

MODEL QUESTION PAPER – 2

with Detailed Solutions

1. Find the product of eigenvalues of the matrix

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

- 1) 45 2) -45
3) 40 4) 65

2. One of the eigen values of $A = \begin{bmatrix} \cos \theta & -\sin \theta \\ -\sin \theta & \cos \theta \end{bmatrix}$ is

- 1) $\cos \theta + \cos^2 \theta$ 2) $\cos \theta - \sin \theta$
3) $\sin \theta - \sin^2 \theta$ 4) $\tan \theta - \operatorname{cosec} \theta$

3. If all the eigen values of a matrix A of a quadratic form are negative then the quadratic form is

- 1) positive definite
2) negative definite
3) positive semidefinite
4) negative semidefinite

4. Find the rank of the matrix
- $$\begin{bmatrix} 10000 \\ 01000 \\ 00100 \\ 00010 \\ 00001 \end{bmatrix}$$

- 1) 1 2) 2
3) 3 4) 5

5. If A is a 10×15 matrix then

- 1) Rank of A ≤ 10 2) Rank of A ≥ 15
3) Rank of A ≤ 10 4) Rank of A is 150

6. The condition for the function $z = f(x, y)$ to have an extremum at (a, b) is

$$\frac{\partial z}{\partial x} = 0 \text{ and } \frac{\partial z}{\partial y} = 0$$

$$A = \frac{\partial^2 z}{\partial x^2}; B = \frac{\partial^2 z}{\partial x \partial y}$$

$$C = \frac{\partial^2 z}{\partial y^2}; \Delta = AC - B^2$$

Then the function z has a maximum value at (a, b) if

- 1) A > 0, $\Delta > 0$ 2) $\Delta > 0, A < 0$
3) $\Delta < 0, A < 0$ 4) $\Delta < 0, A > 0$

7. Particular integral of $(D^2 + 9)y = \sin 3x$ is

1) $\frac{\cos 3x}{6}$ 2) $-\frac{x \cos 3x}{6}$

3) $\frac{x \cos 3x}{6}$ 4) $\frac{x \sin 3x}{6}$

8. Let $u = x^3 y^2 (1-x-y)$ At $\left(\frac{1}{2}, \frac{1}{3}\right)$, u attains

- 1) zero 2) minimum
3) maximum 4) none of these

9. If $u = \sin^{-1}\left(\frac{x}{y}\right) + \tan^{-1}\left(\frac{y}{x}\right)$ then $x \left(\frac{\partial u}{\partial x}\right) + y \frac{\partial u}{\partial y} =$

- 1) 0 2) 1

3) xy 4) $x^2 + y^2$

10. If $x = r \cos \theta$; $y = r \sin \theta$ then $\frac{\partial(x,y)}{\partial(r,\theta)} =$

- 1) θ 2) r
3) r^2 4) $\frac{1}{r\theta}$

11. Evaluate $\int_C \frac{z dz}{z-2}$ where C is the circle $|Z|=1$

- 1) 1 2) 0
3) -1 4) 2

12. If $u(x, y) = e^x (x \cos y - y \sin y)$ then the analytic function $f(z)$ is

1) $ze^z + c$ 2) $z^2 + c$

3) $e^z + c$ 4) $z + c$

13. When the function $f(z) = u + iv$ is analytic, then u = constant and v = constant are

- 1) Orthogonal 2) Parallel
3) Similar 4) None of these

14. $\int_C \log z dz$ where C is the unit circle $|z| = 1$ is

- 1) $2\pi i$ 2) $-2\pi i$
3) 1 4) 0

15. Evaluate $\int_C \frac{dz}{(z^2+1)(z^2-4)}$ where C is the circle $|z| = \frac{3}{2}$

- 1) 2π 2) 4π
3) 0 4) 8

16. Evaluate $\int_C (z-a)^n dz$ ($n \neq -1$) where C is the circle $|z-a|=r$

- 1) $2\pi i$ 2) 0

- 3)1 4) π
 17. Find the fixed point of $W = \frac{3z-4}{z-1}$
- 1) $z=1$ 2) $z=0$
 3) $z=2$ 4) $z=5$
18. If u and v are conjugate harmonic functions, then uv is
 1) harmonic 2) non-harmonic
 3) constant 4) none of these
19. Find $f(z)$ where $f(z) = u + iv$ and $v = 3x^2y - y^3$
- 1) $z^2 + c$ 2) $z^3 + c$
 3) $z^4 + c$ 4) $z + c$
20. The Fourier sine transform of $f(x) = e^{-ax}$ is
- 1) $\sqrt{\frac{2}{\pi}} \frac{s}{a^2 + s^2}$ 2) $\sqrt{\frac{\pi}{3}} \frac{1}{a^2 + s^2}$
 3) $\frac{s}{s+1}$ 4) $\sqrt{\frac{2}{\pi}} \frac{a}{s^2 + a^2}$
21. The iteration formula given by Newton-Raphson method to find the root of the equation $x \sin x + \cos x = 0$ is
- 1) $x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n}$
 2) $x_{n+1} = x_n - \frac{x_n \cos x_n}{x_n \sin x_n + \cos x_n}$
 3) $x_{n+1} = x_n - \frac{x_n \sin x_n + \cos x_n}{x_n \cos x_n + 2 \sin x_n}$
 4) $x_{n+1} = x_n - x_n \cos x_n + 2 \sin x_n$
22. Let $F(s)$ be the complex Fourier transform of $f(x)$, i.e.,
 $F[f(x)] = F(s)$ then $F[x^n f(x)] =$
- 1) $\frac{d^n}{ds^n} F(s)$ 2) $(-i)^n \frac{d^n}{ds^n} F(s)$
 3) $\frac{d}{ds} F(s)$ 4) $\int_0^s F(s) ds$
23. Apply Lagrange's formula to find $f(x)$
- | | | | | |
|------|---|---|----|----|
| x | 0 | 1 | 4 | 5 |
| f(x) | 4 | 3 | 24 | 39 |
- 1) $2x^3 + x^2 - 3x$ 2) $2x^2 - 4x + 6$
 3) $2x^2 - 3x + 4$ 4) $3x^2 - 4x + 5$
24. The value of $\int_0^\infty e^{-2t} \sin t dt$ is

- 1) $\frac{1}{25}$ 2) $\frac{2}{25}$
 3) $\frac{3}{25}$ 4) $\frac{4}{25}$
25. For the following density function $f(x) = ae^{-|x|}$ $-\infty < x < \infty$, find the mean
- 1) 0 2) 1
 3) 3 4) $\frac{1}{2}$
26. Variance of the random variable X is 9. Its mean is
 2. Then $E(X^2) =$
- 1) 13 2) 8
 3) 10 4) 12
27. Find the moment generating function of a random variable X having the p.d.f.
- $$f(x) = \begin{cases} \frac{1}{3}, & -1 < x < 2 \\ 0, & \text{otherwise} \end{cases}$$
- 1) $\frac{e^{2t} - e^{-t}}{t}$ 2) $\frac{e^{-t} + e^{2t}}{t}$
 3) $\frac{1}{3} \left[\frac{e^{2t} - e^{-t}}{t} \right]$ 4) None of these
28. The probability distribution of a discrete random variable X is given by
- | | | | |
|--------|-----|-----|-----|
| x | -2 | 2 | 5 |
| P(X=x) | 1/4 | 1/4 | 1/2 |
- Then $4E(X^2) - \text{Var}(2X) =$
- 1) 25 2) $\frac{25}{4}$
 3) $\frac{29}{2}$ 4) 58
29. The mean and variance of a binomial distribution are 4 and $\frac{4}{3}$. Find $P(X \geq 1)$
- 1) 0.998 2) 0.34
 3) 0.001 4) 0.0119
30. Moment generating function of a binomial distribution about the origin is
- 1) $(p+q)^{nt+1}$ 2) $(p-q)^{1-nt}$
 3) $(q+pe^t)^n$ 4) $(pe^t - p)^{qt}$

ANSWERS

1. 1	2. 2	3. 2	4. 4	5. 1	6. 2	7. 2	8. 3	9. 1	10. 2
11. 2	12. 1	13. 1	14. 1	15. 3	16. 2	17. 3	18. 1	19. 2	20. 1
21. 1	22. 2	23. 3	24. 4	25. 1	26. 1	27. 3	28. 1	29. 1	30. 3

DETAILED SOLUTIONS

1. (1)

Product of eigenvalues of A

$$= |A| = \begin{vmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{vmatrix}$$

$$= -2(-12) - 2(-6) - 3(-4+1)$$

$$= 24 + 12 + 9$$

$$= 45$$

2. (2)

Characteristic equation

$$= |A - \lambda I|$$

$$= \begin{vmatrix} \cos \theta - \lambda & -\sin \theta & 0 \\ -\sin \theta & \cos \theta - \lambda & 0 \\ 0 & 0 & \cos \theta - \lambda \end{vmatrix} = 0$$

$$\Rightarrow (\cos \theta - \lambda)^2 - \sin^2 \theta = 0$$

$$\Rightarrow (\cos \theta - \lambda - \sin \theta)(\cos \theta - \lambda + \sin \theta) = 0$$

$$\Rightarrow [\lambda - (\cos \theta - \sin \theta)][\lambda - (\cos \theta + \sin \theta)] = 0$$

∴ Eigenvalues are

$$\lambda = \cos \theta - \sin \theta \text{ and } \lambda = \cos \theta + \sin \theta$$

3. (2)

A quadratic form is called Negative definite if all the eigenvalues are negative.

4. (4)

Result:

Rank of the unit matrix $I_n = n$ Given matrix is I_5

$$\therefore \text{Rank of } I_5 = 5$$

5. (1)

For any m × n matrix A

$$\text{Rank of } A \leq \min(m, n)$$

$$\therefore \text{Rank of } A \leq \min(10, 15) = 10$$

$$\therefore \text{Rank of } A \leq 10$$

6. (2)

f(x, y) has maximum at (a, b) if

$$AC - B^2 > 0, A < 0$$

i.e., A > 0 and A < 0

7. (2)

$$\text{P.I. of } (D^2 + a^2) y = \sin ax \text{ is } \frac{\sin ax}{D^2 + a^2} = \frac{-x \cos ax}{2a}$$

$$\therefore \text{P.I. of } (D^2 + 9)y = \sin 3x$$

$$= \frac{-x \cos 3x}{2 \times 3} = \frac{-x \cos 3x}{6}$$

8. (3)

$$u = x^3 y^2 (1-x-y)$$

$$\frac{\partial u}{\partial x} = 3x^2 y^2 (1-x-y) - x^3 y^2$$

$$\frac{\partial^2 u}{\partial x^2} = 6xy^2 (1-x-y) - 6x^2 y^2$$

$$\frac{\partial u}{\partial x} = 2x^3 y (1-x-y) - x^3 y^2$$

$$\frac{\partial^2 u}{\partial x \partial y} = 6x^2 y (1-x-y) - 2x^3 y - 3x^2 y^2$$

$$\frac{\partial^2 u}{\partial y^2} = 2x^3 (1-x-y) - 4x^3 y$$

$$A = \frac{\partial^2 u}{\partial x^2} \left(\frac{1}{2}, \frac{1}{3} \right) = \frac{-1}{9}$$

$$B = \frac{\partial^2 u}{\partial x \partial y} \left(\frac{1}{2}, \frac{1}{3} \right) = \frac{-1}{12}$$

$$C = \frac{\partial^2 u}{\partial y^2} \left(\frac{1}{2}, \frac{1}{3} \right) = -\frac{1}{8}$$

Clearly AC - B^2 = +ve

Also A < 0

∴ At $\left(\frac{1}{2}, \frac{1}{3} \right)$ u attains maximum.

9. (1)

$$u = \sin^{-1} \left(\frac{x}{y} \right) + \tan^{-1} \left(\frac{y}{x} \right)$$

$$\begin{aligned} \frac{\partial u}{\partial x} &= \frac{1}{\sqrt{1 + \left(\frac{x}{y} \right)^2}} \cdot \frac{1}{y} + \frac{1}{1 + \left(\frac{y}{x} \right)^2} \cdot \left(\frac{-y}{x^2} \right) \\ &= \frac{1}{\sqrt{y^2 - x^2}} - \frac{y}{x^2 + y^2} \end{aligned}$$

$$x \frac{\partial u}{\partial x} = \frac{x}{\sqrt{y^2 - x^2}} - \frac{xy}{x^2 + y^2} \dots (1)$$

$$\begin{aligned} \text{Also } \frac{\partial u}{\partial x} &= \frac{1}{\sqrt{1 + \left(\frac{x}{y} \right)^2}} \cdot \frac{x}{y^2} + \frac{1}{1 + \left(\frac{y}{x} \right)^2} \cdot \left(\frac{1}{x} \right) \\ &= \frac{-x}{y \sqrt{y^2 - x^2}} + \frac{xy}{x^2 + y^2} \end{aligned}$$

$$y \frac{\partial u}{\partial x} = \frac{-x}{\sqrt{y^2 - x^2}} - \frac{xy}{x^2 + y^2}$$

Adding(1)and(2)

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 0$$

10. (2)

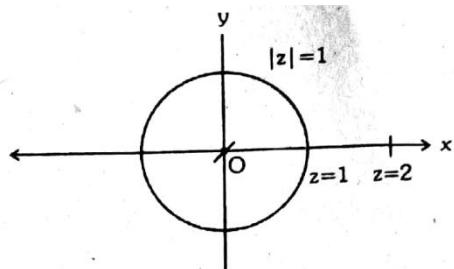
$$x = r \cos \theta; y = r \sin \theta$$

$$\begin{aligned} \frac{\partial(x, y)}{\partial(r, \theta)} &= \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} \\ &= \begin{vmatrix} \cos \theta & -r \sin \theta \\ \sin \theta & r \cos \theta \end{vmatrix} \end{aligned}$$

$$= r \cos^2 \theta + r \sin^2 \theta$$

$$= r$$

11. (2)



Cauchy's integral theorem:

If $f(z)$ is analytic inside and on a simple closed curve

$$\text{Then } \int_C f(z) dz = 0$$

$$\text{Let } f(z) = \frac{z}{z-2}$$

Clearly $f(z)$ is not analytic at $z=2$. But $z=2$ lies outside $|z|=1$

$\therefore f(z)$ is analytic inside and on $|z|=1$

\therefore By Cauchy's integral theorem

$$\int_C f(z) dz = 0 \Rightarrow \int_C \frac{z dz}{z-2} = 0$$

12. (1)

$$u = e^x (x \cos y - y \sin y)$$

$$\phi_1(x, y) = \frac{\partial u}{\partial x} = \cos y (x e^x + e^x) - y \sin y e^x$$

$$\phi_1(z, 0) = \cos 0 (ze^z + e^z) - 0 \sin 0 e^z$$

$$= ze^z + e^z$$

$$\phi_1(x, -y) = \frac{\partial u}{\partial x} = e^x (-x \sin y - \sin y - y \cos y)$$

$$\phi_1(z, 0) = 0$$

By Milne's Thomson method to find $f(z)$ where u is given

$$\begin{aligned} f(z) &= \int [\phi_1(z, 0) - i\phi_2(z, 0)] dz + c \\ &= \int e^z (z+1) dz + c \\ &= \int (ze^z + e^z) dz + c \\ &= ze^z - e^z + e^z + c \\ &= ze^z + c \end{aligned}$$

13. (1)

When $f(z) = u + iv$ is analytic, then $u = \text{constant}$ and $v = \text{constant}$ are orthogonal.

14. (1)

$$|z| = 1$$

$$\Rightarrow z = e^{i\theta} \Rightarrow dz = ie^{i\theta} d\theta$$

For the circle θ varies from 0 to 2π

$$\begin{aligned} \therefore \int_C \log z dz &= \int_0^{2\pi} (\log e^{i\theta}) (ie^{i\theta} d\theta) \\ &= \int_0^{2\pi} i\theta ie^{i\theta} d\theta [\because \log e^x = x] \\ &= - \int_0^{2\pi} \theta e^{i\theta} d\theta \\ &= - \left[\theta \left(\frac{e^{i\theta}}{i} \right) - \left(\frac{e^{i\theta}}{i^2} \right) \right]_0^{2\pi} \end{aligned}$$

(Integration by parts)

$$= [2\pi/1 + 1 - 1] = 2\pi i$$

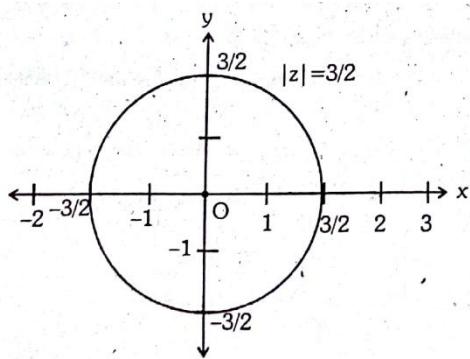
15. (3)

Cauchy's Residue theorem

If $f(z)$ is analytic at all points inside and on a simple closed curve C , except at a finite number of points z_1, \dots, z_n within C , then

$$\int_C f(z) dz \times$$

$$2\pi i \times \text{sum of the residues of } f(z) \text{ at } z_1, z_2, \dots, z_n$$



$$\text{Let } f(z) = \frac{1}{(z^2+1)(z^2-4)}$$

$$= \frac{1}{(z+i)(z-i)(z^2-4)}$$

Poles are $i, -i, 2, -2$.

Clearly the poles $z=i, -i$ lie inside $|z|=\frac{3}{2}$

Residue at the pole $z=a$ is $\lim_{z \rightarrow a} (z-a)f(z)$

Residue at $z=i$

$$= \lim_{z \rightarrow i} (z-i) \frac{1}{(z+i)(z-i)(z^2-4)}$$

$$= \frac{1}{(i+1)((i)^2-4)} = \frac{1}{2i \times -5} = \frac{1}{-10i}$$

Residue at $z=-i$

$$= \lim_{z \rightarrow -i} (z+i) \frac{1}{(z+i)(z-i)(z^2-4)}$$

$$= \frac{1}{(-i-i)((-i)^2-4)}$$

$$= \frac{1}{-2i \times -5} = \frac{1}{10i}$$

$$\therefore \int_C f(z) dz$$

$$= 2\pi i \times \text{sum of the residues of poles inside } |z| = \frac{3}{2}$$

$$= 2\pi i \left[\frac{1}{-10i} + \frac{1}{10i} \right]$$

$$= 0$$

16. (2)

Given $|z-a|=r$

$$\Rightarrow z-a = re^{i\theta}$$

$$\therefore dz = rie^{i\theta} d\theta$$

Now $\int_C (z-a)^n dz$

$$= \int_0^{2\pi} r^n e^{in\theta} ire^{i\theta} d\theta$$

$$= ir^{(n+1)} \int_0^{2\pi} e^{i(n+1)\theta} d\theta$$

$$= ir^{(n+1)} \left[\frac{e^{i(n+1)\theta}}{i(n+1)} \right]_0^{2\pi}$$

$$= \frac{r^{(n+1)}}{n+1} [e^{i2(n+1)\pi} - 1]$$

$$= \frac{r^{(n+1)}}{n+1} [\cos 2(n+1)\pi + i \sin 2(n+1)\pi - 1]$$

$$= \frac{r^{n+1}}{n+1} x[1 + 0i - 1] = 0$$

17. (3)

$$f(z) = \frac{3z-4}{z-1}$$

fixed points are given by

$$f(z)=z$$

$$\Rightarrow \frac{3z-4}{z-1} = z$$

$$\Rightarrow 3z-4=z(z-1)$$

$$3z-4=z^2-z$$

$$\Rightarrow z^2-4z+2=0$$

$$\Rightarrow (z-2)^2 - 0$$

$$\Rightarrow z=2$$

18. (1)

If u and v are harmonic functions, then u is also harmonic function.

19. (3)

$$v=3x^2y-y^3$$

$$\phi_1(x,y)=\frac{\partial v}{\partial y}=3x^2-3y^2$$

$$\phi_1(z,0)=3z^2-0=3z^2$$

$$\phi_2(x, y)=\frac{\partial v}{\partial x}=6xy$$

$$\phi_2(z, 0)=0$$

By Milne's Thomson method.

if v is given then

$$f(z) = \int (\phi_1(z, 0) + i\phi_2(z, 0)) dz + c$$

$$= \int (3z^2 + 0i) dz + c$$

$$= \frac{3z^3}{3} + c$$

$$= z^3 + c$$

20. (1)

Fouriersinetransformof

$$f(x) \text{ is } F(s) = \sqrt{\frac{2}{\pi}} \int_0^{\infty} f(x) \sin sx dx$$

∴ Sinetransformof e^{-ax} is

$$\begin{aligned} F(s) &= \sqrt{\frac{2}{\pi}} \int_0^{\infty} e^{-ax} \sin sx dx \\ &= \sqrt{\frac{2}{\pi}} \frac{e^{-ax}}{a^2 + s^2} [-a \sin sx - s \cos sx]_0^{\infty} \\ &= -\sqrt{\frac{2}{\pi}} \frac{e^{-ax}}{a^2 + s^2} [-a \sin sx - s \cos sx]_0^{\infty} \\ &= \sqrt{\frac{2}{\pi}} \left[-0 + \frac{s}{a^2 + s^2} \right] \\ &= \sqrt{\frac{2}{\pi}} \frac{s}{a^2 + s^2} \end{aligned}$$

21. (1)

NewtonRaphsonmethodis

$$X_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$$

$$f(x) = x \sin x + \cos x$$

$$f'(x) = \sin x + x \cos x - \sin x$$

$$= x \cos x \therefore X_{n+1} = X_n - (X_n \sin X_n + \cos X_n / X_n \cos X_n)$$

22. (2)

Formula:

If $F[fx] = F(s)$, then

$$F[x^n f(x)] = (-i)^n \frac{d^n}{ds^n} F(s)$$

23. (3)

$$\text{Consider } f(x) = 2x^2 - 3x + 4$$

$$\text{then } f(0) = 0 - 0 + 4 = 4$$

$$f(1) = 2(1)^2 - 3(1) + 4 = 3$$

$$f(4) = 2(4)^2 - 3(4) + 4 = 24$$

$$f(5) = 2(5)^2 - 3(5) + 4 = 39$$

Requiredpolynomialis $2x^2 - 3x + 4$

24. (4)

$$L(f(t)) = \int_0^{\infty} e^{-st} f(t) dt$$

$$\text{Consider } \int_0^{\infty} e^{-st} t \sin t dt = L(ts \sin t) = 1$$

$$= \frac{-d}{ds} (L(\sin t)) = \frac{-d}{ds} \left(\frac{1}{s^2 + 1} \right)$$

$$= - \left[\frac{(s^2 + 1) \cdot 0 - 1.2s}{(s^2 + 1)^2} \right] = \frac{2s}{(s^2 + 1)^2}$$

$$\text{In } \int_0^{\infty} e^{-2t} t \sin t dt, s = 2$$

$$\therefore \int_0^{\infty} e^{-2t} t \sin t dt = \frac{2 \times 2}{(2^2 + 1)^2} = \frac{4}{25}$$

25. (1)

$$\text{Giventhat} \int_{-\infty}^{\infty} f(x) dx = 1$$

$$\Rightarrow \int_{-\infty}^{\infty} a e^{-|x|} dx = 1$$

$$\Rightarrow 2a \int_0^{\infty} e^{-|x|} dx = 1 [\because e^{-|x|} \text{ is an evenfunction}]$$

$$\Rightarrow 2a \int_0^{\infty} e^{-x} dx = 1 [\text{in } (0, \infty) e^{-|x|} = e^{-x}]$$

$$\Rightarrow 2a(e^{-x})_0^{\infty} = 1$$

$$\Rightarrow -2a(e^{-\infty} - e^0) = 1$$

$$\Rightarrow 2a(0-1) = 1$$

$$\Rightarrow 2a = 1$$

$$\Rightarrow a = \frac{1}{2}$$

$$\text{Mean} = \int_{-\infty}^{\infty} x f(x) dx = \int_{-\infty}^{\infty} x \frac{1}{2} e^{-|x|} dx$$

$$= \frac{1}{2} \int_{-\infty}^{\infty} x e^{-|x|} dx = 0 \quad [\because x e^{-|x|} \text{ is an odd function}]$$

26. (1)

$$E(X) = \text{mean} = 2$$

$$\text{Given variance} = 9$$

By formula

$$\text{variance} = E(X^2) - (E(X))^2$$

$$\Rightarrow 9 = E(X^2) - (2)^2$$

$$= E(X^2) - 4$$

$$\Rightarrow E(X^2) = 9 + 4 = 13$$

27. (3)

m.g.f. for a continuous random variable 'X' is

$$M_x(t) = \int_{-\infty}^{\infty} e^{tx} f(x) dx$$

$$= \int_{-1}^2 e^{tx} \cdot \frac{1}{3} dx$$

$$= \frac{1}{3} \left[\frac{e^{tx}}{t} \right]_{-1}^2$$

$$= \frac{1}{3} \left[\frac{e^{2t}}{t} - \frac{e^{-t}}{t} \right]$$

$$= \frac{1}{3} \left[\frac{e^{2t} - e^{-t}}{t} \right]$$

28. (1)

$$\begin{aligned} E(X) &= \sum xp(x) \\ &= -2 \times \frac{1}{4} + 2 \times \frac{1}{4} + 5 \times \frac{1}{2} \\ &= -\frac{1}{2} + \frac{1}{2} + \frac{5}{2} \\ &= \frac{5}{2} \end{aligned}$$

$$\begin{aligned} E(X^2) &= \sum x^2 p(x) \\ &= 4 \times \frac{1}{4} + 4 \times \frac{1}{4} + 25 \times \frac{1}{2} \\ &= \frac{29}{2} \end{aligned}$$

$$\begin{aligned} \text{var}(X) &= E(X^2) - [E(X)]^2 \\ &= \frac{29}{2} - \left(\frac{5}{2}\right)^2 = \frac{33}{4} \end{aligned}$$

$$\begin{aligned} \text{var}(2X) &= 4\text{var}(X) \\ \therefore 4E(X^2) - \text{var}(2X) &= 4 \times \frac{29}{2} - 33 \\ &= 58 - 33 = 25 \end{aligned}$$

29. (1)

$$\text{Mean} = np = 4 \quad \dots (1)$$

$$\text{Variance} = npq = \frac{4}{3} \quad \dots (2)$$

$$\begin{aligned} \frac{(2)}{(1)} \Rightarrow \frac{npq}{np} &= \frac{(4/3)}{4} \\ \Rightarrow q &= \frac{1}{3} \\ \therefore p &= 1 - \frac{1}{3} = \frac{2}{3} \\ (1) \Rightarrow n &= \frac{4}{p} \\ &= \frac{4}{2/3} = 6 \end{aligned}$$

$$\begin{aligned} P(X=x) &= nC_x p^x q^{n-x} \\ \therefore p(X \geq 1) &= 1 - p(0) \end{aligned}$$

$$\begin{aligned} &= 1 - nC_0 p^0 q^{6-0} \\ &= 1 - 6C_0 \left(\frac{2}{3}\right)^0 \left(\frac{1}{3}\right)^6 \\ &= 1 - \left(\frac{1}{3}\right)^6 \end{aligned}$$

$$= 0.998$$

30. (3)

Moment generating function of a binomial distribution about origin.

$$=(q+pe^t)^n$$

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