Previous Year Questions and Detailed Solution

1. What are the values of x, y and z respectively in which the matrix $\begin{bmatrix} x & y & z & 0 \\ 0 & 0 & 0 & -1 \\ z & x & -y & 0 \\ -y & z & -x & 0 \end{bmatrix}$ isorthogonal?
   1) 1,0,2 2) 2, 1,-1
   3) 0,1,2 4) $\frac{2}{7}, \frac{3}{7}, \frac{6}{7}$

2. The rank of the matrix A = $\begin{bmatrix} 4 & 21 & 3 \\ 6 & 34 & 7 \\ 2 & 10 & 1 \end{bmatrix}$ is
   1)4 2) 1
   3) 2 4)3

3. Sum of the squares of the eigen values of $\begin{bmatrix} 3 & 4 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{bmatrix}$ is
   1) 10 2) 25
   3) 38 4) 42

4. If some of the eigen values of the quadratic form are positive and others negative then the quadratic form is
   1) positive semidefinite
   2) indefinite
   3) negative semidefinite
   4) negative definite'

5. If $ax + by = lxy, cx + dy = mxy$ and $\Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$, $\Delta_1 = \begin{vmatrix} l & b \\ m & d \end{vmatrix}$, $\Delta_3 = \begin{vmatrix} a & l \\ c & m \end{vmatrix}$, then thevalue of (x, y) are
   1) $\left(\frac{\Delta}{\Delta_1}, \frac{\Delta}{\Delta_2}\right)$ 2) $\left(\frac{\Delta}{\Delta_2}, \frac{\Delta}{\Delta_1}\right)$
   3) $\left(\frac{\Delta_1}{\Delta}, \frac{\Delta_2}{\Delta}\right)$ 4) $\left(\frac{\Delta_2}{\Delta}, \frac{\Delta_2}{\Delta}\right)$

6. Rank of the matrix $\begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 4 & 6 & -2 \\ 3 & 6 & 9 & -3 \end{bmatrix}$ is
   1) 1 2) 2
   3) 3 4) 4

7. If eigen values of the matrix A are 1, -1, 2 then 5, -1, 8 are the eigen values of the matrix.
   1) A+4I 2) A^2+4I
   3) 3A-2I 4) 3A+2I

8. Which of the quadratic forms in three variablesis positive definite?
   1) $4x^2+5y^2$ 2) $2x^2-5y^2+7z^2$
   3) $2x^2+5y^2+5z^2$ 4) $-x^2-2y^2-7z^2$

9. Which of the following Is the factor of the determinant?
   $$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$
   1) a 2) a-b
   3) a + b 4) a+b+c

10. If a+b+c=0, one root of $\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}$=0
    1) x =1 2) x=2
    3)$x=a^2+b^2+c^2$ 4) x=0

11. If A is a 4×4 matrix. Any second order minorof A has its value as 0. Then the rank of A is
    1) <2 2) = 2
    3) >2 4) anything

12. Given A = $\begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$ then the determinantvalue of A^{-1} is
    1) 32 2) $\frac{1}{32}$
    3) $\frac{1}{64}$ 4) 64

13. If $\begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix} X = \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}$ then
    1) X = $\begin{pmatrix} -3 & 4 \\ 14 & 13 \end{pmatrix}$ 2) $\begin{pmatrix} -3 & -4 \\ -14 & 13 \end{pmatrix}$

3) $X=\begin{pmatrix}-3 & 4\\ 14 & -13\end{pmatrix}$ 4) $\begin{pmatrix}-3 & -4\\ -14 & 13\end{pmatrix}$

14. A matrix (a_{ij}) is said to be skew Hermition if
1) $a_{ij} = a_{ji}$ 2) $a_{ij} = -a_{ji}$
3) $a_{ij} = -\bar{a}_{ji}$ 4) $a_{ij} = \bar{a}_{ji}$

15. If the elements of 3 parallel lines of a determinant consist of m, n and p terms respectively, then the determinant can be expressed as
1) the sum of m + n+p determinant
2) the sum of m×n×p determinants
3) the product of m + n+p determinants
4) the product of m×n×p determinants

16. If A and B are orthogonal matrices, then
1) AB is orthogonal
2) AB is singular
3) AB is not orthogonal
4) AB is indeterminate

17. If λ is an eigen value of a matrix A, then
1) $\frac{1}{\lambda}$ is also an eigen value of A.
2) $\frac{1}{\lambda}$ is an eigen value of A^{-1}.
3) λ is an eigen value of A^{-1}
4) None of the above is true

18. The matrix $\frac{1}{2}\begin{pmatrix}1+i & -1+i\\ 1+i & 1-i\end{pmatrix}$ is
1) Hermition 2) Skew Hermitian
3) Unitary 4) None of these

19. Value of the determinant
$$\begin{vmatrix}(a-b-c) & 2a & 2a\\ 2b & (b-c-a) & 2b\\ 2c & 2c & (c-a-b)\end{vmatrix}$$ is
1) (a+b+3) 2) $(a+b+c)^2$
3) $(a+b+c)^3$ 4) abc

20. Solving : $\begin{vmatrix}a+x & a-x & a-x\\ a-x & a+x & a-x\\ a-x & a-x & a+x\end{vmatrix} = 0$ values of xare.
1) 0, 3a 2) 0, 4, -3a
3) 0, 4, 39 4) 0, 4, 4a

21. Value of $\begin{vmatrix}4 & 5 & 6 & x\\ 5 & 6 & 7 & y\\ 6 & 7 & 8 & z\\ x & y & z & 0\end{vmatrix}$
1) $(x+y+z)^2$ 2) (x-y+z)2
3) $(x+y-2z)^2$ 4) $(x-2y+z)^2$

22. Eigen value of the matrix $\begin{vmatrix}2 & 2 & 1\\ 1 & 3 & 1\\ 1 & 2 & 2\end{vmatrix}$ are
1) 5, 5, -3 2) 1, 1, 5
3) 0, 1, 6 4) 1, 2, 4

23. Rank of the matrix $\begin{vmatrix}3 & 5 & 2 & 4\\ 1 & 2 & 5 & 7\\ 6 & 10 & 4 & 8\\ 4 & 7 & 7 & 11\end{vmatrix}$ is
1) one 2) two
3) three 4) four

24. If two matrices A and B are Equivalent, then
1) they are of same order
2) they are of the same rank
3) they have the same number of elements
4) they are of the same order and rank

25. The characteristic equation of the marix $\begin{bmatrix}5 & 2 & -3\\ 0 & 0 & 8\\ 0 & 0 & 7\end{bmatrix}$ is $\lambda^3 - 12\lambda^2 + 35\lambda - k = 0$, then k =
1) 12 2) 18
3) 0 4) 5

26. The value of $\begin{vmatrix}bc & b+c & 1\\ ca & c+a & 1\\ ab & a+b & 1\end{vmatrix}$=
1) (a-b)(b-c)(c-a) 2) abc
3) 2 abc 4) 0

27. The equation $\begin{vmatrix}x-2 & 2x-3 & 3x-4\\ x-4 & 2x-9 & 3x-16\\ x-8 & 2x-27 & 3x-64\end{vmatrix} = 0$ is satisfied if
1) x=1 2) x=2

3) x=3 4) x=4

28. If AA^T =1, then the matrix A is called
1) Idempotent
2) Symmetric matrix
3) Orthogonal matrix
4) Tri-diagonal matrix

29. If A and B are square matrices of the same order then $(A + B)^2$ =
1) $A^2 + B^2 + 2AB$
2) $A^2 + B^2 + AB + BA$
3) $A^2 + B^2 + 2BA$
4) $B^2+A^2+AB+AB$

30. If A = $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$ and A_1, B_1, C_1,.... arecofactors of a_1, b_1, c_1,.... then $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix}$ =
1) Δ 2) Δ^2
3) Δ^3 4) Δ^4

31. If A and B are non-singular matrices and AX=YB, then
1) X=Y 2) $X=YBA^{-1}$
3) $X=A^{-1}YB$ 4) $Y=B^{-1}AX$

32. If the rank of the matrix of order 5×4 is 3, then the value of the determinant of the subsequence matrix of order 4 is
l) 0 2) 1
3) 4 4) 5

33. The product of the eigen values of the matrix $\begin{vmatrix} 1 & 0 & 2 \\ 3 & -1 & 0 \\ 0 & 1 & 2 \end{vmatrix}$ is
1) 0 2) 2
3) 4 4) 8

34. If 1,-1, 5 are eigen values of the matrix A, then eigen values of the matrix A^{-1}+ 21 are
1) 3, 1, 7 2) 3, 1, $\frac{11}{5}$
3) 3,-1,$\frac{1}{5}$ 4) 1,-1, 7

35. If A and B are two square matrices of the same order than (A+B)(A-B) equals
1) A^2-B^2 2) $A^2-AB-BA+B^2$
3) $A^2-BA+AB-B^2$ 4) $^{A2}-AB+BA-B^2$

36. The equation x+y=l, 2x+2y-2=0 have
1) no solution
2) unique solution
3) only two solutions
4) infinite number of solutions

37. If the rank of $\begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 4 \\ 6 & 8 & x \end{bmatrix}$ is 2, then the value ofx is
1) 0 2) 1
3) 5 4) 10
E) None of these

38. If A= $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{bmatrix}$ then the sum and product ofthe eigen values are respectively.
1) 6 and 1 2) 1 and 6
3) 2 and 3 4) 1 and 3

39. The value of the determinant $\begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$ is
1) (x-y)(y-x)(z-x) 2) xyz.
3) x+y+z
4) xyz(x-y)(y-z)(z-x)

40. If A is a square matrix of order n, then $A\left(\frac{\text{adj } A}{|A|}\right)$ is
1) A 2) A^{-1}
3) 1 4) | A|I

41. If A and B are two square matrices of the same order, then (A+B)(A-B) equals
1) A^2-B^2
2) $A^2-AB-BA+B^2$

3) A^2-BA+AB-B^2
4) A^2-AB + BA-B^2

42. If W is a cube root of unit, then a root of theequation $\begin{vmatrix} x+1 & W & W^2 \\ W & x+W^2 & 1 \\ W^2 & 1 & x+W \end{vmatrix}$ is
1) 0 2) 1
3) -1 4) -W

43. The equations x+y=1, 2x+2y-2 = 0 have
1) no solution
2) unique solution
3) only two solutions
4) infinite number of solutions

44. If $\Delta = \begin{bmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{bmatrix}$ thentheva lueof Δ lies in the interval given by
1) 1≤Δ≤1.5 2) 2≤Δ≤4
3) 12≤Δ≤14 4) 0≤Δ≤1

45. If A is a (3×3) matrix such that detA=5, then the value of det (Adj A) is
1) 125 2) 25
3) 625 4) 5

46. If the rank of the matrix $\begin{bmatrix} -1 & 2 & 2 & -2 \\ 2 & -4 & -2 & k \\ 0 & 1 & -3 & 2 \end{bmatrix}$ is2 then K has the value
1) 3 2)4
3) 2 4) 1

47. IfA= $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$ then A(adj A) is
1) $\begin{bmatrix} 1 & 0 & 0 \\ 1 & 2 & 0 \\ 2 & -1 & 3 \end{bmatrix}$ 2) $\begin{bmatrix} 1 & 1 & 1 \\ 0 & 2 & -3 \\ 0 & 0 & 3 \end{bmatrix}$
3) $\begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix}$ 4) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{bmatrix}$

48. The product of the eigen values of the matrix $\begin{bmatrix} 7 & 4 & 9 \\ 2 & -6 & 5 \\ 4 & 3 & -2 \end{bmatrix}$ is
1) zero 2) 275
3) 84 4) 345

49. $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$
1) $(a+b+c)^3$ 2) $2(a+b+c)^3$
3) 0 4) $a^3+b^3+c^3$

50. $\begin{vmatrix} \log_3 512 & \log_4 3 \\ \log_3 8 & \log_4 9 \end{vmatrix} \begin{bmatrix} \log_2 3 & \log_8 3 \\ \log_3 4 & \log_3 4 \end{bmatrix}$ is equal to
1) 1 2) 6
3) 10 4) 12

51. If, a, b, c have all different values and $\begin{vmatrix} a & a^2 & a^3-1 \\ b & b^2 & b^3-1 \\ c & c^2 & c^3-1 \end{vmatrix}$
1) abc=0 2) abc=l
3) abc= 2 4) abc=2

52. If A= $\begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 0 \\ 6 & 0 & 4 \end{bmatrix}$ then A.(ad] A) =
1) $\frac{1}{24}\begin{bmatrix} 2 & -1 & 6 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ 2) $\begin{bmatrix} \frac{1}{24} & 0 & 0 \\ 0 & \frac{1}{24} & 0 \\ 0 & 0 & \frac{1}{24} \end{bmatrix}$
3) $\begin{bmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{bmatrix}$ 4) $\begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -1 & \frac{3}{2} & 0 \\ 3 & 0 & 2 \end{bmatrix}$

53. The product of the eigen value of the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ is
1) 45 2) 168
3) 21 4) 0

54. The rank of the matrix $\begin{bmatrix}0 & 2 & 3 & 4\\ 0 & 0 & 0 & 6\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\end{bmatrix}$

1) 4 2) 3
3) 2 4) 1

55. Given that the value of 9 for which the matrix $\begin{bmatrix}\cos\theta & 0 & \sin\theta\\ 0 & 1 & 0\\ -\sin\theta & 0 & \cos\theta\end{bmatrix}$ is orthogonal

1) $\frac{\pi}{2}$ 2) 0
3) $\frac{\pi}{6}$ 4) arbitrary

56. If A= $\begin{pmatrix}1 & -1\\ 1 & 1\end{pmatrix}$ then which of the following is true?

1) $A^2+2A+2I=0$ 2) $A^2+2A-2I=0$
3) $A^2-2A-2I=0$ 4) $A^2-2A+2I=0$

57. Given that A= $\begin{pmatrix}1 & 0\\ 1 & 1\end{pmatrix}$ the sum of the elements in A^n is

1) 2n 2) 3n
3) n+2 4) n+3

58. If the matrix $\begin{bmatrix}K & 1 & 2\\ 1 & 2 & 0.5\\ 1 & 2 & 3\end{bmatrix}$ has an eigen value$\lambda = 1$, then the value of k is

1) 0.5 2) -0.5
3) 2 4) -2

59. Given that $(1\ 2\ 3)^T$ is an eigen vector for the matrix $\begin{bmatrix}2 & 3 & -1\\ 3 & 2 & 1\\ 2 & 2 & 2\end{bmatrix}$ the corresponding eigen

1) 2 2) 5
3) 4 4) 3

60. If A is a square matrix of order 3 and det A=2 then det (Adj A) =

1) 2 2) 4
3) 8 4) 6

DELTAILED SOLUTIONS

1. (4) For orthogonal matrix,

i.e. $\begin{bmatrix}x & y & z & 0\\ 0 & 0 & 0 & -1\\ z & x & -y & 0\\ -y & z & -x & 0\end{bmatrix}\begin{bmatrix}x & 0 & z & -y\\ y & 0 & x & z\\ z & 0 & y & -x\\ 0 & -1 & 0 & 0\end{bmatrix}$

$=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$

$\Rightarrow\begin{bmatrix}x^2+y^2+z^2 & \ldots & \ldots\\ \vdots & \vdots & \vdots\\ \ldots & \ldots & \ldots\end{bmatrix}$

$=\begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{bmatrix}$

From given choices in (4)

if $x=\frac{2}{7}$; $y=\frac{3}{7}$; $z=\frac{6}{7}$

then, $x^2+y^2+z^2$

$=\frac{4}{49}+\frac{9}{49}+\frac{36}{49}=1$

∴Required values:

$x=\frac{2}{7}$; $y=\frac{3}{7}$; $z=\frac{6}{7}$

2. (3) $A=\begin{bmatrix}4 & 2 & 13\\ 6 & 3 & 47\\ 2 & 1 & 01\end{bmatrix}$

$\sim\begin{bmatrix}1 & 2 & 4 & 3\\ 4 & 3 & 6 & 7\\ 0 & 1 & 2 & 1\end{bmatrix} C_1 \leftrightarrow C_3$

$\sim\begin{bmatrix}1 & 2 & 4 & 3\\ 0 & -5 & -10 & -5\\ 0 & 1 & 2 & 1\end{bmatrix} R_2 \rightarrow R_2 - 4R_1$

$\sim\begin{bmatrix}1 & 2 & 4 & 3\\ 0 & -5 & -10 & -5\\ 0 & 0 & 0 & 0\end{bmatrix} R_3 \rightarrow R_3 + \frac{R_2}{5}$

There are two non zero rows.

Therefore the rank = 2

3. (3)

Characteristic equation is$|A-\lambda I|=0$

$$\Rightarrow \begin{vmatrix} 3-\lambda & 4 & 4 \\ 0 & 2-\lambda & 6 \\ 0 & 0 & 5-\lambda \end{vmatrix} = 0$$

$\Rightarrow (3-\lambda)(2-\lambda)(5-\lambda) = 0$

$\therefore \lambda = 3, 2, 5$

∴ Sum of the squares of Eigen values

$= 3^2+2^2+5^2$

$= 9+4+25$

$= 38$

4. (2)If some of the Eigen values of the matrix A of a quadratic form are positive and others negative, then the quadratic form is Indefinite.

5. (2) $ax + y = lxy$

$\Rightarrow \frac{ax}{xy} + \frac{by}{xy} = l$

$\Rightarrow \frac{a}{y} + \frac{b}{x} = l$

$\Rightarrow \frac{b}{x} + \frac{a}{y} = l$ ..... (1)

Similarly

$cx + dy = mxy$

$\frac{d}{x} + \frac{c}{y} = m$ .... (2)

∴ By Cramer s Rule, solutions of (1) and (2) are given by

$$\frac{1}{x} = \frac{\begin{vmatrix} l & a \\ m & c \end{vmatrix}}{\begin{vmatrix} b & a \\ d & c \end{vmatrix}}$$

and $\frac{1}{y} = \frac{\begin{vmatrix} b & l \\ d & m \end{vmatrix}}{\begin{vmatrix} b & a \\ d & c \end{vmatrix}}$

Now, $\frac{1}{x} = \frac{\begin{vmatrix} l & a \\ m & c \end{vmatrix}}{\begin{vmatrix} b & a \\ d & c \end{vmatrix}}$

$\Rightarrow x = \frac{\begin{vmatrix} b & a \\ d & c \end{vmatrix}}{\begin{vmatrix} l & a \\ m & c \end{vmatrix}}$

$= \frac{-\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{-\begin{vmatrix} a & l \\ c & m \end{vmatrix}}$

$= \frac{\Delta}{\Delta_2}$

Also $\frac{1}{y} = \frac{\begin{vmatrix} b & l \\ d & m \end{vmatrix}}{\begin{vmatrix} b & a \\ d & c \end{vmatrix}}$

$\Rightarrow y = \frac{\begin{vmatrix} b & a \\ d & c \end{vmatrix}}{\begin{vmatrix} b & l \\ d & m \end{vmatrix}}$

$= \frac{-\begin{vmatrix} a & b \\ c & d \end{vmatrix}}{-\begin{vmatrix} l & b \\ m & d \end{vmatrix}}$

$= \frac{\Delta}{\Delta_1}$

$\therefore (x, y) = \left(\frac{\Delta}{\Delta_2}, \frac{\Delta}{\Delta_1}\right)$

6. (1)

Determinant of any submatrix of order 2x2 and 3×3 of the given matrix is zero.

Therefore rank of the given matrix is 1.

Method 2 :

$$A = \begin{bmatrix} 1 & 2 & 3 & -1 \\ 2 & 4 & 6 & -2 \\ 3 & 6 & 9 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 3 & -1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

This equivalent matrix is in the Echelon form. Since the number of non-zero rows of the matrix in the Echelon form is 1, implies rank of A = l.

7. (4)

If $\lambda_1, \lambda_2, \ldots, \lambda_n$, are Eigen values of A. then kA has Eigen values

$k\lambda_1, k\lambda_2, \ldots, k\lambda_n$

A+kl has Eigen values

$k+\lambda_1, k+\lambda_2, \ldots k + \lambda_n$

Given A has Eigen values 1, -1, 2

∴3A has Eigen values 3, -3, 6

Now

3A+2I has Eigen values

3 + 2, -3 + 2, 6+2 = 5, -1, 8

8. (3)

Consider $2x^2+5y^2+5z^2$

Clearly this is a quadratic form with three variables. A quadratic form is said to be positive definite if rank of A = number of positive square terms in the canonical form Positive square terms in $2x^2+5y^2+5z^2$ is 3 ... (1)

Now $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix}$

since $|A| = 50 \neq 0$

$\Rightarrow$ rank A = 3

Since (1) = (2) implies the given expression is a positive definite.

9. (4)

$$\begin{vmatrix} a+b+2c & a & b \\ c & b+c+2a & b \\ c & a & c+a+2b \end{vmatrix}$$

=

$$\begin{vmatrix} (a+b+c)c & a & b \\ c & (a+b+c)a & b \\ c & a & (a+b+c)b \end{vmatrix}$$

put a+b+c, = 0

$\Rightarrow \begin{vmatrix} c & a & b \\ c & a & b \\ c & a & b \end{vmatrix}$

= 0 [∵ two rows are identical]

⇒a+b+c is a factor.

10. (4)

$$\begin{vmatrix} a-x & c & b \\ c & b-x & a \\ b & a & c-x \end{vmatrix}$$

put x=0 then

$$\begin{vmatrix} a & c & b \\ c & b & a \\ b & a & c \end{vmatrix}$$

$R_1 \rightarrow R_1+R_2+R_3$

$$\begin{vmatrix} a+b+c & a+b+c & a+b+c \\ c & b & a \\ b & a & c \end{vmatrix}$$

$\Rightarrow \begin{vmatrix} 0 & 0 & 0 \\ c & b & a \\ b & a & c \end{vmatrix} = 0$

[∵ In this problem given a+b+c=0]

∴ x=0 is a root.

11. (1)

Since any second order minor of A=0, the rank of A<2.

12. (3)

$A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 4 & 0 \\ 0 & 0 & 8 \end{bmatrix}$

$|A| = 2\times4\times8 = 64$

Formula$AA^{-1} = 1$

$\Rightarrow |AA^{-1}| = |I|$

$\Rightarrow |A||A^{-1}| = 1$

$\Rightarrow |A^{-1}| = \frac{1}{|A|}$

$\Rightarrow |A^{-1}| = \frac{1}{64}$

13. (3)

$$\begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix} X = \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}$$

$\Rightarrow X = \begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix}^{-1} \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}$ .... (1)

Let $A = \begin{pmatrix} 3 & 1 \\ 4 & 1 \end{pmatrix}$

$|A| = 3 - 4 = -1$

$A^{-1} = \frac{1}{(-1)} \begin{pmatrix} 1 & -1 \\ -4 & 3 \end{pmatrix}$

$= \begin{pmatrix} -1 & 1 \\ 4 & -3 \end{pmatrix}$

∴ (1) ⇒

$X = \begin{pmatrix} -1 & 1 \\ 4 & -3 \end{pmatrix} \begin{pmatrix} 5 & -1 \\ 2 & 3 \end{pmatrix}$

$= \begin{pmatrix} -3 & 4 \\ 14 & -13 \end{pmatrix}$

14. (3)

A square matrix $A = [a_{ij}]$ is said to be skew - Hermition if the $(i, j)^{th}$ element of A is equal to the negative of the conjugate complex of the $(j, i)^{th}$ element of A

i.e.,$a_{ij} = -\bar{a}_{ij}$ for all i, j

15. (1)

Total determinants = m + n + p

16. (1)

A square matrix A is called orthogonal matrix if the product of the matrix and its transpose AT is an identity matrix, i.e., $AA^T = 1$

Suppose A and B are orthogonal matrices

Then $AA^T = A^TA = I$

$BB^T = B^TB = I$

Now

$(AB)(AB)^T = ABB^TA^T$

$= A(BB^T)A^T = AIA^T = AA^T = 1$

17. (2)

If λ is an Eigen value of the matrix A, then $\frac{1}{\lambda}$ is an Eigen value of A^{-1}

18. (3)

$A = \frac{1}{2}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}$

$A^* = \overline{A}^T = \frac{1}{2}\begin{bmatrix} 1-i & 1-i \\ -1-i & 1+i \end{bmatrix}$

$AA^* = \frac{1}{4}\begin{bmatrix} 1+i & -1+i \\ 1+i & 1-i \end{bmatrix}\begin{bmatrix} 1-i & 1-i \\ -1-i & 1+i \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 2+2 & 2-2 \\ 2-2 & 2+2 \end{bmatrix}$

$= \frac{1}{4}\begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}$

$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

∴ A is unitary.

19. (3)

$$\begin{vmatrix} (a-b-c) & 2a & 2a \\ 2b & (b-c-a) & 2b \\ 2c & 2c & (c-a-b) \end{vmatrix}$$

$R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} (a-b-c) & 2a & 2a \\ 2b & (b-c-a) & 2b \\ 2c & 2c & (c-a-b) \end{vmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$C_2 \to C_2 - C_1$

$C_3 \to C_3 - C_1$

$=$

$$(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -a-b-c & 0 \\ 2c & 2c & -a-b-c \end{vmatrix}$$

$= (a+b+c)\,[(a+b+c)^2 - 0]$

$= (a+b+c)^3$

20. (1)

$$\begin{vmatrix} a+x & a-x & a-x \\ a-x & a+x & a-x \\ a-x & a-x & a+x \end{vmatrix} = 0$$

$C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \begin{vmatrix} 3a-x & a-x & a-x \\ 3a-x & a+x & a-x \\ 3a-x & a-x & a+x \end{vmatrix} = 0$$

$$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 1 & a+x & a-x \\ 1 & a-x & a+x \end{vmatrix} = 0$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\Rightarrow (3a-x)\begin{vmatrix} 1 & a-x & a-x \\ 0 & 2x & 0 \\ 0 & 0 & 2x \end{vmatrix} = 0$$

$\Rightarrow (3a - x)(4x^2 - 0) = 0$

$\Rightarrow x = 0$ (or) $x = 3a$

21. (4)

$$\begin{vmatrix} 4 & 5 & 6 & x \\ 5 & 6 & 7 & y \\ 6 & 7 & 8 & z \\ x & y & z & 0 \end{vmatrix}$$

$R_2 \to R_2 - R_1$

$R_3 \to R_3 - R_1$

$$\Rightarrow \begin{vmatrix} 4 & 5 & 6 & x \\ 1 & 1 & 1 & y-x \\ 2 & 2 & 2 & z-x \\ x & y & z & 0 \end{vmatrix}$$

Interchanging R_1 and R_3

$$-\begin{vmatrix} 2 & 2 & 2 & z-x \\ 1 & 1 & 1 & y-x \\ 4 & 5 & 6 & x \\ x & y & z & 0 \end{vmatrix}$$

$R_1 \to R_1 - 2R_2$

$$= -\begin{vmatrix} 0 & 0 & 0 & z+x-2y \\ 1 & 1 & 1 & y-x \\ 4 & 5 & 6 & x \\ x & y & z & 0 \end{vmatrix}$$

Expanding through R_1

$$= +(x+z-2y)\begin{vmatrix} 1 & 1 & 1 \\ 4 & 5 & 6 \\ x & y & z \end{vmatrix}$$

$$(z+x-2y)[5z-6y+6x-4z+4y-5x]$$

$$= (x+z-2y)^2$$

22. (2)

Characteristic equation is

$$\begin{vmatrix} 2-\lambda & 2 & 1 \\ 1 & 3-\lambda & 1 \\ 1 & 2 & 2+\lambda \end{vmatrix} = 0$$

$(2-\lambda)[(3-\lambda)(2-\lambda)-2] - 2(2-\lambda-1)+1(2-3+\lambda) = 0$

$\Rightarrow (2-\lambda)[6-3\lambda-2\lambda+\lambda^2-2]-2+2\lambda-1+\lambda = 0$

$\Rightarrow (2-\lambda)(\lambda^2-5\lambda+4)+3\lambda-3 = 0$

$\Rightarrow 2\lambda^2-10\lambda+8-\lambda^3+5\lambda^2-4\lambda+3\lambda-3 = 0$

$\Rightarrow \lambda^3+7\lambda^2-11\lambda+5 = 0$

$\Rightarrow -\lambda^3-7\lambda^2+11\lambda-5 = 0$

put $\lambda = 1$ then

$1^3-7(1)^2+11(1)-5 = 0$

$\therefore \lambda = 1$ is a root

Now,

$$\begin{array}{r|l} & \lambda^2-6\lambda+5 \\ \lambda-1 & \lambda^3-7\lambda^2+11\lambda-5 \\ & \lambda^3-\lambda^2 \\ \hline & -6\lambda^2+11\lambda \\ & -6\lambda^2+6\lambda \\ \hline & 5\lambda-5 \\ & 5\lambda-5 \\ \hline & 0 \end{array}$$

$\therefore \lambda^3-7\lambda^2+11\lambda-5 = (\lambda-1)(\lambda^2-6\lambda+5$

Now,

$$\lambda^2-6\lambda+5 = (\lambda-1)(\lambda^2-6\lambda+5)$$

$\Rightarrow \lambda = 1, 5$ are roots

$\therefore$ Roots of $\lambda^3-7\lambda^2+11\lambda-5 = 1, 1, 5$

$\therefore$ Eigen values = 1, 1, 5

23. (2)

$$\begin{vmatrix} 3 & 5 & 2 & 4 \\ 1 & 2 & 5 & 7 \\ 6 & 10 & 4 & 8 \\ 4 & 7 & 7 & 11 \end{vmatrix}$$

$R_2 \leftrightarrow R_1$

$$\sim \begin{bmatrix} 1 & 2 & 5 & 7 \\ 3 & 5 & 2 & 4 \\ 6 & 10 & 4 & 8 \\ 4 & 7 & 7 & 11 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 3R_1$

$R_3 \rightarrow R_3 - 6R_1$

$R_4 \rightarrow R_4 - 4R_1$

$$\sim \begin{bmatrix} 1 & 2 & 5 & 7 \\ 0 & -1 & -13 & -17 \\ 0 & -2 & -26 & -34 \\ 0 & -1 & -13 & -17 \end{bmatrix}$$

$R_3 \rightarrow R_3 - 2R_1$

$R_4 \rightarrow R_4 - R_1$

$$\sim \begin{bmatrix} 1 & 2 & 5 & 7 \\ 0 & -1 & -13 & -14 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

This is in Echelon form

It has two non zero rows.

$\therefore$ Rank = 2

24. (4)

If two matrices A and B are EQUIVALENT, then they are of the same order and rank.

25. (3)

Characteristic equation is

$$\begin{vmatrix} 5-\lambda & 2 & -3 \\ 0 & 0-\lambda & 8 \\ 0 & 0 & 7-\lambda \end{vmatrix} = 0$$

$\Rightarrow (5-\lambda)[-\lambda(7-\lambda)-0]-2[0-0]-3[0-0] = 0$

$(5-\lambda)(\lambda^2-7\lambda) = 0$

$\Rightarrow 5\lambda^2 - 35\lambda - \lambda^3 + 7\lambda^2 = 0$

$\Rightarrow -\lambda^3 + 12\lambda^3 - 35\lambda = 0$

$\Rightarrow \lambda^3 - 12\lambda + 35\lambda = 0$...(1)

But the given characteristic equation is

$\lambda^3 - 12\lambda^2 + 35\lambda - k = 0$ ...(2)comparing (1) and (2)

k = 0

26. (1)

$$\begin{vmatrix} bc & b+c & 1 \\ ca & c+a & 1 \\ ab & a+b & 1 \end{vmatrix}$$

$R_2 \leftrightarrow R_2 - R_1$

$R_3 \leftrightarrow R_3 - R_1$

$$= \begin{vmatrix} bc & b+c & 1 \\ c(a-b) & a-b & 0 \\ b(a-c) & a-c & 0 \end{vmatrix}$$

$$= (a-b)(a-c) \begin{vmatrix} bc & b+c & 1 \\ c & 1 & 0 \\ b & 1 & 0 \end{vmatrix}$$

= (a – b) (a – c) [bc (0 – 0) 0 (b + c) (0 – 0) + (c – b)

= (a – b) (a – c) (c – b)

= (a – b) (b – c) (c – a)

27. (4)

$$\begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ x-4 & 2x-9 & 3x-16 \\ x-8 & 2x-27 & 3x-64 \end{vmatrix} = 0$$

$R_2 \rightarrow R_2 - R_1$

$R_3 \rightarrow R_3 - R_1$

$$\Rightarrow \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ -2 & -6 & -12 \\ -6 & -24 & -60 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} x-2 & 2x-3 & 3x-4 \\ 1 & 3 & 6 \\ 1 & 4 & 10 \end{vmatrix} = 0$$

$\Rightarrow (x-2)(30-24) - (2x-3)(10-6) + (3x-4)(4-3) = 0$

$\Rightarrow (x-2)(6) - (2x-3)(4) + (3x-4).1 = 0$

$\Rightarrow$ x – 4 =0

$\Rightarrow$ x = 4

28. (3)

AA^T = I, then the matrix A is called an orthogonal matrix.

29. (2)

If A and B are square matrices of the same order then

$(A+B)^2 = (A+B)(A+B)$

$= A^2+AB + BA + B^2$

$= A^2+B^2+AB + BA$

30. (2)

Standard Result:

If $\Delta = \begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix}$

and A_1, B_1, C_1, ..., are cofactors of a_1, b_1, c_1,

....then $\begin{vmatrix} A_1 & B_1 & C_1 \\ A_2 & B_2 & C_2 \\ A_3 & B_3 & C_3 \end{vmatrix} = \Delta^2$

31. (3)

Given A and B are non-singular matrices. This implies A^{-1} and B^{-1} exists.

Now AX = YB

$\Rightarrow A^{-1}AX = A^{-1}YB$

$\Rightarrow X = A^{-1}YB$

32. (1)

The rank of a matrix is r, if all the minors of order greater than or equal to (r+1) is zero. Given matrix is of order 5×4 its rank is 3.

∴The value of the minors of order 4 is zero.

33. (3)

Result:

Product of the Eigen value of a matrix A = determinant of A

Given A = $\begin{bmatrix} 1 & 0 & 2 \\ 3 & -1 & 0 \\ 0 & 1 & 2 \end{bmatrix}$

∴Product of Eigen value

= |A| = 1 (-2 – 0)

= -0 + 2 (3 – 0)

= -2 + 6 = 4

34. (2)

If $\lambda_1, \lambda_2, \ldots, \lambda_n$ are the Eigen values of a matrix A of order n then $\frac{1}{\lambda_1}, \frac{1}{\lambda_2}, \ldots . \frac{1}{\lambda_n}$,are the Eigen valuesof A^{-1}

If 1, -1, 5 are Eigen values of matrix A, then $\frac{1}{1}, \frac{1}{-1}, \frac{1}{5}$(i.e.) 1, -1, $\frac{1}{5}$ are Eigen values of A^{-1}

∴Eigen values of $A^{-1}+2I$ are

$1+2, -1+2, \frac{1}{5}+2 = 3, 1, \frac{11}{5}$

35. (4)

A and B are two square matrices of the same order 'n' then

$(A+BMA-B) = A^2-AB+BA-B^2$

36. (4)

Given equations are

x + y = 1

2x + 2y = 2

$\therefore \Delta = \begin{vmatrix} 1 & 1 \\ 2 & 2 \end{vmatrix} = 0$

∴ The system has infinite number of solutions.

37. (4)

Rank is 2

⇒ Any submatrix of 3×3 = C 2 1 1'

$\therefore \begin{bmatrix} 2 & 1 & 1 \\ 1 & 3 & 4 \\ 6 & 8 & x \end{bmatrix} = 0$

⇒ 2(3x-32)-(x-24) + (8-18)

⇒6x – 64 – x+24 -10 = 0

⇒ 5x-50 = 0

∴x = 10

38. (1)

Sum of Eigen values

= Trace of A

= Sum of the main diagonal elemenis =1+2+3 = 6

Product of Eigen values

= determinant of A

$= \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{vmatrix}$

= (6 - 4) - (3 - 2) + (2-2)

= 2-1+0 = 1

39. (1)

$\Delta = \begin{vmatrix} 1 & 1 & 1 \\ x & y & z \\ x^2 & y^2 & z^2 \end{vmatrix}$

$C_1 \to C_1 - C_3; C_2 \to C_2 - C_3$

$\therefore \Delta = \begin{vmatrix} 0 & 0 & 1 \\ x-z & y-z & z \\ x^2-z^2 & y^2-z^2 & z^2 \end{vmatrix}$

Taking you (x-z) common from C_1 and (y-z)common from c_2

$\Delta = (x-z)(y-z)\begin{vmatrix} 0 & 0 & 1 \\ 1 & 1 & z \\ (x+z) & (y+z) & z^2 \end{vmatrix}$

=(x – z) (y – z) ((y + z) – (x + z))

= (x – z) (y – z) (y – x)

= (x – y) (y – z) (z – x)

40. (3)

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$\therefore A\left(\frac{\text{adj } A}{|A|}\right) = AA^{-1} = 1$

41. (4)

$(A + B) (A – B) = A^2 – AB + BA – B^2$

42. (1)

$$\begin{vmatrix} x+1 & w & w^2 \\ w & x+w^2 & 1 \\ w^2 & 1 & x+w \end{vmatrix}$$

$C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} x+1+w+w^2 & w & w^2 \\ x+1+w+w^2 & w^2 & 1 \\ x+1+w+w^2 & 1 & w \end{vmatrix}$$

$= \begin{vmatrix} 0 & w & w^2 \\ 0 & w^2 & 1 \\ 0 & 1 & w \end{vmatrix} \because [1+w+w^2 = 0]$

= 0

∴ x = 0 is a root

43. (4)

x+y = 1

2x+2y =2 =0

⇒x+y = 1

Both represent same equation

Now x+y= 1 ⇒ y=1-x for different values of x, we get values of y

∴The equations have infinite number of solutions.

44. (2)

$$\Delta = \begin{vmatrix} 1 & \cos\theta & 1 \\ -\cos\theta & 1 & \cos\theta \\ -1 & -\cos\theta & 1 \end{vmatrix}$$

= 1 (1 + $\cos^2\theta$) – cos θ (- cos θ+ cos θ)+1 ($\cos^2\theta$+1)

= 2 (1 + $\cos^2\theta$)

Now,

$0 \le \cos^2\theta \le 1$

⇒ 2 (1+0) ≤ 2 (1 + $\cos^2\theta$) ≤ 2 (1 + 1)

⇒ 2 ≤Δ≤ 4

45. (2)

Result:

If det A = Δ then

det (adj A) = Δ^2

Given det A = 5

∴det (adj A) = 25

46. (2)

Since the rank of the matrix is2, atleast one minor of the given matrix of order 2 is not zero and every minor of the matrix of order greater than 2 is zero, i.e., any minor of order 3 is zero.

$$\therefore \begin{vmatrix} 2 & 1 & -2 \\ -4 & -2 & k \\ 1 & -3 & 2 \end{vmatrix} = 0$$

$$2(-4+3k) - (-8-k) - 2(12+2) = 0$$

$$\Rightarrow -8 + 6k + 8 + k - 24 - 4 = 0$$

⇒ 7k = 28 ⇒k = 4

47. (3)

Formula:

A (adj A) = | A| I_3

$$A = \begin{vmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{vmatrix}$$

|A| = 1 (6 – 3) – 1 (3 + 6)+ 1 (-1 – 4)

= 3 – 9 – 5

= -11

∴ A (adj A) = |A| I_3

$$= (-11)\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -11 & 0 & 0 \\ 0 & -11 & 0 \\ 0 & 0 & -11 \end{bmatrix}$$

48. (4)

Formula:

Product of the Eigen value of a matrix A = determinant of the matrix A

$$A = \begin{bmatrix} 7 & 4 & 9 \\ 2 & -6 & 5 \\ 4 & 3 & -2 \end{bmatrix}$$

$$= 7(12-15) - 4(-4-20) + 9(6+24)$$

$$= -21 + 96 + 270 = 345$$

49. (1)

$$\Delta = \begin{bmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{bmatrix}$$

$$= (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

$$C_2 \to C_2 - C_1$$
$$C_3 \to C_3 - C_1$$

$$= (a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ 2b & -b-c-a & 0 \\ 2a & 0 & -c-a-b \end{vmatrix}$$

= (a + b + c) [(-b – c – a) (-c – a – b)]

= $(a + b + c)^3$

50. (3)

Formula:

$$\log_2 b = \frac{\log_e b}{\log_e a} = \frac{\log b}{\log a}$$

Given product is equal to

$$\begin{vmatrix} \frac{\log 512}{\log 3} & \frac{\log 3}{\log 4} \\ \frac{\log 8}{\log 3} & \frac{\log 9}{\log 4} \end{vmatrix} \begin{vmatrix} \frac{\log 3}{\log 2} & \frac{\log 3}{\log 8} \\ \frac{\log 4}{\log 3} & \frac{\log 4}{\log 3} \end{vmatrix}$$

$$= \left(\frac{\log 512}{\log 3} \times \frac{\log 9}{\log 4} - \frac{\log 3}{\log 4}\frac{\log 8}{\log 3}\right) \times \left(\frac{\log 3}{\log 2} \times \frac{\log 4}{\log 3} - \frac{\log 3}{\log 8}\frac{\log 4}{\log 3}\right)$$

$$= \left(\frac{9\log 2}{\log 3} \times \frac{2\log 3}{2\log 2} - \frac{\log 3}{2\log 2} \times \frac{3\log 2}{\log 3}\right) \times \left(\frac{\log 3}{\log 2} \times \frac{2\log 2}{\log 3} - \frac{\log 3}{3\log 2} \times \frac{2\log 2}{\log 3}\right)$$

$$= \left(9 - \frac{3}{2}\right)\left(2 - \frac{2}{3}\right)$$

$$= \frac{15}{2} \times \frac{4}{3} = 10$$

51. (2)

If a, b, c have all different values and

$$\begin{vmatrix} a & a^2 & a^3 - 1 \\ b & b^2 & b^3 - 1 \\ c & c^2 & c^3 - 1 \end{vmatrix} = 0$$

$$= \begin{vmatrix} a & a^2 & a^3 \\ b & b^2 & b^3 \\ c & c^2 & c^3 \end{vmatrix} + \begin{vmatrix} a & a^2 & -1 \\ b & b^2 & -1 \\ c & c^2 & -1 \end{vmatrix} = 0$$

$$= abc \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} - \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$= (abc - 1) \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = 0$$

$$= (abc - 1) \begin{vmatrix} 1 & 1 & 1 \\ a & b & c \\ a^2 & b^2 & c^2 \end{vmatrix} = 0$$

$$= (abc - 1) \begin{vmatrix} 1 & 1 & 1 \\ a & b - a & c - a \\ a^2 & b^2 - a^2 & c^2 - a^2 \end{vmatrix} = 0$$

$$C_2 \to C_2 - C_1$$
$$C_3 \to C_3 - C_1$$

$$= (abc - 1) \left[(b - a)(c - a) \begin{vmatrix} 1 & 0 & 0 \\ a & 1 & 1 \\ a^2 & b + a & c + a \end{vmatrix} \right]$$

= (abc – 1) (b –a) (c- a) (c + a – b – a) = 0

= (abc – 1) (a –b) (b – c) (c – a) =0

= (abc-1) a b-a c-a *0

a, b, c have all different values ⇒ (a-b)(b-c)(c-a)≠0

= abc-1 = 0

⇒ abc = 1

52. (3)

Formula:

If A is any - n rowed square matrix then A.(adj A) = (adj A) A

= | A | In

$$|A| = \begin{bmatrix} 2 & 0 & 0 \\ -1 & 3 & 0 \\ 6 & 0 & 4 \end{bmatrix}$$

= 2×3×4 = 24

∴A(adj A) = | A | In

$$= 24 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 24 & 0 & 0 \\ 0 & 24 & 0 \\ 0 & 0 & 24 \end{bmatrix}$$

53. (4)

Formula:

Product of Eigen values = determinant of the matrix

$$A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

|A | = 8(21 - 16)+6(-18+8) + (24 - 14)

= 40-60+20 = 0

∴Product of Eigen values

= |A| = 0

54. (1)

A matrix A is said to be of rank r when

I) otleasl one minor of A of order r is not zero

II) every minor of A of order (r+1) is zero

$$|A| = \begin{vmatrix} 0 & 2 & 3 & 4 \\ 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{vmatrix}$$

$$= \begin{vmatrix} 0 & 1 & 0 & 0 \\ 2 & 0 & 1 & 0 \\ 3 & 0 & 0 & 1 \\ 4 & 6 & 0 & 0 \end{vmatrix}$$

$$= 0 - 1 \begin{vmatrix} 2 & 1 & 0 \\ 3 & 0 & 1 \\ 4 & 0 & 0 \end{vmatrix} + 0 - 0$$

$= -1 \times (-1 \times (-4) + 0)$

$= -4$ since $|A| \neq 0$

Rank of the matrix A = 4

55. (4)

If a matrix A is orthogonal then

$|A| = \pm 1$

$$\text{Let } A = \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix}$$

$|A| = \cos\theta (\cos\theta - 0) - 0 + \sin\theta (\sin\theta) = \cos^2\theta + \sin^2\theta = 1$

56. (4)

Cayley-Hamilton Theorem:

Every square matrix satisfies its own characteristic equation.

Characteristic equation is given by

$|A - \lambda I| = 0$

$$\begin{vmatrix} 1-\lambda & -1 \\ 1 & 1-\lambda \end{vmatrix} = 0$$

$$(1-\lambda)^2 + 1 = 0$$

$\Rightarrow \lambda^2 - 2\lambda + 1 + 1 = 0$

$\Rightarrow \lambda^2 - 2\lambda + 2 = 0$

By Cayley Hemilton theorem

$A^2 - 2A + 2I = 0$

57. (3)

$$A = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$A^2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}$$

$$A^3 = A^2 A = \begin{pmatrix} 1 & 0 \\ 2 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}$$

$$A^4 = A^3 A = \begin{pmatrix} 1 & 0 \\ 3 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 \\ 4 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & 0 \\ n & 1 \end{pmatrix}$$

$\therefore$ Sum of elements in $A^n = (n + 2)$

58. (1) Characteristic equation is given by

$$(A - \lambda I) = \begin{vmatrix} k-\lambda & 1 & 2 \\ 1 & 2-\lambda & 0.5 \\ 1 & 2 & 3-\lambda \end{vmatrix} = 0 \ldots$$

(1)

Given 1 is an Eigen value

$\Rightarrow \lambda = 1$ satisfies (1)

$$\therefore \begin{vmatrix} k-1 & 1 & 2 \\ 1 & 2-1 & 0.5 \\ 1 & 2 & 3-1 \end{vmatrix} = 0$$

$$\Rightarrow \begin{vmatrix} k-1 & 1 & 2 \\ 1 & 1 & 0.5 \\ 1 & 2 & 2 \end{vmatrix} = 0$$

$\Rightarrow k - 1 (2 - 1) - 1 (2 - 0.5) + 2(2 - 1) = 0$

$\Rightarrow k - 1 - 1.5 + 2 = 0$

$\Rightarrow k = 0.5$

59. (2)

Let the Eigen vectors be

$X^T = (x_1, x_2, x_3)$then

$(A - \lambda I)X^{-1} = 0$

$$\Rightarrow \begin{pmatrix} 2-\lambda & 3 & -1 \\ 3 & 2-\lambda & 1 \\ 2 & 2 & 3-\lambda \end{pmatrix}\begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = 0$$

$(2-\lambda)x_1 + 3x_2 - x_3 = 0$ ... (1)

$3x_1 + (2-\lambda)x_2 + x_3 = 0$ ... (2)

$2x_1 + 2x_2 - (3-\lambda)x_3 = 0$ ...(3)

Given Eigen vector $(1, 2, 3)^T$

Substituting

$x_1 = 1, x_2 = 2, x_3 = 3$ in (1)

$(2-\lambda) + 6 - 3 = 0$

$\Rightarrow \lambda = 5$

60. (2)

det (A) = 2

det (Adj A) = $[\det A]^2 = 4$