

## MODEL QUESTION PAPER – 1

### with Detailed Solutions

1. Find the sum of the eigen values of  $A = \begin{pmatrix} 3 & 10 & 5 \\ -2 & -3 & -4 \\ 3 & 5 & 7 \end{pmatrix}$

- 1) -3                          2) 4  
3) 7                            4) 9

2. Let  $A = \begin{pmatrix} 1 & 2 \\ 3 & 1 \end{pmatrix}$ . Then

- 1)  $A^3 + 3A^2 - 4I = 0$       2)  $A^4 - 3A^2 + 4I = 0$   
3)  $4I + A + A^2 + A^3 = 0$     4)  $A^2 - 2A - 5I = 0$

3. If all the eigen values of a matrix  $A$  of a quadratic form are positive then the quadratic form is

- 1) Positive definite  
2) Negative definite  
3) Positive Semidefinite  
4) Negative semidefinite

4. In a square matrix  $A$  of order 3,

- $a_1$  = sum of its leading diagonals  
 $a_2$  = sum of the minors of its leading diagonals  $a_3 = |A|$  = determinant of  $A = \det A$   
Then the characteristic equation of  $A$  is  
1)  $\lambda^3 - a_1\lambda^2 + a_2\lambda^2 - a_3 = 0$   
2)  $\lambda^2 + a_1\lambda^2 + a_2\lambda^1 + a_3 = 0$   
3)  $\lambda^2 - a_1\lambda^2 + a_2\lambda + \lambda = 0$   
4)  $a_1 + a_2\lambda + a_3\lambda^2 = 0$

5. For the function  $2(x^2 - y^2) - x^4 + y^4$ , the point  $(0, 0)$  is a

- 1) maximum                    2) minimum  
3) saddle point                4) none of these

6. Particular integral of  $(D^2 - 4D + 13)y = e^{2x}$

- 1)  $\frac{e^{2x}}{9}$                     2)  $\frac{e^{2x}}{3}$   
3)  $\frac{e^x}{4}$                       4)  $e^x$

7. Solve  $(D^2 - 1)y = 0$

- 1)  $y = e^x + c$                 2)  $y = e^x + e^{2x}$   
3)  $y = Ae^x + Be^{-x}$     4)  $y = 1 + x + e^x$

8. If  $A = f_{xx}(a, b)$ ,  $B = f_{xy}(a, b)$  and  $C = f_{yy}(a, b)$  then  $f(x, y)$  has minimum at  $(a, b)$  if  $f_x = 0, f_y = 0$  and

- 1)  $AC < B^2$  and  $A < 0$       2)  $AC > B^2$  and  $A < 0$   
3)  $AC < B^2$  and  $A > 0$       4)  $AC > B^2$  and  $A > 0$

9. Form a partial differential equation from  $x^2 + y^2 + (z - c)^2 = a^2$

- 1)  $xq + yp = 0$                     2)  $x + y + pq = 0$   
3)  $xpq + yq - 2 = 0$     4)  $yp - xq = 0$

10.  $\nabla \left( \frac{1}{r} \right) =$

- 1)  $\vec{r}$                             2)  $\frac{1}{r}$   
3)  $\frac{\vec{r}}{-r^3}$                       4) 0

11. If  $\vec{F}$  is solenoidal then  $\nabla \cdot \vec{F}$  is

- 1)  $\vec{F}$                             2)  $F^2$   
3)  $\nabla^2 \vec{F}$                       4)  $\nabla^4 \vec{F}$

12. If  $\vec{F} = (z+3y)i + (x-2z)j + (x+az)k$  is solenoidal then  $a =$

- 1) 0                              2) 1  
3) 3                              4) 2

13. The temperature at any point in space is given by  $T = xy + yz + zx$ . Determine the directional derivative of  $T$  in the direction of the vector  $\vec{r} = 3\vec{i} - 4\vec{k}$  at the point  $(1, 1, 1)$

- 1)  $\frac{2}{5}$                             2)  $-\frac{2}{5}$   
3)  $\frac{2}{7}$                             4)  $-\frac{2}{7}$

14. Evaluate  $\int_C \frac{dz}{z^2 e^z}$  where  $C$  is  $|z| = 1$

- 1)  $2\pi i$                             2)  $-2\pi i$   
3)  $3\pi i$                             4)  $\pi i$

15. Find the residue of  $\frac{z}{(z-2)(z+1)^2}$  at  $z = 2$

- 1)  $\frac{2}{9}$                             2)  $-\frac{2}{9}$   
3)  $\frac{4}{3}$                               4)  $\frac{3}{11}$

16.  $\int_0^{1+i} (x^2 - iy) dz$  along the path  $y = x$

- 1)  $\frac{5}{6}$                             2)  $\frac{5}{6} + i$

- 3)  $\frac{5}{6} - \frac{i}{6}$       3)  $\frac{5+i}{7}$
17. The critical point of the transform  $z^2 + 6z$   
 1) 3      2) -3  
 3) 2      4) 6
18. If only the magnitude of the angle is preserved then the transform is called  
 1) conformal      2) isomorphism  
 3) uniform      4) isogonal
19. The value of  $L(t^2 e^{-2t})$  is  
 1)  $\frac{1}{(s+2)^2}$       2)  $\frac{2}{(s+2)^2}$   
 3)  $\frac{2}{(s+2)^3}$       4)  $\frac{2}{s+2}$
20.  $s^2 L[f(t)] - sf(0) - f'(0) =$   
 1)  $f'(0)$       2)  $L(f''(0))$   
 3)  $L(f'(t))$       4)  $L(f(t))$
21. Find  $L(\sin 3t)$   
 1)  $\frac{3}{s^2 + 9}$       2)  $\frac{s}{s^2 + 9}$   
 3)  $\frac{1}{s^2 + 9}$       4)  $\frac{3}{s^2 + 3}$
22. The inverse Laplace transform for the differential equation  $y'' + 2y' - 3y = \sin t$  given  $y=0$ ,  $y'(0) = 0$  when  $t = 0$   
 1)  $L^{-1}\left[\frac{1}{(s+1)(s+2)(s+3)}\right]$   
 2)  $L^{-1}\left[\frac{1}{(s-1)(s+3)(s^2+1)}\right]$   
 3)  $L^{-1}\left[\frac{1}{(s+1)(s-3)(s^2-1)}\right]$   
 4)  $L^{-1}\left[\frac{s^2+1}{(s^2-1)(s^3+1)(s-1)}\right]$
23. Newton's algorithm for finding the  $p^{\text{th}}$  root of a number  $N$   
 1)  $x_k^p + N$       2)  $\frac{x_k^p + N}{px_k^{p-1}}$   
 3)  $\frac{(p-1)x_k^p + N}{2p_k}$       4)  $x_{k+1} = \frac{(p-1)x_k^p + N}{px_k^{p-1}}$
24. Suppose that the random variable 'X' assumes three values 0, 1 and 2 with

probabilities  $\frac{1}{3}$ ,  $\frac{1}{6}$  and  $\frac{1}{2}$  respectively. Obtain  $F(1)$  where  $F(X)$  is the distribution function of  $X$ .

- 1)  $\frac{1}{2}$       2)  $\frac{1}{3}$   
 3)  $\frac{1}{4}$       4)  $\frac{1}{6}$

25. If a random variable 'X' has the p.d.f.

$$f(x) = \begin{cases} \frac{x+1}{2} & \text{if } -1 < x < 1 \\ 0 & \text{otherwise} \end{cases}$$

Find the mean

- 1)  $\frac{2}{3}$       2)  $\frac{1}{3}$   
 3)  $\frac{4}{5}$       4)  $\frac{1}{4}$

26. Moment generating function of a binomial distribution about mean is

- 1)  $(qe^{-tp} + pe^{tq})^n$       2)  $(pe^t + qe^{-t})^n$   
 3)  $(pe^t + qe^{-t})^n$       4) None of these

27. If  $E(X)=2$  and  $E(X^2)=4$  then  $E(X-2)^2 =$

- 1) 8      2) 2  
 3) 0      4) 4

28. If the probability density function of a random variable  $X$  is  $f(x)=2x$  ( $0 < x < 1$ ) then variance of  $X$  is

- 1) 1      2) 2  
 3)  $\frac{5}{12}$       4)  $\frac{1}{3}$

29. A binomial distribution has mean 4 and variance 3 find  $n$ .

- 1) 10      2) 12  
 3) 121      4) 16

30. Moment generating function of the Poisson distribution is

- 1)  $e^{\lambda(e^t-1)}$       2)  $e^{e^t-\lambda}$   
 3)  $e^{\lambda e^t-1}$       4)  $\lambda e^t - e^\lambda$

### ANSWERS

1. 3	2. 4	3. 1	4. 1	5. 3	6. 1	7. 3	8. 4	9. 4	10. 3
11. 4	12. 1	13. 2	14. 2	15. 1	16. 3	17. 2	18. 4	19. 3	20. 3
21. 1	22. 2	23. 4	24. 1	25. 2	26. 1	27. 1	28. 3	29. 4	30. 1

## DETAILED SOLUTIONS

1. (3)

Sum of the eigenvalues = trace of A  
 = sum of the main diagonal elements of A  
 $= 3 + (-3) + 7$   
 $= 7$

2. (4)

Characteristic polynomial of A

$$|A - xI| = \begin{vmatrix} 1-x & 2 \\ 3 & 1-x \end{vmatrix}$$

$$(1-x)^2 - 6 = x^2 + 1 - 2x - 6$$

$$= x^2 - 2x - 5$$

By Cayley-

Hamilton theorem every non-singular matrix A satisfies its characteristic equation.

$$\Rightarrow A^2 - 2A - 5I = 0$$

3. (1)

The quadratic form is positive definite if all the eigenvalues are positive.

4. (1)

Required characteristic equation is

$$\lambda^3 - a_1\lambda^2 + a_2\lambda - a_3 = 0$$

5. (3)

$$u = 2(x^2 - y^2) - x^4 + y^4$$

$$\frac{\partial u}{\partial x} = 4x - 4x^3$$

$$\frac{\partial^2 u}{\partial x^2} = 4 - 12x^2$$

$$\frac{\partial u}{\partial y} = -4x + 4y^3$$

$$\frac{\partial^2 u}{\partial y^2} = -4 + 12y^2$$

$$\frac{\partial^2 u}{\partial x \partial y} = 0$$

$$A = \frac{\partial^2 u}{\partial x^2}(0,0) = 4$$

$$B = \frac{\partial^2 u}{\partial x \partial y}(0,0) = 0$$

$$C = \frac{\partial^2 u}{\partial y^2}(0,0) = -4$$

Now AC - B<sup>2</sup>

$$= 4(-4) - 0$$

$$= 16 < 0$$

i.e., AC - B<sup>2</sup> < 0

Therefore (0,0) is a saddle point.

6. (1)

$$\text{P.I.} = \frac{e^{2x}}{D^2 - 4D + 13}$$

$$= \frac{e^{2x}}{(2)^2 - 4(2) + 13}$$

$$= \frac{e^{2x}}{9}$$

7. (3)

$$\text{Auxiliary equation is } m^2 - 1 = 0 \Rightarrow m = \pm 1$$

 $\therefore$  Solution is  $y = Ae^x + Be^{-x}$ 

8. (4)

f(x,y) has minimum at (a,b) if  $f_x = 0, f_y = 0$  and  $AC - B^2 > 0$  and  $A > 0 \Rightarrow AC > B^2$  and  $A > 0$ 

9. (4)

$$x^2 + y^2 + (z-c)^2 = a^2 \quad \dots (1)$$

$$2x + 2(z-c)\frac{\partial z}{\partial x} = 0$$

$$x + (z-c)p = 0$$

$$(z-c) = -\frac{x}{p} \quad \dots (2)$$

Also differentiating (1) partially w.r.t. y

$$2y + 2(z-c)\frac{\partial z}{\partial y} = 0$$

$$\Rightarrow y + (z-c)q = 0$$

$$\Rightarrow (z-c) = -\frac{y}{q} \quad \dots (3)$$

From (2) and (3)

$$-\frac{x}{p} = -\frac{y}{q}$$

$$\Rightarrow yp - xq = 0$$

10. (3)

$$\nabla f(r) = f'(r) \begin{pmatrix} \bar{r} \\ r \end{pmatrix}$$

$$\therefore \nabla \left( \frac{1}{r} \right) = -\frac{1}{r^2} \cdot \frac{\bar{r}}{r}$$

$$= -\frac{\bar{r}}{r^3}$$

11. (4)

$$\nabla \times (\nabla \times \bar{F}) = \nabla(\nabla \cdot \bar{F}) - \nabla^2 \bar{F}$$

$$= -\nabla^2 \bar{F}$$

 $\because \bar{F}$  is solenoidal  $\Rightarrow \nabla \cdot \bar{F} = 0$ Let  $V = \nabla \times \nabla \times \bar{F}$ then  $\bar{V} = -\nabla^2 \bar{F}$ Now  $\nabla \times \nabla \times \nabla \times \bar{F} = \bar{V}$

$$\begin{aligned}
 &= \nabla \times \nabla \times \bar{V} = \nabla (\nabla \cdot \bar{V}) - \nabla^2 \bar{V} \\
 &= \nabla (\nabla \cdot (\nabla \times \nabla \times F)) - \nabla^2 V \\
 &= \nabla (\operatorname{div}(\operatorname{curl} \nabla \times F)) - \nabla^2 V \\
 &= -\nabla^2 V [\because \operatorname{div} \operatorname{curl} \bar{F} = 0] \\
 &= -\nabla^2 (-\nabla^2 \bar{F}) \\
 &= \nabla^4 \bar{F}
 \end{aligned}$$

12. (1)

A vector  $\bar{F}$  is solenoidal if  $\nabla \cdot \bar{F} = 0$ 

$$\nabla \times \bar{F} = \left( \frac{\partial}{\partial x} i + \frac{\partial}{\partial y} j + \frac{\partial}{\partial z} k \right)$$

$$[(z+3y)i + (x-2z)j + (x+az)k]$$

$$= 0$$

$$\Rightarrow \frac{\partial}{\partial x} (z+3y) + \frac{\partial}{\partial y} (x-2z) +$$

$$\frac{\partial}{\partial z} (x+az) = 0$$

$$\text{i.e., } 0+0+a=0$$

$$\text{i.e., } a=0$$

13. (2)

$$T = xy + yz + zx$$

$$\nabla \phi = \left( i \frac{\partial}{\partial x} + j \frac{\partial}{\partial y} + k \frac{\partial}{\partial z} \right) (xy + yz + zx)$$

$$= i(y+z) + j(x+z) + k(x+y)$$

$$\nabla \phi (1,1,1) = 2i + 2j + 2k$$

 $\hat{e}$  = unit vector along  $3i - 4k$ 

$$= \frac{3i - 4k}{\sqrt{9+16}}$$

$$= \frac{3i - 4k}{5}$$

$\therefore$  Directional derivative at  $(1,1,1)$  in the direction of  $3i - 4k$

$$= (2i + 2j + 2k) \left( \frac{3i - 4k}{5} \right)$$

$$= \frac{1}{5}(6 - 8)$$

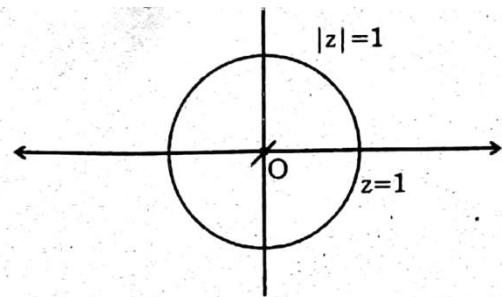
$$= \frac{-2}{5}$$

14. (2)

Cauchy's integral formula for higher derivatives :

If a function  $f(z)$  is analytic within and on a simple closed curve  $C$  and is analytic in it then

$$f(a) = \frac{1}{2\pi i} \int \frac{f(z)dz}{(z-a)^2}$$



$$\int_C \frac{dz}{z^2 e^z} = \int_C \frac{e^{-z} dz}{(z-0)^2}$$

$$\text{Let } f(z) = e^{-z}$$

$$f(z) = -e^{-z}$$

$$f(0) = -e^0 = -1$$

$$z=0 \text{ lies inside } |z|=1$$

$\therefore$  By Cauchy's theorem

$$\int_C \frac{e^{-z} dz}{(z-0)^2} = \int \frac{f(z)dz}{(z-0)^2}$$

$$= 2\pi i f(0)$$

$$= 2\pi i(-1) = -2\pi i$$

15. (1)

$$\text{Residue of } f(z) \text{ at } z=a \text{ is } \lim_{z \rightarrow a} (z-a)f(z)$$

$\therefore$  Residue at  $z=2$  is

$$\lim_{z \rightarrow 2} (z-2) \frac{z}{(z-2)(z+1)^2} = \frac{2}{(2+1)^2} = \frac{2}{9}$$

16. (3)

$$z = x+iy \Rightarrow dz = dx+idy$$

$$\int_0^{1+i} (x^2 - iy) dz = \int_0^{1+i} (x^2 - iy) (dx + idy)$$

[put  $y=x$ ;  $dy=dx$ ]

$$= \int_0^1 (x^2 - ix) (dx + idx)$$

$$= \int_0^1 (x^2 - ix) (1+i) dx$$

$$= (1+i) \left[ \frac{x^3}{3} - i \frac{x^2}{2} \right]_0^1$$

$$= (1+i) \left( \frac{1}{3} - \frac{i}{2} \right)$$

$$= \frac{5}{6} - \frac{i}{6}$$

17. (2)

$$\text{Let } W = f(z) = z^2 + 6z$$

$$\text{then } f'(z) = 2z + 6$$

Critical points of  $w = f(z)$  are given by  $f'(z) = 0$

$$\Rightarrow 2z + 6 = 0$$

$$\Rightarrow z+3=0$$

$$\Rightarrow z=-3$$

18. (4)

If only the magnitude of the angle is preserved then the transformation is called isogonal.

19. (3)

Formula:

$$L[e^{-at}f(t)] = F(s+a) \text{ where}$$

$$F(s) = L[f(t)]$$

$$\text{Here } f(t) = t^2$$

$$\therefore L(f(t)) = \frac{2}{s^3}$$

$$\therefore L[e^{-2t}t^2] = \frac{2}{(s+2)^3}$$

20. (3)

$$L[f'(t)] = s^2 L[f(t)] - sf(0) - f'(0)$$

21. (1)

$$L(\sin at) = \frac{a}{s^2 + a^2}$$

$$\therefore L(\sin 3t) = \frac{3}{s^2 + 3^2}$$

$$= \frac{3}{s^2 + 9}$$

22. (2)

$$y''(t) + 2y'(t) - 3y(t) = \sin t$$

Taking Laplace transform

$$L(y''(t)) + 2L(y'(t)) - 3L(y(t)) = L(\sin t)$$

$$\Rightarrow s^2 L[y(t)] - s y(0) - y'(0) + 2[s L[y(t)] - y(0)] - 3L[y(t)]$$

$$= \frac{1}{s^2 + 1}$$

$$\Rightarrow s^2 L[y(t)] + 2s L[y(t)] - 3L[y(t)]$$

$$= \frac{1}{s^2 + 1}$$

$$\Rightarrow (s^2 + 2s - 3)L[y(t)] = \frac{1}{s^2 + 1}$$

$$L[y(t)] = \frac{1}{(s^2 + 2s - 3)(s^2 + 1)}$$

$$= \frac{1}{(s-1)(s-3)(s^2 + 1)}$$

$$\therefore y(t) = L^{-1} \frac{1}{(s-1)(s+3)(s^2 + 1)}$$

23. (4)

$$\text{Let } x = \sqrt[p]{N}$$

$$x^p = N$$

$$\text{Let } f(x) = x^p - N$$

$$f(x) = px^{p-1}$$

Newton's formula:

$$\begin{aligned} x_{k+1} &= x_k - \frac{f(x_k)}{f'(x_k)} \\ &= x_k - \frac{x_k^p - N}{px_k^{p-1}} \\ &= \frac{(p-1)x_k^p + N}{px_k^{p-1}} \end{aligned}$$

24. (1)

$$F(x) = P(X \leq x)$$

$$\therefore F(1) = P(X \leq 1)$$

$$= P(0) + P(1)$$

$$= \frac{1}{3} + \frac{1}{6} = \frac{1}{2}$$

25. (2)

$$\text{Mean} = \int_{-1}^1 xf(x) dx$$

$$= \frac{1}{2} \int_{-1}^1 x(x+1) dx$$

$$= \frac{1}{2} \int_{-1}^1 (x^2 + x) dx$$

$$= \frac{1}{2} \left[ \frac{x^3}{3} + \frac{x^2}{2} \right]_{-1}^1$$

$$= \frac{1}{2} \left[ \left( \frac{1}{3} + \frac{1}{2} \right) - \left( \frac{-1}{3} + \frac{1}{2} \right) \right]$$

$$= \frac{1}{2} \left[ \frac{1}{3} + \frac{1}{3} \right]$$

$$= \frac{1}{2} \times \frac{2}{3}$$

$$= \frac{1}{3}$$

26. (1)

m.g.f. of a binomial distribution about mean

$$= (qe^{-tp} + pe^{tq})^n$$

27. (1)

$$E(X-2)^2 = E(X^2 - 2X + 4)$$

$$= E(X^2) - 2E(X) + E(4)$$

$$= 4 - 2 \times 2 + 4$$

$$[\because E(4) = 4]$$

$$= 8$$

28. (3)

$$E(X) = \int_0^1 f(x) dx$$

$$\begin{aligned}
 &= \int_0^1 x \cdot 2x dx \\
 &= 2 \int_0^1 x^2 dx \\
 &= 2 \left( \frac{x^3}{3} \right)_0^1 \\
 &= 2 \left( \frac{1}{3} \right) = \frac{2}{3} \\
 E(x^2) &= \int_0^1 x^2 f(x) dx \\
 &= \int_0^1 x^2 2x dx \\
 &= 2 \int_0^1 x^3 dx \\
 &= 2 \left( \frac{x^4}{4} \right)_0^1 \\
 &= 2 \left( \frac{1}{4} \right) = \frac{1}{2}
 \end{aligned}$$

Variance(X)=E(X<sup>2</sup>)-(E(X))<sup>2</sup>

$$\begin{aligned}
 &= \frac{2}{3} - \left( \frac{1}{2} \right)^2 \\
 &= \frac{2}{3} - \frac{1}{4} = \frac{8-3}{12} \\
 &= \frac{5}{12}
 \end{aligned}$$

29. (4)

$$\begin{aligned}
 \text{Mean}=np &= 4 & \dots (\text{i}) \\
 \text{variance}=npq &= 3 & \dots (\text{ii})
 \end{aligned}$$

$$\begin{aligned}
 \frac{(ii)}{(i)} &\Rightarrow \\
 \frac{npq}{np} &= \frac{3}{4} \\
 \Rightarrow q &= \frac{3}{4} \\
 \therefore p &= 1 - q \\
 &= 1 - \frac{3}{4} \\
 &= \frac{1}{4}
 \end{aligned}$$

$\therefore (1) \Rightarrow$

$$np = 4$$

$$\begin{aligned}
 \therefore n &= \frac{4}{p} \\
 &= \frac{4}{(1/4)} \\
 &= 4 \times 4 = 16
 \end{aligned}$$

30. (1)

Moment generating function for Poisson distribution  
 $= M_x(t) = e^{\lambda(e^t-1)}$

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