

MODEL QUESTION PAPER – 3

with Detailed Solutions

1. A and B are symmetric. If AB is also symmetric if and only if

- 1) AB=BA 2) AB+BA=0
3) A-B=0 4) A²+B²=0

2. Every diagonal element of a Hermitian matrix is

- 1) purely imaginary 2) purely real
3) 0 4) 1

3. A square matrix A is nilpotent of order 5 then

- 1) A¹⁰=0 2) A⁶=0
3) A²=0 4) A⁵=0

4. If at least one eigenvalue is zero and the remaining are positive, then the quadratic form is

- 1) positive definite
2) negative definite
3) positive semidefinite
4) negative semidefinite

5. Rank of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$

- 1) 0 2) 1
3) 2 4) 3

6. Let u=2(x-y)²-x⁴-y⁴. At ($\sqrt{2}, -\sqrt{2}$), u attains

- 1) maximum
2) minimum
3) neither maximum nor minimum
4) none of these

7. $u = e^{x^3+y^3}$, then $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$

- 1) log u 2) e^u
3) 3u log u 4) 3ue^u

8. Particular integral of $(D^2 - 4D + 3)y = \cos 2x$ is

- 1) $\frac{8 \sin 2x + \cos 2x}{-65}$ 2) $\frac{\sin x + \cos 2x}{64}$
3) $\frac{8 \sin x + 8 \cos 3x}{65}$ 4) $\frac{8 \sin 2x - \cos x}{-65}$

9. The condition for the function $z=f(x,y)$ to have a extreme value at (a,b) is

$$\frac{\partial z}{\partial x} = 0 \text{ and } \frac{\partial z}{\partial y} = 0$$

$$A = \frac{\partial^2 z}{\partial^2 x^2}; B = \frac{\partial^2 z}{\partial x \partial y}$$

$$C = \frac{\partial^2 z}{\partial y^2}; \Delta = AC - B^2$$

Then the function z has a minimum value at (a,b) if

- 1) $\Delta < 0, \Delta < 0$ 2) $\Delta < 0, A > 0$
3) $\Delta > 0, A > 0$ 4) $\Delta > 0, A < 0$

10. If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x+y}}\right)$ then $x = \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} =$

- 1) $\frac{\tan u}{2}$ 2) $\tan u$
3) $\frac{\sin u}{2}$ 4) $\frac{\cos u}{2}$

11. If $\bar{F} = (axy-z^2)i + (x^2+2yz)j + (y^2-axz)k$ is irrotational then a =

- 1) 0 2) 1
3) 2 4) 3

12. $\int_C \vec{r} \cdot \vec{dr} =$

- 1) 0 2) 1
3) 2 4) 3

13. $\frac{1}{2} \int_C x dy - y dx =$

- 1) the volume enclosed by the curve C
2) 0
3) 1
4) the area bounded by a simple closed curve

14. Find $\int_C \vec{F} \cdot \vec{dr}$ where $\vec{F} = x^2 i + xy j$ taken round the square in the xy plane whose sides are x=0, y=0, y=a is

- 1) $\frac{a^3}{8}$ 2) $\frac{a^3}{2}$
3) $\frac{a^3}{4}$ 4) $\frac{a^3}{5}$

15. Evaluate $\int_C \frac{\sin 3z}{z + \frac{\pi}{2}} dz$ where C is the circle $|z|=5$

- 1) $2\pi i$ 2) πi
3) $34i$ 4) $-2\pi i$

16. Find the residue of $\frac{1-e^{2z}}{z^4}$

- 1) $\frac{-4}{3}$ 2) $\frac{2}{3}$

- 3) $\frac{1}{3}$ 4) 1
17. Find the image of $|z-3| = 3$ under the mapping $w = \frac{1}{z}$
- 1) $u+v=0$ 2) $6v+l=0$
 3) $v+3=0$ 4) $6u+v=0$
18. One of the poles of $\frac{\cot \pi z}{(z-6)^5}$ is
- 1) 5 2) $\frac{1}{2i}$
 3) $\pm \frac{3}{7i}$ 4) $\pm \frac{4i}{9}$
19. The mapping $w = z^2 + z + 1$ is
- 1) conformal everywhere
 2) not conformal at $z = -\frac{1}{2}$
 3) not conformal at $z = 1$
 20. 4) not conformal at $z = 0$
20. If $L[f(t)] = F(s)$ then $L[tf(t)] =$
- 1) $\frac{-d}{ds} F(s)$ 2) $\frac{d}{ds} F(s)$
 3) $\frac{d}{ds} F(s+a)$ 4) $\int_0^s F(s) ds$
21. Find $L^{-1}\left[\frac{s}{(s+2)^2+4}\right]$
- 1) $\cos 2t + \sin 2t$ 2) $e^{-2t} \cos 2t \sin 2t$
 3) $e^{-2t}, (\cos 2t - \sin 2t)$ 4) None of these
22. Given $e^0 = 1$, $e^1 = 2.72$, $e^2 = 7.39$, $e^3 = 20.09$, $e^4 = 54.60$. Find the approximate value of $\int_0^4 e^x dx$.
- 1) 53.8733 2) 43.8523
 3) 24.1321 4) 68.3348
23. If $L[f(t)] = F(s)$ then $L[e^{-at}f(t)] =$
- 1) $F(sa)$ 2) $F(s)$
 3) $F\left(\frac{s}{a}\right)$ 4) $F(s+a)$
24. Find $L^{-1}\left[\frac{1}{(s+1)(s+3)}\right]$

- 1) $\frac{e^t + e^{3t}}{2}$ 2) $\frac{e^{-t} - e^{-4t}}{2}$
 3) $\frac{e^t + e^{3t}}{5}$ 4) $\frac{e^{-t} - e^{-3t}}{2}$
25. $E(X^2) = 276$ and variance = 20 then $E(X)$ is
- 1) 0 2) 16
 3) 20 4) 256
26. Assuming that the probability of a child being a male and being a female are the same and the probability that in a family of n children ($n > 1$) are females is $\frac{5}{2^9}$. What is the value of n ?
- 1) 9 2) 11
 3) 10 4) 8
27. The moment generating function of $p(x) = \frac{1}{2x}$; $x = 1, 2$
- 1) e^t 2) e^{-t}
 3) $\frac{e^t}{2-e^t}$ 4) $\frac{e^t}{2+t}$
28. A bag contains 7 white, 6 red and 5 black balls. Two balls are drawn at random. Find the probability that they will both be white.
- 1) $\frac{6}{23}$ 2) $\frac{4}{53}$
 3) $\frac{7}{47}$ 4) $\frac{7}{51}$
29. Mean and variance of a Poisson distribution is
- 1) Mean = λ ; Variance = λ^2
 2) Mean = 1; Variance = λ
 3) Mean = λ ; Variance = λ
 4) Mean = λ ; Variance $\neq \lambda$
30. 10 coins are thrown simultaneously. Find the probability of getting at least 7 heads.
- 1) $\frac{11}{2^6}$ 2) $\frac{13}{2^7}$
 3) $\frac{11}{2^7}$ 4) $\frac{11}{25}$

ANSWERS

1. 1	2. 2	3. 4	4. 3	5. 2	6. 1	7. 3	8. 1	9. 3	10. 1
11. 3	12. 1	13. 4	14. 2	15. 1	16. 1	17. 2	18. 1	19. 2	20. 1
21. 3	22. 1	23. 4	24. 4	25. 2	26. 3	27. 3	28. 4	29. 3	30. 1

DETAILED SOLUTIONS

1. (a)

If A and B are symmetric then A is also symmetric if and only if $AB = BA$.

2. (2)

Every diagonal element of a Hermitian matrix is purely real.

3. (4)

A square matrix A is nilpotent of order 5 then $A^5 = 0$

4. (3)

If at least one eigenvalue is zero and the remaining eigenvalues are positive then the quadratic form is positive semidefinite.

5. (2)

Clearly $|A|=0$

Also any determinant of submatrix of order 2 = 0

Also A is not a zero matrix

\therefore Rank of $A=1$

6. (1)

$$u=2(x-y)^2-x^4-y^4$$

$$\frac{\partial u}{\partial x}=4(x-y)-4x^3$$

$$\frac{\partial^2 u}{\partial x^2}=4-12x^2$$

$$\frac{\partial u}{\partial y}=-4(x-y)-4y^3$$

$$\frac{\partial^2 u}{\partial y^2}=-4-12y^2$$

$$\frac{\partial^2 u}{\partial x \partial y}=-4$$

$$A=\frac{\partial^2 u}{\partial x^2}(\sqrt{2}, -\sqrt{2})=4-12(\sqrt{2})^2=-20$$

$$B=\frac{\partial^2 u}{\partial x \partial y}(\sqrt{2}, -\sqrt{2})=-4$$

$$C=\frac{\partial^2 u}{\partial y^2}(\sqrt{2}, -\sqrt{2})=4-12(-\sqrt{2})^2=-20$$

$$\text{Now } AC-B^2=400-16=384>0$$

$$\text{Also } A=-20<0$$

\therefore At $(\sqrt{2}, -\sqrt{2})$, u attains maximum

7. (3)

$$u=e^{x^3+y^3}$$

$$\text{Let } f=\log u=\log e^{x^3+y^3}=x^3+y^3$$

$\therefore f$ is a homogeneous function of degree 3.

By Euler's theorem

$$x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = 3f$$

$$\text{i.e., } x \frac{\partial}{\partial x}(\log u) + y \frac{\partial(\log u)}{\partial y} = 3 \log u$$

$$\Rightarrow \frac{x}{u} \cdot \frac{\partial u}{\partial x} + \frac{y}{u} \cdot \frac{\partial u}{\partial y} = 3 \log u$$

$$\therefore x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 3u \log u$$

8. (1)

$$\text{P.I.} = \frac{\cos 2x}{d^2 - 4D + 3}$$

$$= \frac{\cos 2x}{-(2)^2 - 4D + 3} = \frac{\cos 2x}{-4 - 4D + 3}$$

$$= \frac{\cos 2x}{-4D - 1} = \frac{-\cos 2x}{4D + 1}$$

$$= \frac{-\cos 2x}{4D + 1} \times \frac{4D - 1}{4D - 1}$$

$$= \frac{-\cos 2x(4D - 1)}{16D^2 - 1}$$

$$= \frac{-4D(\cos 2x) + \cos 2x}{-16(2)^2 - 1}$$

$$= \frac{8 \sin 2x + \cos 2x}{-65}$$

9. (3)

The function $z=f(x, y)$ has a minimum at (a, b) if

$$AC-B^2 > 0, A > 0$$

$$\text{i.e., } \Delta > 0, A > 0$$

10. (1)

$$\text{Let } f=\sin u=\frac{x+y}{\sqrt{x}+\sqrt{y}}$$

Clearly f is a homogeneous function of degree $\frac{1}{2}$

$$\text{By Euler's theorem } x \frac{\partial f}{\partial x} + y \frac{\partial f}{\partial y} = \frac{1}{2} f$$

$$= x \frac{\partial}{\partial x} (\sin u) + y \frac{\partial}{\partial y} (\sin u) = \frac{\sin u}{2}$$

$$\Rightarrow x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{\tan u}{2}$$

11. (3)

A vector \vec{F} is irrotational if $\nabla \times \vec{F} = 0$

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ axy - z^2 & x^2 - 2yz & y^2 - axz \end{vmatrix} = 0$$

$$\Rightarrow i(2y-2y) - j(-az+2z) + k(2x-ax) = 0$$

Coefficient of i, j, k=0

$$\Rightarrow -az+2z=0$$

$$\Rightarrow a=2$$

12. (1)

$$\nabla \times \vec{r} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x & y & z \end{vmatrix}$$

$$= 0$$

By Stokes theorem

$$\int_c \vec{r} \cdot d\vec{r} = \iint_s (\nabla \times \vec{r}) \cdot \hat{n} dS$$

$$= 0$$

13. (4)

By Green's theorem deductions

The area bounded by a simple closed curve C

$$= \frac{1}{2} \int_s x dy - y dx$$

14. (2)

$$\nabla \times \vec{F} = \begin{vmatrix} i & j & k \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ x^2 & xy & 0 \end{vmatrix}$$

$$= y \hat{k}$$

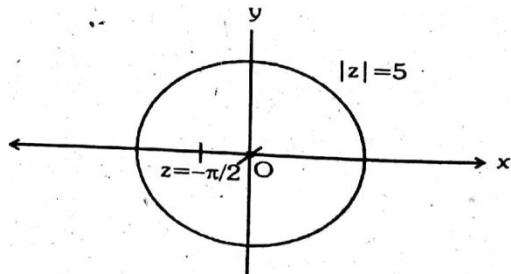
By Stokes theorem

$$\int_c \int_0^a (y \hat{k}) \cdot \hat{k} dx dy$$

[\because the given square is in the xy plane. \hat{n} = perpendicular unit normal to xy $\therefore \hat{n} = \hat{k}$]

$$\begin{aligned} & \int_c^a \int_0^a y dx dy \\ &= \int_0^a [yx]_0^a dy = a \int_0^a y dy \\ &= a \left[\frac{y^2}{2} \right]_0^a = a \left(\frac{a^2}{2} \right) = \frac{a^3}{2} \end{aligned}$$

15. (1)



Cauchy's integral formula

If f(z) is analytic inside and on a simple closed curve C

and z_0 is any point inside C

then

$$f(z_0) = \frac{1}{2\pi i} \int_C \frac{f(z) dz}{z - z_0}$$

Let f(z) = sin 3z

$$z_0 = \frac{-\pi}{2} = \frac{-3.14}{2}$$

$$= -1.57$$

$$\text{Clearly } z_0 = \frac{-\pi}{2} = -1.57$$

lies inside |z|=5

 \therefore By Cauchy's integral formula

$$\begin{aligned} & \int_C \frac{f(z)}{z - z_0} dz = \int_C \frac{\sin 3z}{z + \frac{\pi}{2}} dz \\ &= 2\pi i f\left(-\frac{\pi}{2}\right) \\ &= 2\pi i \times \sin 3\left(\frac{-\pi}{2}\right) \\ &= 2\pi i \times \sin\left(\frac{-3\pi}{2}\right) \\ &= -2\pi i \times \sin\left(\frac{3\pi}{2}\right) \\ &= -2\pi i(-1) \\ &= 2\pi i \end{aligned}$$

16. (1)

Formula:

Residue at a pole of order m is

$$= \lim_{z \rightarrow a} \frac{1}{(m-1)!} \frac{d^{m-1}}{dz^{m-1}} ((z-a)^m f(z))$$

$\ln \frac{1-e^{2z}}{z^4}$, $z=0$ is a pole of order 4

\therefore Residue at $z=0$ = 11

$$\begin{aligned} &= \lim_{z \rightarrow a} \frac{1}{(4-1)!} \frac{d^{4-1}}{dz^{4-1}} \left((z - 0)^4 \left(\frac{1 - e^{2z}}{z^4} \right) \right) \\ &= \lim_{z \rightarrow a} \frac{1}{3!} \frac{d^3}{dz^3} (1 - e^{2z}) \\ &= \frac{1}{6} = \lim_{z \rightarrow 0} (-8e^{2z}) \\ &= \frac{-4}{3} \end{aligned}$$

17. (2)

$$\text{Given } w = \frac{1}{2} \Rightarrow z = \frac{1}{w}$$

$$\therefore |z - 3i| = 3 \Rightarrow \left| \frac{1}{w} - 3i \right| = 3$$

$$\Rightarrow |1-3iw| = 3|w|$$

$$\Rightarrow |1-3i(u+iv)| = 3|u+iv|$$

$$\Rightarrow |1+3v-3iu| = 3|u+iv|$$

$$\Rightarrow (1+3v)^2 + 9u^2 = 9(u^2 + v^2)$$

$$\Rightarrow 1+9v^2+6v+9u^2=9u^2+9v^2$$

$$\Rightarrow 1+6v=0$$

18. (1)

$$\text{Let } f(z) = \frac{\cot \pi z}{(z-6)^5} = \frac{\cos \pi z}{(z-6)^2 (\sin \pi z)}$$

$$(z-6)^3 = 0 \Rightarrow z=6 \text{ is a pole of order 3}$$

$\sin \pi z = 0 \Rightarrow z=0, \pm 1, \pm 2, \dots$ are simple poles

$\therefore z=5$ is one of the poles off (z)

19. (2)

A mapping $f(z)$ is analytic and

$f'(z) \neq 0$, then the map $w=f(z)$ is conformal

$$\text{Let } f(z) = w = z^2 + z + 1$$

$$f(z) = 2z + 1 = 0$$

$$f(z) = 0 \Rightarrow 2z + 1 = 0$$

$$\Rightarrow z = \frac{-1}{2}$$

$$\text{Clearly } f' \left(\frac{-1}{2} \right) = 0$$

\therefore w is not conformal at $z = \frac{-1}{2}$

20. (1)

Formula:

If $L[f(t)] = F(s)$

then $L[tf(t)] = \frac{-d}{ds} F(s)$

21. (3)

Formula:

$$L^{-1}[sF(s)] = \frac{d}{dt} L^{-1}[F(s)]$$

$$= L^{-1}\left[\frac{s}{(s+2)^2+4}\right] = \frac{d}{dt} L^{-1}\left[\frac{1}{(s+2)^2+4}\right]$$

$$\begin{aligned} &= \frac{d}{dt} e^{-2t} L^{-1}\left(\frac{1}{s^2+2^2}\right) \\ &= \frac{d}{dt} \left(e^{-2t} \frac{\sin 2t}{2} \right) \end{aligned}$$

$$= \frac{1}{2} [2e^{-2t} \cos 2t + \sin 2t e^{-2t}(-2)]$$

$$= e^{-2t} (\cos 2t - \sin 2t)$$

22. (1)

X	0	1	2	3	4
$y = e^x$	1	2.72	7.39	20.09	54.6

By Simpson's $\frac{1}{3}$ rule

$$\int_0^4 e^x dx = \frac{h}{3} [y_0 + y_4 + 2(y_1 + y_3)]$$

$$= \frac{1}{3} [55.60 + 14.78 + 4(2.72 + 20.09)]$$

$$= \frac{1}{3} [70.38 + 91.24]$$

$$= 53.8733$$

23. (4) Formula:

If $L[f(t)] = F(s)$ then $L[e^{-at}f(t)] = F(s+a)$

24. (4)

$$\begin{aligned} \frac{1}{(s+1)(s+3)} &= \frac{A}{s+1} + \frac{B}{s+3} \\ &= \frac{A(s+3) + B(s+1)}{(s+1)(s+3)} \end{aligned}$$

$$\Rightarrow 1 = A(s+3) + B(s+1)$$

$$\text{put } s = -1 \Rightarrow A = \frac{1}{2}$$

$$\text{put } s = -3 \Rightarrow B = -\frac{1}{2}$$

$$\begin{aligned}\therefore L^{-1} \left(\frac{1}{(s+1)(s+3)} \right) \\ &= L^{-1} \left[\frac{\left(\frac{1}{2}\right)}{s+1} - \frac{\left(\frac{1}{2}\right)}{s+3} \right] \\ &= \frac{1}{2} \left[L^{-1} \left(\frac{1}{s+1} \right) - L^{-1} \left(\frac{1}{s+3} \right) \right] \\ &= \frac{1}{2} [e^{-t} - e^{-3t}]\end{aligned}$$

25. (2)

$$\text{Variance} = E(X^2) - E(X)^2$$

$$20 = 276 - (E(X))^2$$

$$[E(X)]^2 = 276 - 20$$

$$= 256$$

$$\therefore E(X) = 16$$

26. (3)

Let P be the probability of a child being female

$$\therefore p = q = \frac{1}{2}$$

Binomial distribution $P(X=x) = {}^n C_x p^x q^{n-x}$

$${}^n C_1 \left(\frac{1}{2}\right)^n = \frac{5}{2^9} \Rightarrow \frac{n}{2^n} = \frac{5}{2^9} = \frac{10}{2^{10}}$$

$$\therefore n = 10$$

27. (3)

$$M_x(t) = E(e^{tx})$$

$$\begin{aligned}&= \sum_{x=1}^{\infty} e^{tx} \left(\frac{1}{2^x}\right) \\ &= \sum_{x=1}^{\infty} \left(\frac{e^t}{2}\right)^x \\ &= \frac{e^t}{2} + \frac{e^{2t}}{2^2} + \frac{e^{3t}}{2^3} + \dots \\ &= \frac{e^t}{2} \left[1 + \left(\frac{e^t}{2}\right) + \left(\frac{e^t}{2}\right)^2 + \dots \right] \\ &= \frac{e^t}{2} \left[\frac{1}{1 - \frac{e^t}{2}} \right] \\ &= \frac{e^t}{2} \left[\frac{2}{2 - e^t} \right] \\ &= \frac{e^t}{2 - e^t}\end{aligned}$$

28. (4)

$$\text{Required probability} = \frac{7C_2}{18C_2}$$

$$= \frac{\frac{7 \times 6}{1 \times 2}}{\frac{18 \times 17}{1 \times 2}} = \frac{7}{51}$$

29.

(3)

In a poisson distribution

mean $= \lambda$ variance $= \lambda$

30.

(1) In tossing a coin

$$p = \frac{1}{2}; q = \frac{1}{2}$$

$$n = 10$$

$$P(X=x) = nC_x p^x q^{n-x}$$

P (getting atleast 7 heads)

$$= P(x \geq 7)$$

$$= P(7) + P(8) + P(9) + P(10)$$

$$= 10C_7 \left(\frac{1}{2}\right)^7 \left(\frac{1}{2}\right)^3 + 10C_8 \left(\frac{1}{2}\right)^8 \left(\frac{1}{2}\right)^2 +$$

$$10C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right)^1 + 10C_{10} \left(\frac{1}{2}\right)^{10}$$

$$= \frac{1}{2^{10}} [10C_7 + 10C_8 + 10C_9 + 10C_{10}]$$

$$= \frac{1}{2^{10}} [120 + 45 + 10 + 1]$$

$$= \frac{176}{2^{10}} = \frac{11}{2^6}$$

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