(z) = z\overline{z}$

= (x + iy) (x - iy)

$= x^2 + y^2 = u + iv$

$\Rightarrow u = x^2 + y^2$

v = 0

$\Rightarrow u_x = \frac{\partial u}{\partial x} = 2x$

$u_y = \frac{\partial u}{\partial y} = 2y$

$v_x = \frac{\partial v}{\partial x} = 0$

$v_y = \frac{\partial v}{\partial y} = 0$

Clearly $u_x \neq v_y$

$u_y \neq v_x$

$\therefore$ C.R equations are not satisfied.

$\therefore$ f(z) is nowhere analytic.

30. (3)

If f(z) is analytic

then $f'(i) = u_x + iv_x$

$= v_y - iu_y$

31. (3)

Method -1

v = xy

$$\frac{\partial v}{\partial x} = y; \frac{\partial v}{\partial y} = x$$

$$\frac{\partial v}{\partial x}(z,0) = 0; \frac{\partial v}{\partial y}(z,0) = z$$

By Milne – Thompson method

$f(z) = \int \left[\frac{\partial v}{\partial y}(z,0) + i\frac{\partial v}{\partial x}(z,0)\right] dz$

$\int (z + i0)\, dz$

$= \frac{z^2}{2}$

Method : 2

From the given choices

if $f(z) = \frac{z^2}{2}$

then $f(z) = \frac{z^2}{2} = \frac{(x+iy)^2}{2}$

$= \frac{x^2-y^2+2ixy}{2}$

$= \frac{x^2-y^2}{2} + i2xy$

$= u + iv$

$\Rightarrow v = xy$

So required analytic function

$f(z) = \frac{z^2}{2}$

32. (3)

Let $f(z) = P(r, \theta) + i(Q(r, \theta))$

then Polar form of Cauchy-Riemann equations are

$$\frac{\partial P}{\partial r} = \frac{1}{r}\frac{\partial Q}{\partial \theta}$$
$$\frac{\partial Q}{\partial r} = \frac{1}{r}\frac{\partial P}{\partial \theta}$$

33. (3)

Result :

If $z = x + iy$ and $w = f(z)$

then $\frac{dw}{dz} = \frac{\partial w}{\partial x}$

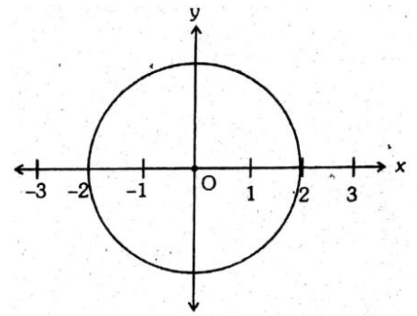
$\therefore \frac{dw}{dx} = \frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}$

34. (3)

Result:

If f(z) = u+iv is an analytic function then both u and v are harmonic functions.

35. (1)



Cauchy's Theorem :

If a function f(z) is analytic at all points inside and on a closed curve C

then $\int_c f(z)dz = 0$

Let $f(z)= \frac{z^2}{z+3}$

clearly f(z) is not analytic at z = -3

But this point lies outside the circle |z|=2

Hence, f(z) is analytic at all points interior and on

the closed curve C

Where C is |z|=2

Hence, by Cauchy's theorem

$\int_c f(z)dz = 0$

i.e., $\int_c \frac{z^2}{z+3} dz = 0$

36. (2)

$$e^{t/z} = 1 + \frac{\left(\frac{1}{z}\right)}{1!} + \frac{\left(\frac{1}{z}\right)^2}{2!} + \frac{\left(\frac{1}{z}\right)^3}{3!}$$

$= 1 + \frac{1}{z} + \frac{1}{2x^2} + \frac{1}{6z^2} + \cdots$

Residue of $e^{t/z}$ at z = 0

= coefficient of $\frac{1}{(z-0)} = \frac{1}{z}$

in the series expansion of $e^{t/z} = 1$

37. (4)

Let $f(z) = |z|^2$

where $z = x + iy$

$|z|^2 = x^2 + y^2$

then $f(z) = |z|^2$

$= x^2 + y^2$

$= (x^2 + y^2) + i\,0$

Let f(z) = u + iv
then u = $x^2 + y^2$; v=0

$$\frac{\partial u}{\partial x} = 2x; \frac{\partial u}{\partial y} = 2y$$

$$\frac{\partial v}{\partial x} = 0; \frac{\partial v}{\partial y} = 0$$

$\therefore \frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$

and $\frac{\partial u}{\partial y} \neq \frac{\partial v}{\partial x}$

C.R. equation is not satisfied

So f(z)= $|z|^2$ is not analytic.

38. (1)

If f(x) = u + iv = $-\frac{iz^2}{2} + ic$

then

$f(z) = \frac{-i(x+iy)^2}{2} + ic$

$= \frac{-i(x^2-y^2+2ixy)}{2} + ic$

$= xy + i\left[\frac{y^2-x^2}{2}\right] + c$

$\therefore$ if $f(z) = \frac{-iz^2}{2} + ic$

Then, the real part is xy

39. (2)

If $w = \frac{i}{z}$ then

when z = 1, $w = \frac{i}{1} = i$

when z = i, $w = \frac{i}{i} = 1$

when z = ∞, $w = \frac{i}{\infty} = 0$

$\therefore$ Required bilinear transform is $w = \frac{i}{z}$

40. (1)

By Cauchy - Reimann equations if w = u + iv is an analytic function then,

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial x}$ and

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

41. (2)

Invariant points are given by

$z = \frac{1+iz}{1-iz}$

$\Rightarrow 1 + iz = z(1-iz)$

$\Rightarrow 1 + iz = z - iz^2$

$\Rightarrow iz^2 + iz - z + 1 = 0$

$\Rightarrow iz^2 + z(i-1) + 1 = 0$

$\therefore z = \frac{-(i-1)\pm\sqrt{(i-1)^2-4i}}{2i}$

$= \frac{-(i-1)\pm\sqrt{-1+1-2i-4i}}{2i}$

$= \frac{1}{2}\left[\frac{-(i-1)\pm\sqrt{-6i}}{i}\right]$

$= \frac{1}{2}\left[-(i-1)\times(-i) \pm (-i)\sqrt{-6i}\right]$

$$\left(\because \frac{1}{i} = -i\right)$$

$= -\frac{1}{2}\left[(-i^2 + i) \pm \sqrt{i^2(-6i)}\right]$

$= -\frac{1}{2}\left[(1+i) \pm \sqrt{6i}\right]$

42. (1)

Critical points are given by $\frac{dw}{dz}$ =0 and

$\frac{dw}{dz} = \infty$

$w = z + \frac{1}{z}$

$\frac{dw}{dz} = 1 - \frac{1}{z^2}$

$\frac{dw}{dz} = 0 \Rightarrow 1 - \frac{1}{z^2} = 0$

$\Rightarrow z^2 = 1$

$\Rightarrow z = 1$

$\frac{dw}{dz} = \infty \Rightarrow 1 - \frac{1}{z^2} = \infty$

$\Rightarrow \frac{1}{z^2} = \infty$

$\Rightarrow z = 0$

$\therefore$ 1 and 0 are the critical points.

43. (4)

$$\frac{1}{(z-2)} e^{\frac{1}{z-1}}$$

$$\frac{1}{(z-2)^2}\left[1 + \frac{1}{(z-1)} + \frac{1}{2!\,(z-1)^2} + \frac{1}{3!\,(z-1)^3} + \cdots\right]$$

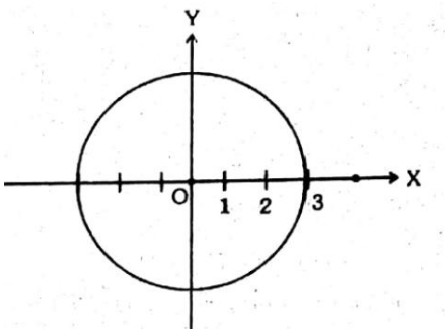
The principal part contains infinite number of terms. So, z = l is an essential singularity.

44. (3)

Cauchy's Residue theorem :

$\int_c f(z)dz = 2\pi i \times$ sum of all residues of f(z) inside C

Let $f(z) = \frac{z^2+1}{(z-1)(z-2)}$



Poles of f(z) are z=l and z=2

Clearly z=l and z = 2 lies inside the circle $|z| = 3$

Residue of f(z) at z = l

$= \underset{z\to 1}{\text{it}}(z-1)f(z)$

$= \underset{z\to 1}{\text{it}}(z-1)\frac{z^2+1}{(z-1)(z-2)}$

$= \frac{1+1}{1-2} = \frac{2}{-1}$

Residue at z=2

$= \underset{z\to 2}{\text{it}}(z-2)f(z)$

$= \underset{z\to 2}{\text{it}}(z-2)\frac{(z^2+1)}{(z-1)(z-2)}$

$= \frac{4+1}{2-1} = 5$

By Cauchy Residue theorem

$\therefore \int_c f(z)dz = 2\pi i \times$ sum of residues inside $|z|=3$

$= 2\pi i \times (-2+5)$

$= 6\pi i$

$\therefore \int_{|x|=3} \frac{z^2+1}{(z-1)(z-2)} = 6\pi i$

45. (2)

If $z = e^x \sin y$ dz

then $\frac{\partial z}{\partial x} = e^x \sin y$

$\frac{\partial^2 z}{\partial x^2} = e^x \sin y$

$\frac{\partial z}{\partial y} = e^x \cos y$

$\frac{\partial^2 z}{\partial y^2} = e^x \sin y$

$\therefore \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^x \sin y - e^x \sin y = 0$

$\therefore e^x \sin y$ is a harmonic function

46. (1)

$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = 0$ is not true

47. (2)

If $w = \frac{i}{z}$ then

z	$w = \frac{i}{z}$
1 i ∞	i 1 0

$\therefore \; w = \frac{i}{z}$ is the required transform.

48. (3)

$w = z^2 - 3z + 2$

$\frac{dw}{dz} = 2z-3$

Critical points is given by

$\frac{dw}{dz} = 0$

$\Rightarrow 2z - 3 = 0$

$\Rightarrow z = \frac{3}{2}$

49. (2)

$u = x^2 - y^2$

$$\frac{\partial u}{\partial x} = 2x; \frac{\partial u}{\partial x}(z,0) = 2z$$

$$\frac{\partial u}{\partial y} = -2y; \frac{\partial u}{\partial y}(z,0) = 0$$

By Milne Thompson method analytic function

f(z)

having $u = x^2 - y^2$ as real part

$f(z) = \int \left[\frac{\partial u(z,0)}{\partial x} - i\frac{\partial u}{\partial y}(z,0)\right] dz$

$= \int (2z - 0)dz$

$= z^2$

Let $f(z) = u + iv$

then $u + iv = z^2$

$= (x + iy)^2$

$= x^2 - y^2 + i2xy$

$\therefore v = 2xy$

50. (2)

Formula:

If $w = f(z)$

then $\frac{dw}{dz} = \frac{\partial w}{\partial x}$ and

$$\frac{dw}{dz} = -i\frac{\partial w}{\partial y}$$

$\therefore \frac{\partial w}{\partial x} = -i\frac{\partial w}{\partial y}$

51. (1)

Let $f(z) = u + iv$

If $f(z) = \frac{-iz^c}{2} + ic$

then $f(z) = \frac{-i(x+iy)^2}{2} + ic$

$= \frac{-i(x^2-y^2+2ixy)}{2} + ic$

$= xy + i\left(\frac{y^2-x^2}{2} + c\right)$

$\therefore u = xy$

So if $f(z) = \frac{-iz^2}{2} + ic$ then the real part is xy.

52. (3)

If $(w_1, w_2, w_3, w_4) = (z_1, z_2, z_3, z_4)$

then

$\frac{(w_1 - w_2)(w_3 - w_4)}{(w_2 - w_3)(w_4 - w_1)} = \frac{(z_1 - z_2)(z_3 - z_4)}{(z_2 - z_3)(z_4 - z_1)}$

we have $(z, 0, 1, \infty) = (w, -1, -i, 1)$

$\Rightarrow \frac{(z-0)(1-\infty)}{(z-\infty)(1-0)} = \frac{(w+1)(-i-1)}{(w-1)(-i-1)}$

$\Rightarrow \frac{(w+1)(-i-1)(1+i)}{(w-1)(1-i)(1+i)} = \frac{z(1-\infty)}{(z-\infty)}$

Now

$\frac{1-\infty}{z-\infty} = \lim_{t \to \infty}\left(\frac{1-t}{z-t}\right)$

$= \lim_{t \to \infty} \frac{\frac{1}{t}-1}{\frac{z}{t}-1}$

$= 1$

Now (1) $\Rightarrow$

$\left(\frac{w+1}{w-1}\right)\left(\frac{-(1+i)^2}{2}\right) = z$

$\Rightarrow \frac{w+1}{w-1}(-i) = z$

$\Rightarrow \frac{w+1}{w-1} = iz$

$\Rightarrow w + 1 = iz(w - 1)$

$\Rightarrow w = \frac{iz+1}{iz-1}$

$= \frac{i\left(z+\frac{1}{i}\right)}{i\left(z-\frac{1}{i}\right)}$

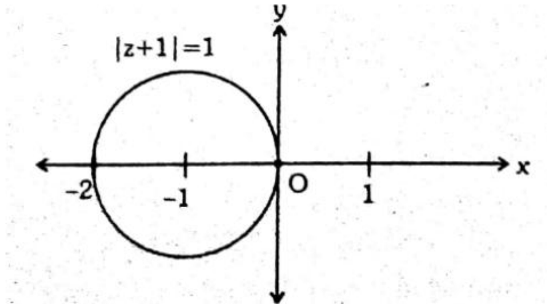
$= \frac{z-i}{z+i}$

$\therefore$ Required bilinear transform $w = \frac{z-i}{z+i}$

53. (4)

Cauchy's Residue theorem

$\int_c f(z)dz = 2\pi i \times$ sum of residues inside C



$$|z + 1| = 1 \Rightarrow |z - (-1)| = 1$$

is a circle with centre z = -l and radius = 1

Let $f(z) = \frac{z^2+1}{z^2-1}$

$= \frac{z^2+1}{(z+1)(z-1)}$

Poles -z =1,-1

z = -l lies inside |z+1! = 1

Residue at z = -1

$= \lim_{z \to -1} (z + l)f(z)$

$= \lim_{z \to -1} (z+1)\frac{z^2+1}{(z-1)(z+1)}$

$= \lim_{z \to -1} \frac{z^2+1}{z-1}$

$= \frac{2}{-2} = -1$

By Cauchy's residue theorem

$\therefore \int_c f(z)dz) = 2\pi i \times -1$

$= -2\pi i$

54. (1)

w = z-1

then z-4 = w-3

$f(z) = \frac{1}{z-4} = \frac{1}{w-3}$

$= \frac{1}{w\left(1-\frac{3}{w}\right)}$

$= \frac{1}{2}\left(1-\frac{3}{w}\right)^{-1}$

$= \frac{1}{w}\left(1+\frac{3}{w}+\frac{3^2}{w^2}+\cdots\right)$

$= \frac{1}{w}+\frac{3}{w^2}+\frac{3^2}{w^3}+\ldots.$

$= \frac{1}{(z-1)}+\frac{3}{(z-1)^2}+\frac{3^2}{(z-1)^3}+\cdots$

$\therefore$ Coefficient of $\frac{1}{(z-1)^2}$ = 3

55. (3)

$w = f(z) = e^x\cos y + ie^x \sin y$

w = u + iv

$= e^x (\cos y + i \sin y)$

$u = e^x\cos y$

$v = e^x \sin y$

$\frac{\partial u}{\partial x} = e^x \cos y$

$\frac{\partial u}{\partial y} = -e^x \sin y$

$\frac{\partial v}{\partial x} = e^x \sin y$

$\frac{\partial v}{\partial y} = e^x \cos y$

$\therefore \frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

C.R. equations are satisfied

$\therefore$ f(z) is analytic everywhere in the complex plane.

56. (4)

Let u = P+Q

v = Q-P

by C R. equations

$u_x = v_y$

$u_y = -v_x$

$\therefore \frac{\partial P}{\partial x}+\frac{\partial Q}{\partial y} = \frac{\partial Q}{\partial y}-\frac{\partial P}{\partial y}$ .... (1)

$\frac{\partial P}{\partial y}+\frac{\partial Q}{\partial y} = -\frac{\partial Q}{\partial x}+\frac{\partial P}{\partial x}$

$\Rightarrow \frac{\partial P}{\partial x}-\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}+\frac{\partial Q}{\partial y}$ ....(2)

(1) + (2)

$\Rightarrow 2\frac{\partial P}{\partial x} = 2\frac{\partial Q}{\partial y}$

$\Rightarrow \frac{\partial P}{\partial x} = \frac{\partial Q}{\partial y}$ ....(3)

(1) - (2) $\Rightarrow$

$$2\frac{\partial Q}{\partial x} = -2\frac{\partial P}{\partial y}$$

$\Rightarrow \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ ....(4)

by (3) and (4)

f(z) = P + iQsatisties

C.R. equations

$\therefore$ P + iQ is an analytic function

57. (2)

A function w=f(z) is not confoltnui z_0 then

$\frac{dw}{dz} = f'(z_0)$ if $z_0 = -\frac{d}{a}$

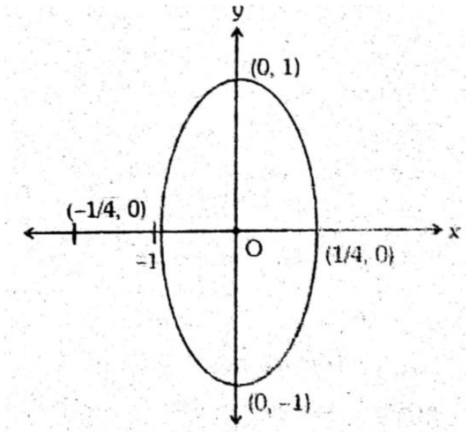
then f(z) is not analytic.

58. (3)

$16x^2 + y^2 = 1$

$$\frac{x^2}{1/16}+y^2 = 1$$

$$\frac{x^2}{\left(\frac{1}{4}\right)^2}+\frac{y^2}{1} = 1$$



Let $\int f(z)\,dz = \int_c \frac{dz}{z^2+9}$

$= \int_c \frac{dz}{(z+3i)(z-3i)}$

The points z =3i, -3i lies outside C

∴ By Cauchy Goursattheoem

$\int_c f(z)dz = 0 \Rightarrow \int_c \frac{dz}{z^2+9} = 0$

59. (3)

$$e^{1/2} = 1 + \frac{1}{z} + \frac{1}{2!\,z^2} + \frac{1}{3!\,z^3} + \cdots$$

$= 1 + \frac{1}{(z-0)} + \frac{1}{2(z-0)^2} = \frac{1}{6(z-0)^3} + \cdots$

since $\frac{1}{z-0}$ terms are infinite, z = 0 is an essential singularity

60. (4)

Consider $f(z) = |z|^2$

$= x^2 + y^2$

if f(z) = u + iv then

$u = x^2 + y^2$; v = 0

$$\frac{\partial u}{\partial x} = 2x; \frac{\partial u}{\partial y} = 2y$$

$$\frac{\partial v}{\partial x} = 0; \frac{\partial v}{\partial y} = 0$$

$\therefore \frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$ and $\frac{\partial u}{\partial y} \neq \frac{-\partial y}{\partial x}$

C.R equation is not statistical

So $f(z) = |z|^2$ is not analytic

61. (1)

Let $J = \frac{\partial(u,v)}{\partial(x,y)} = \begin{vmatrix} \frac{\partial u}{\partial x} & \frac{\partial u}{\partial y} \\ \frac{\partial v}{\partial x} & \frac{\partial v}{\partial y} \end{vmatrix}$

$= \frac{\partial u}{\partial x}\frac{\partial v}{\partial y} - \frac{\partial u}{\partial y}\frac{\partial v}{\partial x}$ ....(1)

Given w = u + iv is analytic

∴ w satisfies C.R. equation

i.e., $\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

$$\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$$

∴ (1) ⇒

$$\frac{\partial u}{\partial x}\frac{\partial u}{\partial x} - \frac{\partial v}{\partial x}\left(-\frac{\partial v}{\partial x}\right)$$

$= \left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial v}{\partial x}\right)^2$

$= \left|\frac{\partial u}{\partial x} + i\frac{\partial v}{\partial x}\right|^2$

$= \left|\frac{\partial w}{\partial x}\right|^2$

$= \left|\frac{dw}{dz}\right|^2 \left[\because \frac{dw}{dz} = \frac{\partial w}{\partial x}\right]$

62. (3)

Given $u = e^x \sin y$

$\frac{\partial u}{\partial x} = e^x \sin y$

$\frac{\partial u}{\partial y} = e^x \cos y$

If $v = -e^x \cos y + c^2$

then $\frac{\partial v}{\partial x} = -e^x \cos y$

$\frac{\partial v}{\partial y} = e^x \sin y$

clearly

$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$

and $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

C.R. equations are satisfied

$v = -e^x \cos y + c^2$

is the required imaginary part.

63. (4)

$w = z^2$

$u + iv = (x + iv)^2$

$= x^2 - y^2 + 2ixy$

$\therefore u = x^2 - y^2$

$v = 2xy$

when $y = 1$

$u = x^2 - 1$

$v = 2x$

Eliminating x

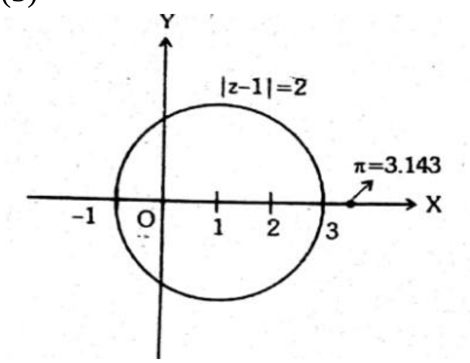
$u = \left(\frac{v}{2}\right)^2 - 1$

$\Rightarrow \left(\frac{v}{2}\right)^2 = u + 1$

$v^2 = 4(u+1)$

This is a parabola with vertex (-1, 0)

64. (3)



Let $f(z) = \frac{e^z}{z(z-\pi)}$

Poles are $z = 0, \pi$

Clearly $z = 0$ lies inside $|z - 1| = 2$

Res at $z = 0 = \lim_{z \to 0} (z - 0)\, f(z)$

$= \lim_{z \to 0} \frac{e^z}{z(z-\pi)}$

$= \frac{e^0}{0-\pi} = \frac{-1}{\pi}$

By Cauchy residue theorem

$\int_c f(z)dz = 2\pi i \times$ sum of the residues inside C

$\therefore \int_c \frac{e^z}{z(z-\pi)} dz = 2\pi i \times \left(\frac{-1}{\pi}\right) = -2i$

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