

## MODEL QUESTION PAPER – 7

### with Detailed Solutions

1. Find the eigen values of  $A = \begin{bmatrix} 3 & 0 & 0 \\ 5 & 4 & 0 \\ 3 & 6 & 1 \end{bmatrix}$
- 1) 9, 16, 1      2) 3, 4, 1  
 3) 1, 2, 3      4) 4, 5, 3
2. If the eigen value of  $A = \begin{bmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{bmatrix}$  are 2, 2, 3 then eigen values of  $A^{-1}$  are
- 1)  $\frac{1}{2}, \frac{1}{2}, \frac{1}{3}$       2) 4, 4, 9  
 3)  $\frac{1}{4}, \frac{1}{4}, \frac{1}{9}$       4) 2, 2, 3
3. If  $\lambda$  is an eigen value of any orthogonal matrix then
- 1)  $\frac{1}{\lambda^2}$  is an eigen value  
 2)  $\lambda+1$  is an eigen value  
 3) is an eigen value  
 4) None of these
4.  $A = \begin{bmatrix} 2000 \\ 0300 \\ 0010 \\ 0007 \end{bmatrix}$
- Product of eigen values of  $A =$
- 1) 13      2) 35  
 3) 8      4) 42
5. Find l and m such that  $\begin{bmatrix} 1 & -1 & 2 & 4 \\ 2 & 1 & -1 & 3 \\ 7 & -1 & l & m \end{bmatrix}$  has rank 2.
- 1) l=4; m = 18      2) l=6; m = 26  
 3) l=22; m = 11      4) l=5; m = 10
6. If  $u=x^3 + y^3 - 3axy$ , where a is positive, then the point (a, a) is
- 1) maximum      2) minimum  
 3) critical point      4) None of these
7. Solve  $P^2-3P+2=0$
- 1)  $(y-x-c_1)(y-2x-c_2) = 0$   
 2)  $(y+x+c_1)(y+2x+c_2) = 0$   
 3)  $(y+c_1)(x+c_2) = 0$   
 4)  $x+y+2xy+c_2 = 0$

8. Particular integral of  $(D^3-3D^2+4D-2)y=e^x$  is
- 1) x      2)  $x^2e^x$   
 3)  $e^x$       4)  $xe^x$
9. If  $u=\log(ax+by)$  then  $\frac{\partial^2 u}{\partial x \partial y} =$
- 1)  $\frac{a}{ax+by}$       2)  $\frac{ab}{ax+by}$   
 3)  $\frac{-ab}{(ax+by)^5}$       4)  $\frac{-ab}{(ax+by)^2}$
10. If  $u=\left(\frac{1}{r}\right)$  and  $r^2 = (x-a)^2+(y-b)^2+(z-c)^2$  where x-a, y-b and z-c are not simultaneously zero, prove that
- $$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} =$$
- 1) 0      2) 1  
 3) xyz      4)  $u^2(x+y+z)$
11. If  $u=x^{10}+y^{10}+z^{10}$  then  $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} =$
- 1) u      2)  $u^2$   
 3) 10 u      4)  $u^{10}$
12. If  $u=\frac{1}{x^2+y^2+z^2}$  then  $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} =$
- 1)  $\frac{2}{(x^2+y^2+z^2)^2}$       2)  $\frac{1}{(x^2+y^2+z^2)^2}$   
 3)  $(x^2 + y^2 + z^2)^2$       4)  $(x^2 + y^2 + z^2)^3$
13. If z is a homogeneous function then
- $$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 z}{\partial x \partial y} + y^2 \frac{\partial^2 z}{\partial y^2} =$$
- 1) nz      2) n(n+1)z  
 3) n(n-1)z      4) n(n-1)
14. Find the directional derivative of  $x^2+2xy$  at (1, -1, 3) in the direction of  $\bar{i} + 2\bar{j} + 2\bar{k}$
- 1)  $\frac{4}{3}$       2)  $\frac{7}{9}$   
 3)  $\frac{14}{9}$       4)  $\frac{14}{11}$
15. Find the unit normal to the surface  $x^2+y^2+z^2=3$  at (1, 1, 1)
- 1)  $\frac{i+j+k}{\sqrt{3}}$       2)  $\frac{i-j-2k}{\sqrt{3}}$   
 3)  $\frac{ii-2j-2k}{\sqrt{3}}$       4)  $\frac{i+j+k}{\sqrt{5}}$

16. If  $\hat{n}$  is the unit normal vector drawn outward to any closed surface  $S$  enclosing a volume  $V$ , then the value of  $\iint_S \bar{r} \cdot \hat{n} dS =$
- $V$
  - $2V$
  - $3V$
  - $4V$
17. Divergence of curl of any vector is equal to
- 1
  - 0
  - $\infty$
  - not defined
18. Obtain the directional derivative of  $\phi = xy^2 + yz^3$  at the point  $(2, -1, 1)$  in the direction of  $i + 2j + 2k$
- $\frac{10}{3}$
  - $\frac{11}{3}$
  - $-\frac{11}{3}$
  - $-\frac{10}{3}$
19. If  $\nabla\phi = 2xyz^3i + x^2z^3j + 3x^2yz^2k$  find if  $\phi$  if  $\phi(1, -2, 3) = 4$
- $xyz+3$
  - $x^2yz^3+20$
  - $xy^2z^3+30$
  - $x^3y^2z+10$
20. Find the unit normal to the surface  $x=t$ ,  $y=t^2$ ,  $z=t^3$  at  $t=1$
- $\frac{i+j+k}{\sqrt{3}}$
  - $\frac{-i-j+k}{\sqrt{3}}$
  - $\frac{i-j-k}{\sqrt{3}}$
  - $\frac{-i+j-k}{\sqrt{3}}$
21. The maximum rate of change of the temperature  $T=x+y+z$  at  $(1, 1, 1)$  is
- $\sqrt{3}$
  - $\sqrt{2}$
  - 1
  - 0
22. A vector function  $\bar{F}$  is called solenoidal if
- $\nabla \times \bar{F} = 0$
  - $\nabla \bar{F} = 0$
  - $\bar{F} = 0$
  - $\nabla \cdot \bar{F} = 0$
23.  $\iint_S \text{curl } \bar{F} \cdot \hat{n} dS$  where  $S$  is the surface of a sphere of radius 5 is
- 1
  - 0
  - 1
  - $\pi$
24.  $\iiint_V \nabla \cdot \hat{n} dv$  is equal to
- 1
  - 0
  - 1
  - $v$
25. The value of the integral  $\int_c \frac{3z^5 + 4z - 20}{z-1} dz$  where  $|z|=0.5$  is
- 0
  - $\frac{\pi i}{2}$

- 3)  $\pi I$
- 4)  $2\pi i$
26. Find the Taylor series for  $f(z)=e^z$  at  $z=0$
- $1+z+z^2+z^3+\dots$
  - $1 + \frac{1}{z} + \frac{1}{z^2} + \frac{1}{z^3} + \dots$
  - $1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$
  - $1 - \frac{z}{1} + \frac{z^2}{2} - \frac{z^3}{3} + \dots$
27. Evaluate  $\int_C e^z dz$  where  $C$  is  $|z|=10$
- 0
  - 1
  - $\pi$
  - $2\pi i$
28. If  $\int_0^{0.6} f(x) dx = k[f(0) + 4f(0.3) + f(0.6)]$  then the value of  $k$  where Simpson's rule is applied is
- 0.1
  - 0.2
  - 0.3
  - 0.4
29. The interpolation polynomial for the data
- | x | -1 | 0 | 1 | 2  |
|---|----|---|---|----|
| f | 1  | 1 | 1 | -1 |
- then  $f(x) =$
- $x^3 - x - 1$
  - $1+x+x^2$
  - $1+x-x^2$
  - $x+x^2+x^3$
30. Find  $L(t \sin 2t)$
- $\frac{4s}{s^2+4}$
  - $\frac{4s}{(s^2+4)^2}$
  - $s^2 + 4$
  - $s^2 - 4$
31. Find  $L^{-1}\left(\frac{1}{(s+1)^2}\right)$
- $\frac{t^3}{3}$
  - $e^{-t}$
  - $e^{-t}$
  - $e^{-st}t^3$
32. Let the Fourier sine transform of  $f(x)$  and  $g(x)$  be  $F_s(s)$  and  $G_s(s)$ . Then
- $$\frac{2}{\pi} \int_0^\infty F_s(s) G_s(s) ds = 1 \quad (2)$$
- $\int_0^\infty f(x) f(x) dx$
  - $\int_0^\infty [f(x) + g(x)] dx$
33. Find the z-transform of  $\frac{a^k}{k!}$ ,  $k \geq 0$
- $e^{az}$
  - $e^z$
  - $e^{z/a}$
  - $e^{a/z}$
34. From a group of 3 Indians 4 Pakistanis and 5 Americans, a subcommittee of four people is selected by lots. Find the probability that the

subcommittee will consists of 2 Indians and 2 Pakistanis

1)  $\frac{1}{495}$

3)  $\frac{4}{201}$

2)  $\frac{2}{55}$

4)  $\frac{7}{47}$

35. (z) Is the probability for the normal distribution in the Standard form. It is given that  $\int_0^{1.5} \phi(z)dz = 0.4332$ . Then the probability that the standard normal variate is greater than -1.5 is
- 1) 0.4332      2) 0.5668  
3) 0.9332      4) 0.0668

36. Which one of the following is not true?

- 1) In Poisson distribution, mean = variance  
2) In a distribution function  $\lim_{x \rightarrow -\infty} F(x) = 1$   
3) Distribution function is an increasing function  
4) In a normal distribution, mean = median

37. If the probability distribution of a random variable X is

X	0	1	2	3
P(x)	3k	1/3	2k	1/2

then the value of k is

1)  $\frac{1}{12}$

3)  $\frac{1}{25}$

2)  $\frac{1}{6}$

4)  $\frac{1}{30}$

38. Four persons are chosen at random from a group containing 3 men, 2 women and 4 children. Find the chance that exactly two of them will be children

1)  $\frac{10}{21}$

3)  $\frac{13}{21}$

2)  $\frac{20}{21}$

4)  $\frac{4}{21}$

#### ANSWERS

1. 2	2. 1	3. 3	4. 4	5. 1	6. 2	7. 1	8. 4	9. 4	10. 1
11. 3	12. 1	13. 3	14. 1	15. 1	16. 3	17. 2	18. 3	19. 2	20. 2
21. 1	22. 4	23. 2	24. 1	25. 1	26. 3	27. 1	28. 1	29. 3	30. 2
31. 2	32. 3	33. 4	34. 2	35. 3	36. 2	37. 4	38. 1		

## DETAILED SOLUTIONS

1. (2)

Characteristic equation of A

$$= |A - \lambda I|$$

$$= \begin{vmatrix} 3-\lambda & 0 & 0 \\ 5 & 4-\lambda & 0 \\ 3 & 6 & 1-\lambda \end{vmatrix} = 0$$

$$\Rightarrow (3-\lambda)(4-\lambda)(1-\lambda) = 0$$

$$\therefore \lambda = 1, 3, 4$$

2. (1)

Result:

If the eigen values of a square matrix A are  $\lambda_1, \lambda_2, \dots, \lambda_n$  then eigen values of  $A^{-1}$  are

$$\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \dots, \frac{1}{\lambda_n}$$

Given eigen values of A are 2, 2, 3

$$\therefore \text{Eigen values of } A^{-1} \text{ are } \frac{1}{2}, \frac{1}{2}, \frac{1}{3}$$

3. (3)

If  $\lambda$  is an eigen value of an orthogonal matrixA then  $\frac{1}{\lambda}$  is also an eigen value of A.

4. (4)

For a diagonal matrix the diagonal matrix, the eigen values are the diagonal elements.

Clearly A is a diagonal matrix

 $\therefore$  Eigen values are 2, 3, 1, 7Product of eigen values =  $2 \times 3 \times 1 \times 7 = 42$ 

5. (1)

$$A = \begin{bmatrix} 1 & -1 & 2 & 4 \\ 2 & 1 & -1 & 3 \\ 7 & -1 & l & m \end{bmatrix}$$

 $R_2 \rightarrow R_2 - 2R_1$  $R_3 \rightarrow R_3 - 7R_1$ 

1-12 4

0 3-5 -5

$$A \sim \begin{bmatrix} 1 & -1 & 2 & 5 \\ 0 & 3 & -5 & -5 \\ 0 & 0 & l-4m-18 & \end{bmatrix}$$

 $R_3 \rightarrow R_3 - 2R_2$ 

$$\Rightarrow A \sim \begin{bmatrix} 1 & -1 & 2 & 4 \\ 0 & 3 & -5 & -5 \\ 0 & 6 & l-14m-28 & \end{bmatrix}$$

Since rank = 2, each entries of 3rd row = 0

 $\Rightarrow 1-4=0$  and  $m-18=0$  $\Rightarrow l=4$  and  $m=18$ 

6. (2)

$$u = x^3 + y^3 - 3axy$$

$$\frac{\partial u}{\partial x} = 3x^2 - 3ay$$

$$\frac{\partial^2 u}{\partial x^2} = 6x$$

$$\frac{\partial u}{\partial y} = 3y^2 - 3ax$$

$$\frac{\partial^2 u}{\partial y^2} = 6y$$

$$\frac{\partial^2 u}{\partial x \partial y} = -3a$$

$$\frac{\partial^2 u}{\partial y^2} = 6y$$

At (a, a)

$$A = \frac{\partial^2 u}{\partial x^2} = 6a$$

$$B = \frac{\partial^2 u}{\partial x \partial y} = -3a$$

$$C = \frac{\partial^2 u}{\partial y^2} = 6a$$

Clearly  $AC-B^2 = 36a^2 - 9a^2 = 27a^2$ Also  $A=6a = \text{positive}$  [ $\because a$  is positive] $\therefore AC-B^2 > 0 \quad A > 0$ 

This implies (a, a) is minimum point.

7. (1)

$$P^2 - 3P + 2 = 0$$

$$\Rightarrow (P-2)(P-1) = 0$$

$$\therefore P = 1, 2$$

Let  $P = 1$ 

$$\Rightarrow \frac{dy}{dx} = 1 \Rightarrow dy = dx$$

$$\int dy = \int dx$$

$$\Rightarrow y = x + c_1$$

... (1)

Let  $P = 2$ 

$$\Rightarrow \frac{dy}{dx} = 2 \Rightarrow dy = 2dx$$

$$\Rightarrow \int dy = 2 \int dx$$

$$\Rightarrow y = 2x + c_2$$

... (2)

From (1) and (2) solution is

$$(y-x-c_1)(y-2x-c_2) = 0$$

8. (4)

$$\begin{aligned} P.I. &= \frac{e^x}{D^3 - 3D^2 + 4D - 2} \\ &= \frac{e^x}{(1)^3 - 3(1)^2 + 4(1) - 2} \\ &= \frac{e^x}{0} \end{aligned}$$

$$\text{Now } \frac{e^x}{D^3 - 3D^2 + 4D - 2}$$

$$\begin{aligned} &= \frac{e^x}{(D^2 - 2D + 2)(D - 1)} \\ &= x \left[ \frac{e^x}{D^2 - 2D + 2} \right] \\ &= x \left[ \frac{e^x}{(1)2 - 2(1) + 2} \right] \end{aligned}$$

$$= xe^x$$

9. (4)

$$u = \log(ax+by)$$

$$\begin{aligned} \frac{\partial u}{\partial y} &= \frac{b}{ax+by} \\ \frac{\partial^2 y}{\partial x \partial y} &= \frac{\partial}{\partial x} \left( \frac{b}{ax+by} \right) \\ &= \frac{-ab}{(ax+by)^2} \end{aligned}$$

10. (1)

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{\partial u}{\partial r} \cdot \frac{\partial r}{\partial x} \\ &= \frac{-1}{r^2} \frac{\partial r}{\partial x} \end{aligned}$$

$$\text{Now } r^2 = (x-a)^2 + (y-b)^2 + (z-c)^2$$

$$2r \frac{\partial r}{\partial x} = 2(x-a)$$

$$\begin{aligned} \therefore \frac{\partial r}{\partial x} &= \frac{x-a}{r} \\ \Rightarrow \frac{\partial u}{\partial x} &= \frac{-1}{r^2} \cdot \frac{\partial r}{\partial x} \\ &= \frac{-(x-a)}{r^3} \\ \frac{\partial^2 u}{\partial x^2} &= \frac{-r^3 + (x-a) \cdot 3r^2 \frac{\partial r}{\partial x}}{r^6} \end{aligned}$$

$$\begin{aligned} &= \frac{-r^3 + 3(x-a)r^2 \frac{(x-a)}{4}}{r^6} \\ &= \frac{3(x-a)^2 - r^2}{r^5} \end{aligned}$$

Similarly

$$\begin{aligned} \frac{\partial^2 u}{\partial y^2} &= \frac{3(y-b)^2 - r^2}{r^5} \\ \frac{\partial^2 u}{\partial z^2} &= \frac{3(z-c)^2 - r^2}{r^5} \end{aligned}$$

Therefore

$$\begin{aligned} &\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \\ &= \frac{3[(x-a)^2 + (y-b)^2 + (z-c)^2 - r^2]}{r^5} \\ &= \frac{3(r^2 - r^2)}{r^5} = 0 \end{aligned}$$

11. (3)

$u=x^{10}+y^{10}+z^{10}$  is a homogeneous function of degree 10. Therefore by Euler's theorem

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z} = 10u$$

12. (1)

$$\begin{aligned} u &= \frac{1}{x^2 + y^2 + z^2} \\ \frac{\partial u}{\partial x} &= \frac{-2x}{(x^2 + y^2 + z^2)^2} \\ \frac{\partial^2 u}{\partial x^2} &= \frac{-(x^2 + y^2 + z^2)^2 2 + 2x \cdot 2(x^2 + y^2 + z^2) 2x}{(x^2 + y^2 + z^2)^3} \\ &= \frac{-2(x^2 + y^2 + z^2)^2 + 8x^2(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)^4} \end{aligned}$$

Similarly

$$\begin{aligned} \frac{\partial^2 y}{\partial y^2} &= \frac{-2(x^2 + y^2 + z^2)^2 + 8y^2(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)^3} \\ \frac{\partial^2 u}{\partial x^2} &= \frac{-2(x^2 + y^2 + z^2)^2 + 8z^2(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)^4} \end{aligned}$$

$$\begin{aligned} & \therefore \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} \\ &= \frac{-6(x^2 + y^2 + z^2)^2 + 8(x^2 + y^2 + z^2)(x^2 + y^2 + z^2)}{(x^2 + y^2 + z^2)^4} \\ &= \frac{-6(x^2 + y^2 + z^2)^2 + 8(x^2 + y^2 + z^2)^2}{(x^2 + y^2 + z^2)^4} \\ &= \frac{2(x^2 + y^2 + z^2)^2}{(x^2 + y^2 + z^2)^4} \\ &= \frac{2}{(x^2 + y^2 + z^2)^2} \end{aligned}$$

13. (3)

By Extended Euler's theorem

$$\begin{aligned} & x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2} \\ &= n(n-1)z \end{aligned}$$

14. (1)

Let  $\phi = x^2 + 2xy$ 

$$\begin{aligned} \nabla \phi &= \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) (x^2 + 2xy) \\ &= i \frac{\partial}{\partial x} (x^2 + 2xy) + j \frac{\partial}{\partial y} (x^2 + 2xy) \\ &\quad + k \frac{\partial}{\partial z} (x^2 + 2xy) \end{aligned}$$

$$= (2x + 2y)i + (2x)j + 0k$$

$$\nabla \phi = (1, -1, 3)$$

$$= [2(1) + 2(-1)]i + 2(1)j$$

$$= 2j$$

 $\hat{e}$  = unit vector along  $i + 2j + 2k$ 

$$= \frac{i + 2j + 2k}{\sqrt{1+4+4}} = \frac{i + 2j + 2k}{3}$$

Directional derivative

$$\begin{aligned} & \nabla \phi \cdot e \\ &= 2j \left( \frac{i + 2j + 2k}{3} \right) \\ &= \frac{4}{3} \end{aligned}$$

15. (1)

Let  $\phi = x^2 + y^2 + z^2 - 3$ 

$$\begin{aligned} \nabla \phi &= \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) (x^2 + y^2 + z^2 \\ &\quad - 3) \end{aligned}$$

$$= 2xi + 2yj + 2zk$$

$$\begin{aligned} & \nabla \phi \text{ at } (1, 1, 1) \\ &= 2(1)i + 2(1)j + 2(1)k \\ &= 2i + 2j + 2k \end{aligned}$$

Unit normal to the surface

$$\begin{aligned} & = \frac{\nabla \phi}{|\nabla \phi|} \\ &= \frac{2i + 2j + 2k}{\sqrt{4+4+4}} \\ &= \frac{2i + 2j + 2k}{2\sqrt{3}} \\ &= \frac{i + j + k}{\sqrt{3}} \end{aligned}$$

16. (3)

$$\begin{aligned} \nabla \cdot \bar{r} &= \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) (xi + yj + zk) \\ &= 3 \end{aligned}$$

By Gauss Divergence theorem

$$\begin{aligned} \iint_x \bar{F} \cdot \bar{n} \, ds &= \iiint_v \nabla \cdot \bar{F} \, dv \\ &\therefore \iint_s \bar{r} \cdot \bar{n} \, ds \iiint_v \nabla \cdot \bar{r} \, dv \\ &= 3 \iiint_v \, dv \\ &3 \times \text{volume} = 3V \end{aligned}$$

17. (2)

$$\begin{aligned} \text{Divergence of curl of any vector } \bar{F} &= \nabla \cdot (\nabla \times \bar{F}) \\ &= 0 \end{aligned}$$

18. (3)

$$\begin{aligned} \phi &= xy^2 + yz^3 \\ \nabla \phi &= \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) (xy^2 + yz^3) \\ &= y^2 i + (2xy + z^3) + 3yz^2 k \\ \nabla \phi(2, -1, 1) &= (-1)^2 i + [2(2)(-1) + 1^2]j \\ &\quad + 3(-1)(1)^2 k \\ &= i - 3j - 3k \end{aligned}$$

 $\hat{e}$  = unit vector in the direction of  $i + 2j + 2k$ 

$$= \frac{i + 2j + 2k}{\sqrt{1+4+4}} = \frac{i + 2j + 2k}{3}$$

 $\therefore$  Required directional derivative

$$= \nabla \phi \cdot \hat{e}$$

$$\begin{aligned}
 &= (\mathbf{i} - 3\mathbf{j} - 3\mathbf{k}) \cdot \left( \frac{i+2j+2k}{3} \right) \\
 &= \frac{1}{3} [1 - 6 - 6] \\
 &= \frac{-11}{3}
 \end{aligned}$$

19. (2)

$$\begin{aligned}
 \nabla \phi &= i \frac{\partial \phi}{\partial x} + j \frac{\partial \phi}{\partial y} + k \frac{\partial \phi}{\partial z} \\
 &= 2xyz^3 \mathbf{i} + x^2z^3 \mathbf{j} + 3x^2yz^2 \mathbf{k} \\
 \frac{\partial \phi}{\partial x} &= 2xyz^3 \quad \dots (1) \\
 \frac{\partial \phi}{\partial y} &= x^2z^3 \quad \dots (2) \\
 \frac{\partial \phi}{\partial z} &= 3x^2yz^2 \quad \dots (3)
 \end{aligned}$$

Integrating (1), (2) and (3) with respect to x, y, z respectively we get

$$\phi(x, y, z) = x^2yz^3 + f(y, z) \quad \dots (4)$$

$$\phi(x, y, z) = x^2yz^3 + g(x, z) \quad \dots (5)$$

$$\phi(x, y, z) = x^2yz^3 + h(x, y) \quad \dots (6)$$

From (4), (5) and (6)

$$\phi(x, y, z) = x^2yz^3 + k \text{ where } k \text{ is a constant}$$

Given  $\phi(1, -2, 2) = 4$

$$\Rightarrow (1)^2(-2)(2)^3 + k = 4$$

$$\Rightarrow -16 + k = 4 \Rightarrow k = 20$$

$$\therefore \phi(x, y, z) = x^2yz^3 + 20$$

20. (2)

$$t^3 = t \cdot t^2$$

$$\Rightarrow z = xy$$

Given surface  $\phi = z - xy$

$$\begin{aligned}
 \nabla \phi &= \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) (z - xy) \\
 &= -y\mathbf{i} - x\mathbf{j} + \mathbf{k} \\
 \nabla(1, 1, 1) &= -\mathbf{i} - \mathbf{j} + \mathbf{k} \\
 \text{unit normal} &= \frac{\nabla \phi}{|\nabla \phi|} \\
 &= \frac{-i - j + k}{\sqrt{(-1)^2 + (-1)^2 + 1^2}} \\
 &= \frac{-i - j + k}{\sqrt{3}}
 \end{aligned}$$

21. (1)

$$\nabla T = \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right) (x + y + z)$$

$$\begin{aligned}
 &= i \frac{\partial}{\partial x} (x + y + z) + j \frac{\partial}{\partial y} (x + y + z) + k \frac{\partial}{\partial z} (x + y + z) \\
 &= i + j + k \\
 \nabla T(1, 1, 1) &= i + j + k \\
 \text{Maximum rate of change} \\
 |\Delta T| &= \sqrt{1^2 + 1^2 + 1^2} \\
 &= \sqrt{3}
 \end{aligned}$$

22. (4)

A vector function is called solenoidal if  
 $\operatorname{div} \bar{F} = 0$   
i.e.  $\nabla \cdot \bar{F} = 0$

23. (2)

$$\nabla \cdot (\operatorname{curl} \bar{F}) = \nabla \cdot (\nabla \times \bar{F}) = 0$$

By Gauss divergence theorem

$$\begin{aligned}
 \iint_S \bar{F} \cdot \hat{n} \, ds &= \iiint_V \nabla \cdot \bar{F} \, dv \\
 \therefore \iint_S \operatorname{curl} \bar{F} \cdot \hat{n} \, ds &= \iiint_V \nabla \cdot (\operatorname{curl} \bar{F}) \, dv \\
 &= 0
 \end{aligned}$$

24. (1)

By Gauss divergence theorem

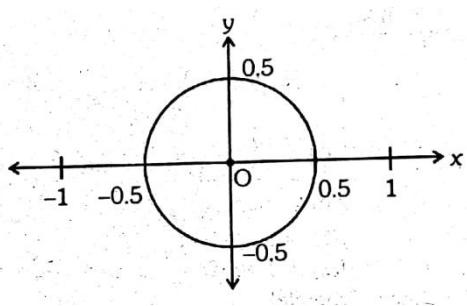
$$\begin{aligned}
 \iint_S \bar{F} \cdot \hat{n} \, ds &= \iiint_V \nabla \cdot \bar{F} \, dv \\
 \therefore \iiint_S \nabla \cdot \hat{n} \, dv &= \iint_S \hat{n} \cdot \hat{n} \, ds = S
 \end{aligned}$$

(surface of the volume V)

25. (1)

Cauchy's theorem

If a function  $f(z)$  is analytic at all points inside and on a closed contour  $C$  then  $\int_C f(z) dz = 0$



$$\text{Let } f(z) = \frac{3z^5 + 4z - 20}{z - 1}$$

Clearly  $f(z)$  is not analytic at  $z = 1$

But  $z=1$  lies outside of the circle  $|z| = 0.5$

$$\therefore f(z) = \frac{3z^5 + 4z - 20}{z - 1}$$

is analytic at all points inside and on the closed contour  $|z| = 0.5$

$\therefore$  By Cauchy's theorem

$$\int_C f(z) dz = \int_C \frac{3z^5 + 4z - 20}{z - 1} dz = 0$$

26. (3)

Taylor series expansion of  $e^z$  at  $z=0$  is

$$1 + \frac{z}{1!} + \frac{z^2}{2!} + \frac{z^3}{3!} + \dots$$

27. (1)

If  $f(z)$  is analytic inside and on the curve  $C$  then

$$\int_C f(z) dz = 0$$

Clearly  $f(z) = e^z$  is analytic everywhere. Therefore  $f(z)$  is analytic within and on  $|z| = 10$

$\Rightarrow \int_C e^z dz = 0$  by Cauchy's theorem.

28. (1)

$h = \text{interval length} = 0.3$

By Simpson's  $\frac{1}{3}$  rule

$$k = \frac{h}{3} = \frac{0.3}{3} = 0.1$$

29. (3)

Consider  $f(x) = 1+x-x^2$

$$f(-1) = 1 - 1 - (-1)^2 = -1$$

$$f(0) = 1+0-0^2 = 1$$

$$f(1) = 1 + 1 - 1 = 1$$

$$f(2) = 1+2-2^2 = -1$$

Required polynomial  
 $= 1+x-x^2$

30. (2)

Formula

If  $L[f(t)] = F(s)$  then

$$L[t f(t)] = \frac{-d}{ds} F(s)$$

Let  $f(t) = \sin 2t$

$$L(f(t)) = L(\sin 2t) = \frac{2}{s^2 + 4}$$

$$\therefore L(t \sin 2t) = \frac{-d}{ds} \left( \frac{2}{s^2 + 4} \right)$$

$$= - \left[ \frac{(s^2 + 4)0 - 2.2s}{(s^2 + 4)^2} \right]$$

$$= \frac{4s}{(s^2 + 4)^2}$$

31. (2)

Formula:

$$L^{-1}[F(s+a)] = e^{-at} L^{-1}[F(s)]$$

$$\text{Now } L^{-1}\left(\frac{1}{(s+1)^2}\right)$$

$$= e^{-t} L^{-1}\left(\frac{1}{s^2}\right)$$

$$= e^{-t} \cdot t$$

32. (3)

By Parseval's identity for Fourier sine transform

$$\frac{2}{\pi} \int_0^\infty F_s G_s(s) ds = \int_0^\infty f(x) g(x) dx$$

33. (4)

Formula:

$$Z[f(t)] = \sum_{k=-\infty}^{\infty} \frac{f(k)}{z^k}$$

$$\therefore z \left[ \frac{a^k}{k!} \right] = \sum_{k=0}^{\infty} \frac{\left(\frac{a}{z}\right)^k}{k!} (\because k \geq 0)$$

$$= 1 + \frac{\left(\frac{a}{z}\right)}{1!} + \frac{\left(\frac{a}{z}\right)^2}{2!} + \frac{\left(\frac{a}{z}\right)^3}{3!} \dots$$

$$= e^{\frac{a}{z}}$$

34. (2)

Required probability

$$\begin{aligned}
 &= \frac{\text{Number of favourable cases}}{\text{Total number of cases}} \\
 &= \frac{3C_2 \times 4C_2}{12C_4} \\
 &= \frac{\frac{3 \times 2}{1 \times 2} \times \frac{4 \times 3}{1 \times 2}}{\frac{12 \times 11 \times 10 \times 9}{1 \times 2 \times 3 \times 4}} = \frac{18}{495} = \frac{2}{55}
 \end{aligned}$$

35. (3)

$$\begin{aligned}
 P(X > -1.5) &= P(-1.5 < X < \infty) \\
 &= P(-1.5 < X < 0) + P(0 < X < \infty) \\
 &= P(0 < X < 1.5) + P(0 < X < \infty) \\
 &= 0.4332 + 0.5 \\
 &= 0.9332
 \end{aligned}$$

36. (2)

$\lim_{x \rightarrow -\infty} F(x) = 1$  is wrong  
correct answer is  $\lim_{x \rightarrow -\infty} F(x) = 0$

37. (4)

For a probability mass function  
 $= 1$

Total probability

$$\Rightarrow 3k + 1 + 2k + \frac{1}{2} = 1$$

$$\begin{aligned}
 \Rightarrow 5k + \frac{5}{6} &= 1 \\
 \Rightarrow 5k &= 1 - \frac{5}{6} = \frac{1}{6} \\
 \therefore k &= \frac{1}{30}
 \end{aligned}$$

38. (1)

Out of 4 children two children can be selected in  $4C^2$  ways

The remaining, two persons can be chosen from 5 persons (3 men + 2 women) in  $5C_2$  ways.

$\therefore$  Required probability

$$\begin{aligned}
 &= \frac{4C_2 \times 5C_2}{9C_4} \\
 &= \frac{\frac{4 \times 3}{1 \times 2} \times \frac{5 \times 4}{1 \times 2}}{\frac{9 \times 8 \times 7 \times 6}{1 \times 2 \times 3 \times 4}} \\
 &= \frac{10}{21}
 \end{aligned}$$

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