MODEL QUESTION PAPER – 5
with Detailed Solutions

1. Find the sum of the eigen values of $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$
   1) -1 2) -2
   3) -3 4) 5
2. Two eigen values of a 3×3 non singular matrix A are 2, 3 and |A| =36. Then the third eigen value
   1) 5 2) 6
   3) 7 4) 8
3. If some of the eigen values of a matrix A of the quadratic form are positive and others are negative then the quadratic form is
   1)positive definite
   2)indefinite
   3)positive semidefinitive
   4)negative semidefinitive
4. Final k so. that the rank of the matrix $\begin{bmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ 3 & 5 & k \end{bmatrix}$ is 2
   1) 2 2) 4
   3) 4 5) 1
5. Find the nature of the quadratic form $6x^2+3y^2+14z^2+4yz+18xz+4xy$
   1) positive definite
   2) indefinite
   3) negative definite
   4) positive semidefinite
6. If u= $\frac{1}{r}$; $r^2=x^2+y^2+z^2$ then $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}+\frac{\partial^2 u}{\partial z^2} =$
   1) 0 2) 1
   3) 2 4) 3
7. The function f(x, y) has a minimum at (a, b) if $\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} = 0$ and
   1) $\frac{\partial^2 f}{\partial x^2}.\frac{\partial^2 f}{\partial y^2} - \frac{\partial^2 f}{\partial x \partial y} < 0$
   2) $\frac{\partial^2 f}{\partial x^2}.\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 < 0$
   3) $\frac{\partial^2 f}{\partial x^2}.\frac{\partial^2 f}{\partial y^2} - \frac{\partial^2 f}{\partial x \partial y} < 0$ *and* $\frac{\partial^2 f}{\partial x^2} > 0$
   4) $\frac{\partial^2 f}{\partial x^2}.\frac{\partial^2 f}{\partial y^2} - \frac{\partial^2 f}{\partial x \partial y} < 0$ *and* $\frac{\partial^2 f}{\partial x^2} > 0$
8. Solve: yq-xp=z
   1) $f(x^2,y^2)=0$ 2) $f\left(xy, \frac{y}{z}\right) = 0$
   3) $f(x^2+y^2, p^2)=0$ 4) None of these
9. Eliminating x from $\frac{dx}{dt} + 2y = 0$, $\frac{dy}{dt} - 2x = 0$ is
   1) $\frac{d^2y}{dt^2} + 4y = 0$ 2) $\frac{d^2y}{dt^2} - 4y = 0$
   3) $\frac{dy}{dt} + 4yx^2 = 0$ 4) $\frac{dy}{dt} + x + y = 0$
10. The equation $(e^y + 1)\cos x\, dx+e^y \sin x dy = 0$ is
    1) exact 2) variable separable
    3) linear 4)solvable for x
11. The vector $\vec{F}$ = yzi + 2xj +xyk is
    1) irrotational 2) solenoidal
    3) 0 4)1
12. If r =xi+yj+zk and r = $|\vec{r}|$ then ∇r^5 =
    1) $5r^3\vec{r}$ 2) $4r^2\vec{r}$
    3) $5r^4\vec{r}$ 4) $6r^5\vec{r}$
13. If A and B are irrotationalthen $\vec{A} \times \vec{B}$ is
    1) solenoidal 2) irrotational
    3) 1 4)0
14. Find the directional derivative of $\phi=x^2+y^2+z^2$ at the point (2, 2, 1) in the direction of 2i+2j+k.
    1) 3 2) 6
    3) 9 4) 12
15. Evaluate $\int_c \frac{\cos \pi z^2}{(z-1)(z-2)} dz$ where C is thecircle $|z| = \frac{3}{2}$
    1) 2π i 2) π i
    3) -3π i 4) 0
16. Find the residue of f(z)=cot z at the pole z=0
    1) 10 2) 3
    3) 5 4) 1
17. Polar form of Cauchy-Riemann equation of f(z) = u+ivis

1) $\frac{\partial u}{\partial r} = \frac{1}{r}.\frac{\partial v}{\partial \theta}$ $\frac{\partial v}{\partial r} = \frac{-1}{r}\frac{\partial u}{\partial \theta}$
2) $\frac{\partial u}{\partial r} = r\frac{\partial v}{\partial \theta}$ $\frac{\partial v}{\partial r} = -r\frac{\partial u}{\partial \theta}$
3) $\frac{\partial u}{\partial \theta} = \frac{1}{r^2}\frac{\partial u}{\partial r}$ $\frac{\partial v}{\partial \theta} = \frac{1}{r^2}\frac{\partial u}{\partial r}$
4) $\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial r}$ $\frac{\partial v}{\partial \theta} = \frac{1}{r}\frac{\partial v}{\partial \theta}$

18. The function f(z)= $\bar{r}$ is
1)analytic
2)nowhere differentiable
3)analytic at 0
4) none of these

19. Find the residue of f(z)= $\frac{z}{z^2+4}$ at z =2i
1) 2
2) 1
3) 0
4) $\frac{1}{2}$

20. Error in the trapezoidal rule is of the order
1) h
2) h^2
3) h^3
4) $\frac{1}{2}$

21. Simpson's $\frac{1}{3}$ rule is called
1) open formula
2) regular formula
3) indefinite formula
4) closed formula

22. In Simpson's $\frac{1}{3}$ rule the interval must bedivided into an _____number of subintervals of width h.
1) odd
2) 5
3) even
4) $\frac{1}{3}$

23. Find the interpolation formula for

x	0	1	2	5
f(x)	2	3	12	147

1) x^3+x^2+X+2
2) x^2+x+2
3) x^3+x^2-x+2
4) $x^4+x^3-x^2+x-2$

24. By Simpson's $\frac{1}{3}$ rule to get a close approximation h will be
1) very large
2) 0
3) 1
4) very small

25. Trapezoidal rule and Simpson's rule are ued to find
1)Numerical integration
2)Numerical partial differentiation
3)Curve fitting
4)Interploting polynomial

26. What is the chance that a leap year selected at random will contain 53 Sundays
1) $\frac{1}{7}$
2) $\frac{4}{7}$
3) $\frac{2}{7}$
4) $\frac{1}{365}$

27. A husband and wife appear In an Interview for two vacancies in the same post. The probability of husband selection is$\frac{1}{7}$ and wife's selection is$\frac{1}{5}$. Find the probability of only one of them will be selected.
1) $\frac{2}{7}$
2) $\frac{1}{7}$
3) $\frac{6}{7}$
4) $\frac{4}{5}$

28. If A and B are mutually exclusive events, then P(A∪B)=
1) $P\left(\frac{A}{B}\right)$
2) $P\left(\frac{B}{A}\right)$
3) P(A) + P(B)
4) None of these

29. P(A+B) = $\frac{3}{4}$; P(AB)=$\frac{1}{4}$ and P($\bar{A}$) = $\frac{2}{3}$ Find P(B)
1) $\frac{1}{3}$
2) $\frac{2}{3}$
3) $\frac{3}{4}$
4) $\frac{4}{5}$

30. A book contains 100 misprints distributed randomly throughout its 100 pages. Assuming Poisson distribution of the number of misprints in a page, find the probability that a page observed at random contains atleast 2 misprints.
1) $1 + \frac{2}{e}$
2) $1 - \frac{2}{e^2}$
3) $1 - \frac{2}{e}$
4) $1 + \frac{2}{e}$

ANSWERS

1. 1	2. 2	3. 2	4. 4	5. 2	6. 1	7. 3	8. 2	9. 1	10. 1
11. 1	12. 1	13. 1	14. 2	15. 1	16. 4	17. 1	18. 2	19. 4	20. 2
21. 1	22. 3	23. 3	24. 4	25. 1	26. 3	27. 1	28. 3	29. 2	30. 3

DETAILED SOLUTIONS

1. (a)
Sum of the eigen values = trace of A
= sum of the main diagonal elements of A
= -2 + 1+0
= -1

2. (2)
Product of eigen values of a matrix A = |A|
Let a be the third eigen value
then 2×3×α = 36
$$\Rightarrow \alpha = \frac{36}{6} = 6$$

3. (2)
If some eigen values are positive and some eigen value are negative then the quadratic form is called indefinite.

4. (4)
If rank of 3×3 matrix is 2 then |A| =0
i.e. $\begin{vmatrix} 2 & 1 & -1 \\ 1 & 4 & 2 \\ 3 & 5 & k \end{vmatrix} = 0$
⇒2(4k-10)-l(k-6)-l(5-12)=0
= 8k-20-k+6-5+12=0
= 7k-7=0 ⇒ k = 1

5. (2)
Matrix of the quadratic form A=$\begin{bmatrix} 6 & 2 & 9 \\ 2 & 3 & 2 \\ 9 & 2 & 14 \end{bmatrix}$
Consider the principal sub determinants
D_1 = |6| = 6 > 0
D_2 = $\begin{vmatrix} 6 & 2 \\ 2 & 3 \end{vmatrix}$= 18-4= 14>0
= 6(42-4)-2(28-18)+9(4-27)
= 218-20-207
= -9<0
D_1>0, D_3<0 and D_2>0
Hence the quadratic form is indefinite.

6. (1)
f(r) = u = $\frac{1}{r}$
f'(r) = $-\frac{1}{r^2}$
f"(r) = $\frac{2}{r^3}$

Formula:
$$\frac{\partial^2 y}{\partial x^2} + \frac{\partial^2 y}{\partial y^2} + \frac{\partial^2 u}{\partial z^2} = f''(r) + \frac{2f'(r)}{r}$$
$$= \frac{2}{r^3} + \frac{2}{r}\left(\frac{-1}{r^2}\right)$$
$$= \frac{2}{r^3} - \frac{2}{r^3} = 0$$

7. (2)
The function f(x, y) has a minimum at (a, b) if
$\frac{\partial f}{\partial x} = 0, \frac{\partial f}{\partial y} > 0 \ and\ \frac{\partial^2 f}{\partial x^2} \cdot \frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \, \partial y}\right)^2 > 0$
and $\frac{\partial^2 f}{\partial x^2} > 0$

8. (2)
Formula:
-xp+yq = z
$$\Rightarrow \frac{dx}{-x} = \frac{dy}{y} = \frac{dz}{z}$$
From first two equations
$$\frac{dx}{-x} = \frac{dy}{y} = \frac{dz}{z}$$
$$\Rightarrow -\int \frac{dx}{x} = \int \frac{dy}{y}$$
⇒log x = log y-log a
⇒xy = a
From last two equations
$$\frac{dy}{y} = \frac{dz}{z}$$
$$\Rightarrow \int \frac{dy}{y} = \int \frac{dz}{z}$$
⇒log y = log z+log b
$$\Rightarrow \frac{y}{z} = b$$
The solution is f$\left(xy, \frac{y}{z}\right)$ = 0

9. (1)
$\frac{dy}{dt}$- 2x = 0
$$\Rightarrow x = \frac{1}{2}\frac{dy}{dt}$$
Now $\frac{dx}{dt} + 2y = 0$
$$\Rightarrow \frac{d}{dt}\left(\frac{1}{2} \cdot \frac{dy}{dt}\right) + 2y = 0$$

$$\Rightarrow \frac{1}{2}\frac{d^2y}{dt^2} + 2y = 0$$

$$\Rightarrow \frac{d^2y}{dt^2} + 4y = 0$$

10. (1)

Compare with Mdx+Ndy

$M=(e^y+1)\cos x$; $N=e^y \sin x$

$$\frac{\partial M}{\partial y} = e^y \cos x = \frac{\partial N}{\partial x} = e^y \cos x$$

clearly $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

∴Given equation is exact

11. (1)

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ yz & zx & xy \end{vmatrix}$$

i(x-x)-j(y-y)+k(z-z) = 0

∴$\vec{F}$ is irrotational

12. (1)

Formula:

$\nabla r^n = nr^{n-2}\vec{r}$

$\nabla r^5 - 5r^3\vec{r}$

13. (1)

$\vec{A}$ and $\vec{B}$ are irrotational

$\Rightarrow \nabla\times\vec{A} = 0$ and $\nabla\times\vec{B} = 0$

Now $\nabla.(A\times B) = \vec{B}.(\nabla\times\vec{A})-\vec{A}(\nabla\times\vec{B}) = 0$

∴$\vec{A}\times\vec{B}$ is solenoidal.

14. (2)

$\nabla\phi=\left(i\frac{\partial}{\partial x} + j\frac{\partial}{\partial j} + k\frac{\partial}{\partial z}\right)(x^2+y^2+z^2)$

$= i\frac{\partial}{\partial x}(x^2 + y^2 + z^2) + j\frac{\partial}{\partial y}(x^2+y^2+z^2) + k\frac{\partial}{\partial z}(x^2+y^2+z^2)$

= 2xi+2yj+2zk

$\nabla\phi$ (2, 2, 1) = 2(2)i+2(2)j+2(1)k = 4i+4j+2k

ê = unit vector in the direction of 2i+2j + k

$$= \frac{2i + 2j + k}{\sqrt{4 + 4 + 1}}$$

$$= \frac{2i + 2j + k}{3}$$

Directional derivative of ϕ in the direction 2i+2j+2k

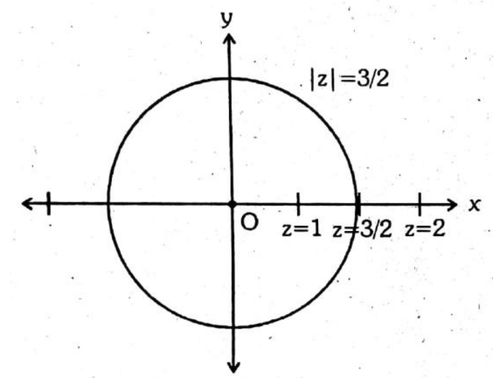
$= \nabla\phi$.ê

$= (4i + 4j + 2k).\left(\frac{2i+2j+k}{3}\right)$

$$= \frac{1}{3}(8 + 8 + 2) = \frac{18}{3}$$

= 6

15. (1)



Cauchy's Residue theorem:

If f(z) is analytic at all points inside and on a simple closed curve C except at a finite number of points $z_1, \ldots, z_n$ within C, then

$\int_c$ f(z)dz = 2 7πi × sum of the residue of f(z) at $z_1, \ldots, z_n$

Let f(z)dz = $\frac{\cos \pi z^2}{(z-1)(z-2)}$

poles = z= 1, 2

z=1 lies inside |z| = $\frac{3}{2}$

Residue of f(z) at z = 1

$$= \lim_{z \to 1}(z-1)f(z)$$

$$= \lim_{z \to 1}(z-1)\frac{\cos \pi z^2}{(z-1)(z-2)} = \frac{\cos \pi (1)^2}{1-2}$$

$$= \frac{-1}{-1} = 1$$

$\int_c$ f(z)dz = 2πi× sum of residues

= 2πi× 1 = 2πi

16. (4)

Formula:

Iff(z)=$\frac{\phi(z)}{\psi(z)}$ where

$\psi(a)=0$ but $\phi(a)\neq 0$

then Residue at z=a is $\frac{\phi(a)}{\psi(a)}$

Let f(z) = cot z = $\frac{\cos z}{\sin z} = \frac{\phi(z)}{\psi(z)}$

ψ(z) = sin z

ψ'(z) = cos z

Now ψ(0) = sin 0 = 0

(0) = cos 0 = 1≠ 0

∴Residue of f(z) at z=0

$$= \frac{\phi(0)}{\psi'(0)} = \frac{1}{\cos 0}$$

$$= \frac{1}{1} = 1$$

17. (1)

Polar form of Cauchy-Rieman equation is

$$\frac{\partial u}{\partial r} = \frac{1}{r} \cdot \frac{\partial v}{\partial \theta}$$

$$\frac{\partial u}{\partial r} = \frac{-1}{r} \cdot \frac{\partial v}{\partial \theta}$$

18. (2)

f(z) = $\bar{z} = \overline{x + iy}$ = x - iy

∴ u=x; v=-y

$$\frac{\partial v}{\partial x} = 0; \frac{\partial v}{\partial y} = -1$$

Clearly $\frac{\partial u}{\partial x} \neq \frac{\partial v}{\partial y}$

∴Cauchy - Riemann equation is not Satisfied.

So f(z) = $\bar{z}$is nowhere analytic.

19. (4)

Formula:

$$f(z) = \frac{z}{z^2 + 4} = \frac{z}{(z + 2i)(z - 2i)}$$

Residue at z=a is $\lim_{z \to a} (z - a)f(z)$

∴Residue at z=2i is $\lim_{z \to a} (z - 2i)\frac{z}{(z+2i)(z-2i)}$

$$= \frac{2i}{2i + 2i} = \frac{2i}{4i} = \frac{1}{2}$$

20. (2)

Error in the trapezoidal rule is of the order h^2.

21. (1)

Simpson's $\frac{1}{3}$ rule is called a closed formula. Since the end point indicate also enter the formula.

22. (3)

In Simpson's $\frac{1}{3}$ rule the interval must be divided into an even number of subintervals of width h.

23. (3)

Consider $f(x)=x^3+x^2-x+2$

f(0) = 0 + 0-0 + 2 = 2

f(1) = 1 + 1-1+2 = 3

f(2) = $2^3+2^2-2+2 = 12$

f(5) = $5^3+5^2-5 + 2 = 147$

∴ Required polynomial is f(x) = x^3+x^2-x+2

24. (4)

The smaller h is, the closer will be the approximation in Simpson's $\frac{1}{3}$ rule.

25. (1)

Trapezoidal and Simpson's rule are used in numerical integration.

26. (3)

A leap year contains 366 days. In 366 days there are 52 full weeks (hence 52 Sundays definitely) and

2 more days.

Those 2 days will be

i) Monday andTuesday

ii) Tuesday and Wednesday

iii) Wednesday and Thursday

iv) Thursday and Friday

v)Friday and Saturday

vi)Saturday and Sunday

vii) Sunday and Monday

Out of these 7 cases (vi) and (vii) are two favourable cases.

∴ Required probability = $\frac{2}{7}$

27. (1)

Let A and B be the event that the husband and wife selection.

P(A) =$\frac{1}{7}$and P(B) = $\frac{1}{5}$

P($\bar{A}$) = 1 - $\frac{1}{7} = \frac{6}{7}$

P($\bar{B}$) = 1 - $\frac{1}{5} = \frac{4}{5}$

Required probability

= P(A)P($\bar{B}$) + P($\bar{A}$)P(B)

$$=\frac{1}{7}\cdot\frac{4}{5}+\frac{6}{7}\cdot\frac{1}{5}$$

$$=\frac{4}{35}+\frac{6}{35}=\frac{10}{35}=\frac{2}{7}$$

28. (3)

If A and B are mutually exclusive events, then

P(A∪B) = P(A) + P(B)

29. (2)

P(A) = l-P($\bar{A}$) = l-$\frac{2}{3}$ = $\frac{1}{3}$

Now P(A+B) = P(A) + P(B)-P(AB)

$$\Rightarrow\frac{3}{4}=\frac{1}{3}+P(B)-\frac{1}{4}$$

$$\therefore P(B)=\frac{3}{4}-\frac{1}{3}+\frac{1}{4}=\frac{2}{3}$$

30. (3)

$$\lambda=\frac{100}{100}=1$$

P(X=x) = $e^{-\lambda}\frac{\lambda^x}{x!}$

P(X≥2) = 1-(P(X=0)+P(x=l)

$$=1-\left[\frac{e^{-\lambda}\lambda^0}{0!}+\frac{e^{-\lambda}\lambda^1}{1!}\right]$$

= 1 – [$e^{-\lambda}$ + $\lambda e^{-\lambda}$]

= 1 – e^{-1} (1+1)

$$=1-\frac{2}{e}$$