PREVIOUS YEAR QUESTIONS & DETAILED SOLUTIONS

1. If Z=log $(x^3+y^3-x^2y-xy^2)$ then $\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}=$

1) $\frac{2}{x+y}$ 2) $\frac{1}{x+y}$
3) $\frac{2}{x-y}$ 4) $\frac{1}{2x+y}$

2. If $A=f_{xx}(a, b)$, $B=f_{xy}(a, b)$ $c=f_{yy}(a, b)$ then f(x, y) has maximum at (a, b) if $f_x=0$, $f_y=0$ and

1) $AC<B^2$ and $A<0$ 2) $AC>B^2$ and $A<0$
3) $AC<B^2$ and $A>0$ 4) $AC>B^2$ and $A>0$

3. The solution of xp+yq=z is

1) f(x,y)=0 2) $f(x^2,y^2)=0$
3) f(xy, yz)=0 4) $f\left(\frac{x}{y},\frac{y}{z}\right)=0$

4. The particular integral of $(D^2+a^2)y=\sin ax$ is

1) $\frac{ax}{2}\cos ax$ 2) $\frac{x}{-2a}\cos ax$
3) $-\frac{ax}{2}\cos ax$ 4) $\frac{x}{2a}\cos ax$

5. If f(x, y, z) = 0, then $\frac{\partial x}{\partial y}.\frac{\partial y}{\partial z}.\frac{\partial z}{\partial x}$ is

1) 1 2) 0
3) -1 4) 2

6. If $x=r\sin\theta\cos\phi$, $y=r\sin\theta\sin\phi$, $z=r\cos\phi$ then $\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)}$ is

1) $r\sin^2\theta$ 2) $r^2\sin\theta$
3) $r^2\sin\phi$ 4) $r\sin^2\phi$

7. If $\frac{dx}{dt}+y=t$ and $\frac{dy}{dt}+x=0$ then the value of y is

1) $y=Ae^t+Be^{-t}+t$
2) $y=A\cos t+B\sin t+t$
3) $y=Ae^t+Be^{-t}-t$
4) $y=A\cos t+B\sin t-t$

8. Let z=ax+by+xy where a and b are arbitrary constants and x, y are independent variables. The differential equation which is related to the above is

1) Ordinary differential equation with order 2
2) Ordinary differential equation with order 1
3) Partial differential equation with order 2
4) Partial differential equation with order

9. The complete integral of the partial differential equation p + q=x+y is

1) $2z=(a+x)^2+(y-a)^2+b$
2) $2z=(a-x)^2+(y-a)^2+b$
3) $z=(a+x)^2+(y-a)^2$
4) $z=(a-x)^2+(y-a)^2$

10. The solution of $\frac{\partial^2 z}{\partial x^2}-2\frac{\partial^2 z}{\partial x\,\partial y}+\frac{\partial^2 z}{\partial y^2}=e^{2x+y}$

1) $z=f(x+y)+g(x+y)+e^{2x+y}$
2) $z=f(x+y)+xg(x+y)+e^{2x+y}$
3) $z=f(x+y)-g(x+y)+\frac{1}{9}e^{2x+y}$
4) $z=f(x-y)+xg(x-y)=e^{2x+y}$

11. The area of the cardiod $r=a(1+\cos\theta)$ is

1) $\frac{3\pi a^2}{4}$ 2) $\frac{4\pi a^2}{3}$
3) $\frac{3\pi a^2}{2}$ 4) $\frac{2\pi a^2}{3}$

12. The volume bounded by the cylinder $x^2+y^2=4$ and the plane 2 = 3 and z=0 is

1) 12π 2) 6π
3) 3π 4) π

13. The formula for the radius of curvature in cartesian coordinate is

1) $\frac{\left[1+(y')^2\right]^{1/2}}{y''}$ 2) $\frac{\left[1+(y')^2\right]^{3/2}}{y''}$
3) $\frac{\left[1+(y')^2\right]^{3/2}}{(y'')^2}$ 4) $\frac{\left[1+(y')^2\right]^{3/2}}{(y'')^2}$

14. The condition for the function z=f(x, y) to havea extremum at (a, a) is $\frac{\partial z}{\partial x}=0$ and $\frac{\partial z}{\partial y}=0$. $A=\frac{\partial^2 z}{\partial x^2}, B=\frac{\partial^2 z}{\partial x\,\partial y}, C=\frac{\partial^2 z}{\partial y^2}$ $\Delta=AC-B^2$ the function z has a maximum value at (a, a) If

1) $\Delta>0, A<0$ 2) $\Delta>0, A=0$

3) $\Delta<0, A>0$ 4) $\Delta>0, A>0$

15. The stationary point of $f(x, y)=x^2-xy + y^2-2x + y$ is

1) (0, 1) 2) (1,0)
3) (-1,0) 4) (1,-1)

16. $\int_0^{\pi/2} \frac{\sin x}{\sin x+\cos x} dx$ is

1) $\frac{\pi}{2}$ 2) π
3) $\frac{\pi}{4}$ 4) 2π

17. $\int x\cos x\, dx$ is

1) $x\sin x - \cos x$ 2) $x\sin x + \cos x$
3) $x\sin x - x\cos x$ 4) $x\sin x + x\cos x$

18. The solution of $\left(x\tan\frac{y}{x} - y\sec^2\frac{y}{x}\right)dx + x\sec^2\frac{y}{x}dy = 0$ is

1) $x\tan\frac{y}{x} = c$ 2) $x\tan\frac{x}{y} = c$
3) $x\tan\frac{y}{x} = c$ 4) $x\tan\frac{x}{y} = c$

19. The necessary and sufficient conditions for the differential equation M dx+N dy=0 to be exact is

1) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ 2) $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$
3) $\frac{\partial M}{\partial x} = \frac{\partial N}{\partial y}$ 4) $\frac{\partial M}{\partial y} = \frac{-\partial N}{\partial x}$

20. The value of $\frac{e^{4x}}{D-2}$ is

1) $\frac{e^{4x}}{4}$ 2) $\frac{-e^{4x}}{4}$
3) $\frac{1}{2}e^{4x}$ 4) $\frac{-e^{4x}}{2}$

21. Divide 24 into 3 parts such that the continued 1st; square of the second and cube of the 3rd may be minimum

1) 12, 3, 9 2) 14, 6, 4
3) 4, 8, 12 4) 4, 3, 17

22. Formation of the differential equation from $y=(Ax+2)e^x$ is

1) $(D^2-2D + 1)y = 0$ 2) $(D^2+2D+1)y = 0$
3) $(D^2+ 1)y = 0$ 4) $(D^2-l)y = 0$

23. Solution of Euler's equation $(x^2y''-2xy'-4y)=0$

1) $Ax+\frac{B}{x^4}$ 2) $Ax+Bx^4$
3) $Ax^4 + \frac{B}{x}$ 4) $\frac{A}{x} + \frac{B}{x^4}$

24. A particle of mass m falls under gravity through a medium whose resistance is equal to μ times its velocity. The corresponding differential equation is, if x is the distance in 't', then

1) $mx''+\mu x'=0$ 2) $mx''+\mu x'=-mg$
3) $mx''+\mu x'=mg$ 4) $mx''=\mu x'$

25. If x=f(t) and y=g(t) then the formula for radius of curvature in parametric coordinates is

1) $\frac{(x^2+y^2)^{3/2}}{\dot{x}y-\dot{y}\ddot{x}}$ 2) $\frac{(\dot{x}^2+\dot{y}^2)^{3/2}}{\dot{x}\ddot{y}-\dot{y}\ddot{x}}$
3) $\frac{(\dot{x}^2-\dot{y}^2)^{1/2}}{\dot{x}\dot{y}-\ddot{x}\ddot{y}}$ 4) $\frac{(\dot{x}^2-\dot{y}^2)^{3/2}}{\dot{x}\ddot{y}-\dot{y}\ddot{x}}$

26. Divide a length 'a' into three equal parts x, y and z such that their product is maximum.

1) $x = \frac{a}{3}, y = \frac{a}{3}, z = \frac{a}{3}$ 2) $x = \frac{a}{2}, y = \frac{a}{2}, z = \frac{a}{2}$
3) $x = \frac{a}{4}, y = \frac{a}{2}, z = \frac{a}{4}$ 4) $x = \frac{a}{6}, y = \frac{a}{3}, z = \frac{a}{3}$

27. The condition for the function z=f(x, y) to have a extremum at (a, a) is $\frac{\partial z}{\partial x} = 0$ and $\frac{\partial z}{\partial y} = 0$ $A = \frac{\partial^2 z}{\partial x^2}, B = \frac{\partial^2 z}{\partial x \partial y}, C = \frac{\partial^2 z}{\partial y^2}$ $\Delta = AC - B^2$. Then the function z has a minimum value at (a, a) if

1) $\Delta>0, \Delta<0$ 2) $\Delta>0, \Delta>0$
3) $\Delta<0, \Delta<0$ 4) $\Delta<0, \Delta>0$

28. $\int_0^{\pi} \log x\, dx$ js

1) $-\pi \log x+\pi$ 2) $-(\pi +\pi \log \pi)$
3) $\pi \log \pi-\pi$ 4) $\pi \log \pi-1$

29. $\int_0^{\pi/2} \log(\tan\theta) d\theta$ is

1) $\pi/4$ 2) 1
3) π 4) 0

30. The radius of curvature of the curve $x^2+y^2+4x-21=0$ is

1) 2 2) 5

3) $\frac{1}{2}$ 4) $\frac{1}{5}$

31. The centre of curvature at (1, 1) on $\sqrt{x}+\sqrt{y}=2$ is
1) (3,3) 2) (-3,-3)
3) $\left(\frac{1}{3},\frac{1}{3}\right)$ 4) $\left(\frac{-1}{3},\frac{-1}{3}\right)$

32. The minimum value of $x^2+y^2+6y+12$ is
1)-3 2) 3
3) 12 4) 20

33. The general solution of the differential equation $(x^2D^2+xD)y=5$ is
1) $y=A+\frac{5x^2}{2}$ 2) $y=\frac{5}{2}(\log x)^2$
3) $y=Ax+B+\frac{5x^2}{2}$
4) $y=A\log x+B+\frac{5}{1}(\log x)^2$

34. The point (0, 1) is a point at which the function $2(x^2-y^2)-x^4+y^4$ has a
1) local maximum 2) local minimum
3) global maximum 4) saddle point

35. Which of the following is a harmonic function?
1) $e^x x$ 2) $e^x \sin y$
3) x^2+y^2 4) sin x cos y

36. Which of the following satisfies the equation $y''-3y'+2y=2$?
1) $y=2e^x-3e^{2x}+1$ 2) $y=2e^x+3e^{2x}+2$
3) $y=2e^x-3e^{2x}-l$ 4) $y=2e^x+3e^{2x}-2$

37. The set of two positive numbers whose sum is 16 and sum of whose cubes is maximum is
1) (6, 10) 2) (8, 8)
3) (4,12) 4) (9,7)

38. A point of local maximum for the curve $x^3y=3x^4+1$ is
1) (-1,-4) 2) (1, 4)
3) (1,-4) 4) (-1,4)

39. The particular integral of $(D^2+4)y=\cos^2 3x$ is
1) $\frac{1}{8}-\frac{1}{64}$ - cos 6x 2) $\frac{1}{8}+\frac{1}{64}\cos 6x$
3) $\frac{1}{64}-\frac{1}{8}\sin 6x$ 4) $\frac{1}{8}+\frac{1}{64}\sin 6x$

40. The value of the integral $\int\int dxdy$ over the region bounded by x=0, y=0 and x+1 is
1) 1 2) $\frac{3}{2}$
3) $\frac{1}{2}$ 4) $\frac{2}{3}$

41. The value of the integral $\int_0^\infty x^2e^{-3x}dx$ is
1) $\frac{2}{27}$ 2) $\frac{1}{3}$
3) $\frac{2}{9}$ 4) $\frac{7}{27}$

42. If $\phi(x,y)=\log\frac{x^4+y^4}{x+y}$, then which of thefollowing is true?
1) $x\frac{\partial^2\phi}{\partial x^2}+y\frac{\partial^2\phi}{\partial y^2}=3$ 2) $x\frac{\partial\phi}{\partial x}+y\frac{\partial\phi}{\partial y}=3$
3) $x^2\frac{\partial\phi}{\partial x}+y^2\frac{\partial\phi}{\partial y}=3$
4) $x^2\frac{\partial\phi}{\partial x}+y^2\frac{\partial\phi}{\partial y}=3\log\phi$

43. The complementary function of $(1+x)^2\frac{d^2y}{dx^2}+(1+x)\frac{dy}{dx}+y=0$ is
1) $c_1\cos(\log(1+x))+c_2\sin(\log(1+x))$
2) $c_1\cos(1+x)+c_2\sin(1+x)$
3) $c_1\cos(\log x)+c_2\sin(\log x)$
4) $c_1\cos x+c_2\sin x$

DETAILED SOLUTIONS

1. (1)
$z=\log(x^3+y^3-x^2y-xy^2)$
$$\frac{\partial z}{\partial x}=\frac{3x^2-2xy-y^2}{x^3+y^3-x^2y-xy^2}$$
$$\frac{\partial z}{\partial y}=\frac{3y^2-x^2-2xy}{x^3+y^3-x^2y-xy^2}$$
Now,

$$\frac{\partial z}{\partial x}+\frac{\partial z}{\partial y}$$
$$=\frac{3x^2-2xy-y^2+3y^2-x^2-2xy}{x^3+y^3-x^2y-xy^2}$$
$=\frac{2x^2+2y^2-4xy}{x^2(x-y)-y^2(x-y)}$
$=\frac{2(x-y)^2}{(x^2-y^2)(x-y)}=\frac{2(x-y)}{(x^2-y^2)}$
$=\frac{2(x-y)}{(x+y)(x-y)}$
$=\frac{2}{x+y}$

2. (2)
$A=f_{xx}$ (a, b); $B = f_{xy}$ (a, b)
$C=f_{yy}$ (8, b) then f(x, y) has maximum at (a, b)
if $f_x=0$, $f_y=0$ and $AC-B^2>0$, $A<0$
i.e., $AC>B^2$ and $A<0$

3. (4)
xp + yq = z
This is Lagrange's linear equation of the type
Pp + Qq = R
Solution is $\frac{dx}{P}=\frac{dy}{Q}=\frac{dz}{R}$
Therefore solution of xp+yq=z is given by
$$\frac{dx}{x}=\frac{dy}{y}=\frac{dz}{z}$$
From first two equations
$$\frac{dx}{x}=\frac{dy}{y}$$
$$\int\frac{dx}{x}=\int\frac{dy}{y}$$
$\Rightarrow \log x = \log y + \log c_1$
$\Rightarrow x = yc_1$
$\Rightarrow \frac{x}{y} = c_1$
Also from last two equations
$$\frac{dy}{y}=\frac{dz}{z}$$
$$\int\frac{dy}{y}=\int\frac{dz}{z}$$
$\log y = \log z + \log c_2$
$y = zc_2$
$\frac{y}{z} = c_2$ ... (2)
From (1) and (2) the solution is
$$f\left(\frac{x}{y},\frac{y}{z}\right)=0$$

4. (2)
Particular integral for $(D^2+a^2)y=\sin ax$ is $\frac{-x\cos ax}{2a}$

5. (3)
f(x, y, z)=0, then
$$\frac{\partial x}{\partial y}.\frac{\partial y}{\partial z}.\frac{\partial z}{\partial x}=-1$$

6. (2)
$x = r\sin\theta\cos\phi$
$y = r\sin\theta\cos\phi$
$z = r\cos\theta$
Now,
$x = r\sin\theta\cos\phi$
$\frac{\partial x}{\partial r}=\sin\theta\cos\phi$
$$\frac{\partial x}{\partial\theta}=r\cos\theta\cos\phi$$
$$\frac{\partial x}{\partial\phi}=-r\sin\theta\sin\phi$$
$$y = r\sin\theta\sin\phi$$
$$\frac{\partial y}{\partial r}=\sin\theta\sin\phi$$
$$\frac{\partial y}{\partial\theta}=r\cos\theta\sin\phi$$
$$\frac{\partial y}{\partial\phi}=-r\sin\theta\cos$$
$$z = r\cos\theta$$
$$\frac{\partial z}{\partial r}=\cos\theta$$
$$\frac{\partial y}{\partial\theta}=-r\sin\phi$$

$\frac{\partial z}{\partial \phi} = 0$

Now,

$$\frac{\partial(x,y,z)}{\partial(r,\theta,\phi)} = \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} & \frac{\partial x}{\partial \phi} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} & \frac{\partial y}{\partial \phi} \\ \frac{\partial z}{\partial r} & \frac{\partial z}{\partial \theta} & \frac{\partial z}{\partial \phi} \end{vmatrix}$$

$$= \begin{vmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\phi & -r\sin\theta & 0 \end{vmatrix}$$

$$= r^2\sin\theta \begin{vmatrix} \sin\theta\cos\phi & \cos\theta\cos\phi & -\sin\phi \\ \sin\theta\sin\phi & \cos\theta\sin\phi & \cos\phi \\ -\cos\phi & -\sin\theta & 0 \end{vmatrix}$$

$= r^2\sin\theta[\sin\theta\cos\phi(0+\sin\theta\cos\phi) - \cos\theta \cos\phi 0 - \cos\theta \cos\phi - \sin\phi - \sin 2\theta \sin\phi - \cos 2\theta \sin\phi$

$= r^2\sin\theta[\cos^2\phi(\sin^2\theta + \cos^2\theta) + \sin 2\phi \sin 2\theta + \cos 2\theta$

$= r^2\sin\theta[\cos^2\phi + \sin^2\phi]$

$= r^2\sin\theta$

7. (1)

$$\frac{dx}{dt} + y = 1$$

$\frac{dx}{dt} = t - y$ ... (1)

Also given

$$x + \frac{dy}{dt} = 0$$

Differentiating with respect to

$$\frac{dx}{dt} + \frac{d^2y}{dt^2} = 0$$

$\therefore$ (1) $\Rightarrow t - y + \frac{d^2y}{dt^2} = 0$

$\Rightarrow \frac{d^2y}{dt^2} - y = -t$

Auxiliary equation is

$m^2 - 1 = 0$

$m = \pm 1$

$\therefore$Complementary function = C.F

$= Ae^t + Be^{-t}$

Particular integral $= \frac{-t}{D^2-1} = \frac{-t}{-(1-D^2)}$

$= t(1-D^2)^{-1}$

$= t[1 + D^2 + D^4 + \cdots]$

$= t + D^2t + \cdots$

$= t + 0 = t$

Solution

$y = CF + PI$

$= Ae^t + Be^{-t} + t$

8. (4)

$z = ax + by + xy$ ... (1)

$\Rightarrow \frac{\partial z}{\partial x} = a + y$

$\Rightarrow a = \frac{\partial z}{\partial x} - y$

Also, $\frac{\partial z}{\partial y} = b + x$

$\Rightarrow b = \frac{\partial z}{\partial y} - x$

$\therefore$ (1) $\Rightarrow z = x\left(\frac{\partial z}{\partial x} - y\right) + y\left(\frac{\partial z}{\partial y} - x\right) + xy$

$$z = x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} - xy$$

This is a partial differential equation with order 1.

9. (8)

$$p + q = x + y$$
$$\Rightarrow p - x = y - q = a$$
$$\Rightarrow p - x = a;\ y - q = a$$
$$\Rightarrow p = a + x;\ q = y - a$$

The solution is given by

$$dz = pdx + qdy$$

$\Rightarrow \int dz = \int(a + x)dx + \int(y-a)dy$

$\Rightarrow z = \frac{(a+x)^2}{2} + \frac{(y-a)^2}{2} + k$

$\Rightarrow 2z = (a+x)^2 + (y-a)^2 + b$

where b = 2k

10. (8)

Let $z = f(x+y) + g(x+y) + e^{2x+y}$

$$\frac{\partial z}{\partial x} = f'(x+y) + g'(x+y) + 2e^{2x+y}$$

$\frac{\partial^2 z}{\partial x^2} = f''(x+y) + g''(x+y) + 4e^{2x+y}$

Similarly

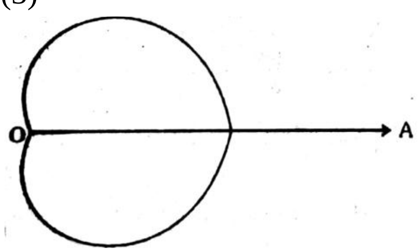
$$\frac{\partial^2 z}{\partial y^2} = f''(x+y) + g''(x+y) + e^{2x+y}$$

Also,

$$\frac{\partial^2 z}{\partial x\,\partial y^2} = f''(x+y) + g''(x+y) + e^{2x+y}$$

$\therefore \frac{\partial^2 z}{\partial x^2} - 2\frac{\partial^2 z}{\partial x\,\partial y} = \frac{\partial^2 z}{\partial y^2} = e^{2x+y}$

11. (3)



Area = $2\int_0^{\pi}\int_0^{a(1+\cos\theta)} r\,dr\,d\theta$

$= 2\int_0^{\pi}\left[\frac{r^2}{2}\right]_0^{a(1+\cos\theta)} d\theta$

$= a^2\int_0^{\pi}(1+\cos\theta)^2\,d\theta$

$= 4a^2\int_0^{\pi}\cos^4 d\theta$

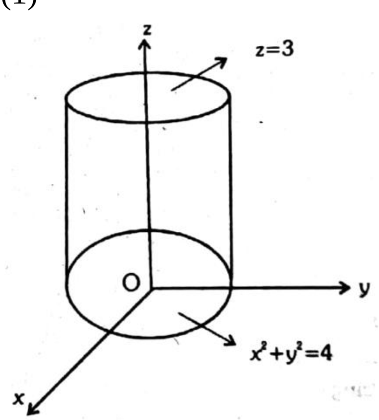
put $\frac{\theta}{2} = \phi$

$\therefore$ Area $= 8a^2\int_0^{\pi/2}\cos^4\phi\,d\phi$

$= 8a^2 \times \frac{3}{4}\times\frac{1}{2}\times\frac{\pi}{2}$

$= \frac{3\pi a^2}{2}$

12. (1)



From the figure

Radis of the base = 2

Height = 3

$\therefore$Volume = $\pi r^2 h$

$= \pi \times 2^2 \times 3 = 12\pi$

13. (2)

Radius of curvature in cartesian coordinates

$$= \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{[1+(y')^2]^{3/2}}{y''}$$

14. (1)

For maximum $\frac{\partial^2 z}{\partial x^2}.\frac{\partial^2 z}{\partial y^2} - \left(\frac{\partial^2 z}{\partial x\,\partial y}\right)^2 > 0$ and $\frac{\partial^2 z}{\partial x^2} < 0$

i.e.$\Delta>0$ and $A<0$

15. (2)

$f = x^2 - xy + y^2 - 2x + y$

$$\frac{\partial f}{\partial x} = 2x - y - 2$$

$$\frac{\partial f}{\partial y} = -x + 2x + 1$$

For stationary points

$$\frac{\partial f}{\partial x} = 0;\ \frac{\partial f}{\partial y} = 0$$

$\Rightarrow$ 2x-y = 2 and x- 2y = 1

Solving (x, y) = (1, 0)

$\therefore$Stationary point is (1, 0)

16. (3)

Let $I = \int_0^{\pi/2}\frac{\sin x\,dx}{\sin x + \cos x}$ ... (1)

Formula: $\int_0^{\pi} f(x)dx = \int_0^{a} f(a-x)dx$

Using the above formula

$I = \int_0^{\frac{\pi}{2}}\frac{\sin\left(\frac{\pi}{2}-x\right)dx}{\sin\left(\frac{\pi}{2}-x\right)+\cos\left(\frac{\pi}{2}-x\right)}$

$= \int_0^{\pi/2} \frac{\cos x\, dx}{\cos x + \sin x}$ ... (2)

Adding (1) and (2)

$2I = \int_0^{\frac{\pi}{2}} \frac{\sin x\, dx}{\sin x + \cos x} + \int_0^{\frac{\pi}{2}} \frac{\cos x\, dx}{\sin x + \cos x}$

$= \int_0^{\frac{\pi}{2}} \frac{\sin x + \cos x}{\sin x + \cos x} dx$

$= \int_0^{\frac{\pi}{2}} dx = [x]_0^{x/2} = \frac{\pi}{2}$

$\therefore I = \frac{\pi}{4}$

17. (2)

$$\int u dv = uv - \int v du$$

Consider

$$u = x; dv = \cos x\, dx$$

$$du = dx; \int dv = \int \cos x\, dx$$

$\Rightarrow v = \sin x$

$\therefore \int x \cos x\, dx = uv - \int v du$

$= x\sin x - \int \sin x dx$

$= x\sin x - (-\cos x)$

$= x\sin x + \cos x$

18. (1)

Consider $x \tan \frac{y}{x} = c$

Differentiating w.r.t. to x

$$x\sec^2\left(\frac{y}{x}\right)\left[\frac{x\frac{dy}{dx} - y}{x^2}\right] + \tan\frac{y}{x} = 0$$

$\Rightarrow \sec^2\left(\frac{y}{x}\right)\left[x\frac{dy}{dx} - y\right] + x\tan\frac{y}{x} = 0$

$\Rightarrow x\sec^2\left(\frac{y}{x}\right)\frac{dy}{dx} + x\tan\frac{y}{x} - y\sec^2\frac{y}{x} = 0$

$\Rightarrow x\sec^2\left(\frac{y}{x}\right)dy + \left(x\tan\frac{y}{x} - y\sec^2\frac{y}{x}\right)dx = 0$

$\therefore x \tan\left(\frac{y}{x}\right) = c$ is a solution of the given differential equation.

19. (2)

$Mdx + Ndy = 0$ is exact if $\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

20. (3)

P.I. $= \frac{e^{4x}}{D-2} = \frac{e^{4x}}{4-2} = \frac{e^{4x}}{2}$

21. (2)

$x + y + z = 24$ and

xy^2z^3 = minimum

Given choices are

$12 \times 3^2 \times 9^3 = 78732$

$14 \times 6^2 \times 4^3 = 32256$

$4 \times 8^2 \times 12^3 = 442368$

$4 \times 3^2 \times 17^3 = 176868$

$\therefore x=14, y=6, z=4$ gives minimum product.

22. (1)

$y = (Ax+2)e^x$

$\Rightarrow ye^{-x} = Ax + 2$

Differentiate with respect to x

$y'e^{-x} - ye^{-x} = A$

Diff. again with respect to x

$y''e^{-x} - y'e^{x} - (y'e^{-x} - ye^{-x}) = 0$

$\Rightarrow y''e^{-x} - 2y'e^{-x} + ye^{-x} = 0$

$\Rightarrow (y'' - 2y' + y)e^{-x} = 0$

since $e^{-x} \neq 0$

$\Rightarrow y'' - 2y' + y = 0$

$\Rightarrow (D^2 - 2D + 1)y = 0$

23. (3)

$x^2y'' - 2xy' - 4y = 0$

$$x^2\frac{d^2y}{dx^2} - 2x\frac{dy}{dx} - 4y = 0$$

put $z = \log x$ and $\theta = \frac{d}{dz}$

then $\frac{dy}{dx} = \theta\, dx$

$$\frac{d^2y}{dx^2} = \theta(\theta - 1)$$

$\therefore$Given equation is reduced to

$(\theta(\theta-1) - 2\theta - 4)y = 0$

$\Rightarrow(\theta^2 - 3\theta - 4)y = 0$

$\Rightarrow(\theta^2 - 3\theta - 4)y = 0$

Auxiliary equation is

$m^2 - 3m - 4 = 0$

$\Rightarrow(m + l)(m-4) = 0$

$\Rightarrow$ m = -1, 4

Solution:

y = Ae^{4z} + Be^{-z}

= $Ae^{4\log x} + Be^{-\log x}$

= $Ae^{\log x^4} + Be^{\log 1/x}$

= $Ax^4 + \frac{B}{x}$

24. (2)

Required equation is

$$m.\frac{dv}{dt} = -mg - \mu v$$

$\Rightarrow$ mx" + μx' = -mg

25. (2)

Radius of curvature $\rho = \frac{(x^2+y^2)^{3/2}}{\dot{x}\ddot{y}-\dot{y}\ddot{x}}$

26. (1)

Check through choices by (1)

$$x = \frac{a}{3}; y = \frac{a}{3}; z = \frac{a}{3}$$

clearly $\frac{a}{3} + \frac{a}{3} + \frac{a}{3} = a$

and xyz = $\frac{a}{3}.\frac{a}{3}.\frac{a}{3} = \frac{a^3}{27}$

by (2)$x = \frac{a}{2}, y = \frac{a}{4}, z = \frac{a}{4}$

clearly$\frac{a}{2} + \frac{a}{4} + \frac{a}{4} = a$

and xyz = $\frac{a}{2}.\frac{a}{4}.\frac{a}{4} = \frac{a^3}{32}$

by (3)

$x = \frac{a}{4}$, $y = \frac{a}{2}$, $z = \frac{a}{4}$

clearly

$$\frac{a}{4} + \frac{a}{2} + \frac{a}{4} = a$$

and xyz = $\frac{a}{4}.\frac{a}{2}.\frac{a}{4} = \frac{a^3}{32}$

By (2)$x = \frac{a}{6}, y = \frac{a}{3}, z = \frac{a}{3}$

$$x + y + z = \frac{a}{6} + \frac{a}{3} + \frac{a}{3} = \frac{5a}{6} \neq a$$

∴Option (4) is not correct

By (1), (2), (3)

Maximum product is attained in (1)

i.e. when $x = \frac{a}{3}, y = \frac{a}{3}, z = \frac{a}{3}$ product is maximum.

Method: 2

Let f=xyz

Given that$x + y + z = a$

take $g = x + y + z - a$

Let $F = f - \lambda g$

$F = xyz - \lambda(x + y + z - a)$

By Lagrange's multiplier method the values of x,y,z for which f is maximum is obained by

$$\frac{\partial F}{\partial x} = 0, \frac{\partial F}{\partial y} = 0, \frac{\partial F}{\partial z} = 0$$

and $\frac{\partial F}{\partial \lambda} = 0$

$$\frac{\partial F}{\partial x} = 0$$

$\Rightarrow yz + \lambda = 0$

$\Rightarrow \lambda = -yz$

$\Rightarrow \lambda x = -xyz$... (1)

$$\frac{\partial F}{\partial y} = 0$$

$\Rightarrow xz + \lambda = 0$

$\Rightarrow \lambda = -xz$

$\Rightarrow \lambda y = -xyz$... (2)

$$\frac{\partial F}{\partial z} = 0$$

$\Rightarrow xy + \lambda = 0$

$\Rightarrow \lambda x = -xyz$... (3)

$$\frac{\partial F}{\partial \lambda} = 0$$

$\Rightarrow x + y + z = a$... (4)

Adding (1) + (2) + (3)

$\Rightarrow \lambda(x + y + z) = -3xyz$

By (4)

λa = -3xyz

$\Rightarrow \lambda = \frac{-3xyz}{a}$

(1) $\Rightarrow \lambda x = -xyz$

$$x\left(\frac{-3xyz}{a}\right) = -xyz$$
$$\frac{3x}{a} = 1$$

$x = \frac{a}{3}$

Similarly

$$y = \frac{a}{3}; z = \frac{a}{3}$$

27. (2)

If $\Delta>0$, $A>0$, then Z has a minimum.

28. (3)

Let $I = \int_0^{\pi} \log x \, dx$

$u = \log x$; $dv = dx$

$du = \frac{1}{x} dx$; $v = x$

$\therefore 1 = uv - \int v du$

$= (x \log x)_0^{\pi} - \int_0^{\pi} x \frac{1}{x} dx$

$= (\pi \log \pi - 0)(x)_0^{\pi}$

$= \pi \log \pi - \pi$

29. (4)

Let $I = \int_0^{\frac{\pi}{2}} \log(\tan\theta) d\theta$

Adding (1) and (2)

$$\int_0^a f(x)dx = \int_0^a f(a-x)dx$$

So, $I = \int_0^{\frac{\pi}{2}} \log \tan\left(\frac{\pi}{2} - \theta\right) d\theta$

$= \int_0^{\frac{\pi}{2}} \log \cot\theta \, d\theta$ .... (2)

Adding (1) and (2)

$2I = \int_0^{\frac{\pi}{2}} [\log \tan\theta \, d\theta + \log \cot\theta] \, d\theta$

$= \int_0^{\frac{\pi}{2}} \log \tan\theta \cot\theta \, d\theta$

$= \int_0^{\frac{\pi}{2}} \log 1 d\theta = \int_0^{\frac{\pi}{2}} 0 \, d\theta$

$= 0 \Rightarrow 1 = 0$

30. (2)

$x^2+y^2+4x-21 = 0$ is an equation of a circle. The radius of curvature of the circle is the radius of the circle.

Radius = $\sqrt{2^2 + 21} = 5$

31. (1)

$$\sqrt{x} + \sqrt{y} = 2$$

Differentiate w.r. to x

$\frac{1}{2}x^{-\frac{1}{2}} + \frac{1}{2}y^{-\frac{1}{2}}.y_1 = 0$ .... (1)

At (1, 1)

$$\frac{1}{2}(1)^{-\frac{1}{2}} + \frac{1}{2}(1)^{-\frac{1}{2}}.y_1 = 0$$
$$\frac{1}{2} + \frac{1}{2}y_1 = 0$$

$\Rightarrow y_1 = -1$

Diff. (1) w.r. to x

$$-\frac{1}{4}x^{-\frac{3}{2}} - \frac{1}{4}y^{-\frac{3}{2}}y_1.y_1 + \frac{1}{2}y^{-\frac{1}{2}}y_2 = 0$$

at (1, 1)

$$-\frac{1}{4}(1)^{-\frac{3}{2}} - \frac{1}{4}(1)^{-\frac{3}{2}}(-1)(-1) + \frac{1}{2}y^{-\frac{1}{2}}y_2 = 0$$

$\Rightarrow -\frac{1}{4} - \frac{1}{4} + \frac{y^2}{2} = 0$

$\Rightarrow y_2 = 1$

Let (α, β) be the centre of curvature at (1, 1) then

$\alpha = x - \frac{y_1(1+y_1^2)}{y_2}$

$= 1 - \frac{(-1)(1+(-1)^2)}{1}$

$= 3$

$\beta = y + \frac{1+y_1^2}{y_2}$

$= 1 + \frac{1+(-1)^2}{1}$

$= 3$

$\therefore$ Centre of curvature = (3, 3)

32. (2)

Let $F = x^2 + y^2 + 6y + 12$

$$\frac{\partial f}{\partial x} = 2x$$
$$\frac{\partial F}{\partial y} = 2y + 6$$

$\frac{\partial^2 F}{\partial x\, \partial y} = 0$

$$\frac{\partial^2 F}{\partial x^2} = 2; \frac{\partial^2 F}{\partial y^2} = 2$$

Now, $\frac{\partial^2 F}{\partial x^2} = 2$

$$\frac{\partial^2 F}{\partial y^2} = 2$$

$$\frac{\partial F}{\partial x} = 0 \Rightarrow x = 0$$

$$\frac{\partial F}{\partial y} = 0 \Rightarrow y = -3$$

∴ The extreme point is (-0, 3)

$$\frac{\partial^2 F}{\partial x^2} \cdot \frac{\partial^2 F}{\partial y^2} - \left(\frac{\partial^2 F}{\partial x\, \partial y}\right)^2$$

= 4 – 0 = 4 > 0

Also $\frac{\partial^2 F}{\partial x^2} = 2 > 0$

∴ The point (0, -3) gives minimum value

Minimum value is F (0, -3)

$= 0^2 + (-3)^2 + 6(-3) + 12$

= 3

33. (4)

put $x = e^x$

$\Rightarrow z = \log x$

Let $D = x\frac{d}{dx}$

then $x^2\frac{d^2}{dx^2} = D(D-1)$

∴ Given equation becomes

$$[D(D-1) + D]y = 5$$

$$\Rightarrow D^2 y = 0$$

$A.E = m^2 = 0$

$\Rightarrow m = 0, 0$

∴ C.F. $= (Az + B)e^{0z}$

= A log x +B

P.I. $= \frac{5}{D^2} = \frac{5e^{0z}}{D^2}$

$= \frac{5}{2}z^2$

$= \frac{5}{2}(\log x)^2$

∴ y = C.F. + P.I.

$= A \log x + B + \frac{5}{2}(\log x)^2$

34. (2)

Let $f(x, y) = 2(x^2 - y^2) - x^4 + y^4$

$$\frac{\partial F}{\partial x} = 4x - 4x^3$$

$\frac{\partial^2 F}{\partial x^2} = 4 - 12x^2$

$$\frac{\partial F}{\partial y} = -4y + 4y^3$$

$$\frac{\partial^2 F}{\partial y^2} = -4 + 12y^2$$

$$\frac{\partial^2 F}{\partial x\, \partial y} = 0$$

At (0, 1)

$$\frac{\partial^2 F}{\partial x^2} \cdot \frac{\partial^2 F}{\partial y^2} - \frac{\partial^2 F}{\partial x\, \partial y} = (4)(-4+12) - 0$$

= 32 > 0

Also $\frac{\partial^2 F}{\partial x^2} = 4 - 12x^2$

$$\frac{\partial^2 F}{\partial x^2}(0,1) = 4 > 0$$

$$\frac{\partial^2 F}{\partial x^2} > 0 \textit{ and}$$

$$\frac{\partial^2 F}{\partial x^2} \cdot \frac{\partial^2 F}{\partial y^2} - \frac{\partial^2 F}{\partial x\, \partial y} > 0$$

So, the point (0, 1) is a minimum point.

35. Let $z = e^x \sin y$

$$\frac{\partial z}{\partial x} = e^x \sin y$$

$$\frac{\partial^2 z}{\partial x^2} = e^x \sin y$$

$$\frac{\partial z}{\partial y} = e^x \cos y$$

$$\frac{\partial^2 z}{\partial y^2} = -e^x \sin y$$

$\therefore \frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = e^x \sin y - e^x \sin y$

=0

36. (1)

y'' -3y' + 2y =2

To find C.F.

$$(D^2 - 3D + 2)y = 0$$

A.E. ⇒ $3^2 - 3m + 2 = 0$

⇒$(m-2)(m-1) = 0$

⇒ m = 2, 1

∴ C.F. = $Ae^x + Be^{2x}$

P.I. = $\frac{2}{D^2-3D+2} = \frac{2e^{0x}}{D^2-3D+2}$

$$= \frac{2e^{0x}}{0-0+2} = 1$$

∴ General solution y = CF + PI

⇒ y = $Ae^x + Be^{2x} + 1$

y = $2e^x - 3e^{2x} + 1$ is in the above from. So it is the required solution.

37. (3)

$6^3+10^3 = 1216$

$8^3+8^3 = 1024$

$4^3+12^3 = 1792$

$9^3+7^3 = 1072$

∴The point (4, 12) gives maximum.

38. (1)

$$x^3y = 3x^4 + 1$$

y = $3x+\frac{1}{x^3}$

$$\frac{dy}{dx} = 3 - 3x^4$$

$$\frac{d^2y}{dx^2} = 12x^{-5} = \frac{12}{x^5}$$

$$\frac{dy}{dx} = 0$$

⇒$3 - 3x^4 = 0$

⇒$3\left(1 - \frac{1}{x^4}\right) = 0$

⇒$x^4 = 1$

⇒ x = ± 1

$$\frac{d^2y}{dx^2} = \frac{12}{x^5}$$

$$\frac{d^2y}{dx^2}(x = -1) = \frac{12}{(-1)^5}$$

= -12 < 0

∴ x = -1 gives maximum.

∴ when x = -1

$$y = 3(-1) + \frac{1}{(-1)^3} = -3 - 1$$

= -4

∴$(-1, -4)$ is the local maximum.

39. (1)

$$\cos^2 3x = \frac{1 + \cos 6x}{2}$$

$$(D^2 + 4)y = \frac{1 + \cos 6x}{2}$$

$$\text{P. I.} = \frac{\frac{1 + \cos 6x}{2}}{D^2 + 4}$$

$$= \frac{\left(\frac{1}{2}\right)}{D^2 + 4} + \frac{\left(\frac{\cos 6x}{2}\right)}{D^2 + 4}$$

$= \frac{e^{0x}}{2(D^2+4)} + \frac{\cos 6x}{2(D^2+4)}$

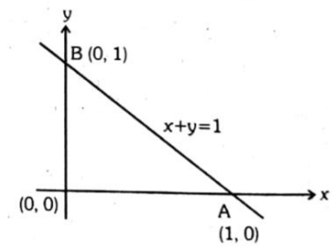
$= \frac{e^{0x}}{2(0^2+4)} + \frac{\cos 6x}{2(-36+4)}$

$= \frac{1}{8} + \frac{\cos 6x}{2(-32)}$

$= \frac{1}{8} - \frac{\cos 6x}{64}$

40. (3)

$\iint_C$ dxdy = Area of the region bounded by C



$\iint_C$ dxdy = Area of Δ OAB

$= \frac{1}{2}(OA \times OB)$

$= \frac{1}{2} \times 1 \times 1 = \frac{1}{2}$

41. (1)

Formula:

By Laplace transformation,

$L(f(t)) = \int_0^\infty e^{-st} f(t)dt$

Also $L(t^n) = \frac{n!}{S^{n+1}}$

Consider $\int_0^\infty x^2 e^{-3x} dx = \int_0^\infty x^2 e^{-St} dt$

where S = 3

$= L(x^2)$

$= \frac{2!}{S^{2+1}} = \frac{2}{S^3} = \frac{2}{3^3} [\because S = 3]$

$= \frac{2}{27}$

Method: 2

$$\int_0^\infty x^2 e^{-3x} dx$$

$u=x^2$; $dv=e^{-3x}dx$

$du=2x\,dx$; $\int e^{-3x}\,dx = \int dv$

$\Rightarrow v = \frac{e^{-3x}}{-3}$

$\therefore \int_0^\infty x^2 e^{-3x} dx = uv - \int vdu$

$= \left(x^2 . \frac{e^{-3x}}{-3}\right)_0^\infty - \int_0^\infty \frac{e^{-3x}}{-3} . 2xdx$

$= 0 + \frac{2}{3}\int_0^\infty x\, e^{-3x}\, dx$

$= \frac{2}{3}\left[\left(\frac{x.e^{-3x}}{3}\right)_0^\infty + \frac{1}{3}\int_0^\infty e^{-3x}\, dx\right]$

$= \frac{2}{3}\left[0 + \frac{1}{3}\left(\frac{e^{-3x}}{-3}\right)_0^\infty\right]$

$= \frac{-2}{27}(e^{-\infty} - e^0)$

$= \frac{2}{27}$

42. (2)

Let $u = e^{\phi}$

$= e^{\log\left(\frac{x^4+y^4}{x+y}\right)}$

$= \frac{x^4+y^4}{x+y}$

∴ u is a homogeneous function of degree 3.

∴ By Euler's theorem

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 3u$$

But $u = e^{\phi}$

$\therefore x\frac{\partial (e^{\phi})}{\partial x} + y\frac{\partial (e^{\phi})}{\partial y} = 3e^{\phi}$

$\Rightarrow xe^{\phi}.\frac{\partial \phi}{\partial x} + ye^{\phi}\frac{\partial \phi}{\partial x} = 3e^{\phi}$

$\Rightarrow x\frac{\partial \phi}{\partial x} + y\frac{\partial \phi}{\partial y} = 3$

43. (1)

put $(1+x) = e^z$

$\Rightarrow \log(1+x) = z$

Also $D = \frac{d}{dz}$

Given equation becomes

$(D(D-1) + D + 1)y = e^z - 1$

Auxiliary equation

$m^2 - m + m + -1 = 0$

$m^2 + 1 = 0$

$\Rightarrow m = \pm i, = 0 \pm i$

Complementary function

$= e^{0z}(c_1 \cos z + c_2 \sin z)$

$= c_1 \cos z + c_2 \sin z$

$= c_1 \cos(\log(1+x)) + c_2 \sin(\log(1+x))$

44. (4)

P.I. $= \frac{\sin 2x - \cos 2x}{D^2+4}$

$= \frac{\sin 2x}{D^2+4} - \frac{\cos 2x}{D^2+4}$

Formula:

$$\frac{\sin \alpha x}{D^2 + \alpha^2} - \frac{-x\cos \alpha x}{2\alpha}$$
$$\frac{\cos \alpha x}{D^2 + \alpha^2} - \frac{x\sin \alpha x}{2\alpha}$$

∴ P.I. $= \frac{-x\cos 2x}{4} + \frac{x\sin 2x}{4}$

45. (1)

$$(D^2 - 3D + 2)y = 2$$

A.E. $= m^2 - 3m + 2 = 0$

$\Rightarrow (m-2)(m-1) = 0$

m = 1, 2

$\Rightarrow$ CF $= Ae^x + Be^{2x}$

P.I. $= \frac{2}{D^2-3D+2}$

$= \frac{2e^{0x}}{D^2-3D+2}$

$= \frac{2e^{0x}}{0-0+2} = \frac{2}{2} = 1$

∴ Solution:

$y'' - 3y + 2y = 2$
is in the form.
$y = Ae^x + Be^{2x} + 1$
since $y = 2e^x + 3e^{2x} + 1$ is in the above from (A = 2; B = -3) it is the solution of
$y'' - 3y + 2y = 2$

46. (4)

$$z(x-y) = x^2 + y^2$$

$\Rightarrow z = \frac{x^2+y^2}{x-y}$
∴ z is a homogeneous function of degree 1
∴By Euler's formula

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = 1.z$$

$$x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} - z = 0$$

47. (3)
$6^3+10^3 = 1216$
$8^3+8^3 = 1024$
$4^3+12^3 = 1792$
$9^3+7^3 = 1072$
∴(4, 12) gives maximum

48. (1)
If $f(x) = xe^{x^2} + c$
then $\frac{df}{dx} = e^{x^2}+2x^2e^{x^2}$
$= e^{x^2}(1+2x^2)$
i.e., derivative of $xe^{x^2}+c$ is xe^{x^2} $(l+2x^2)$
Integration of.e^{x^2} $(1 + 2x^2)$ is $xe^{x^2} + c$

49. (2)
$y^2 = 3x^2+ 1$
Differentiate w.r. to x
$2y\frac{dy}{dx} = 6x$

$$\frac{dy}{dx} = \frac{3x}{y}$$

Slope of the tanget at (1, 2)
$m = \frac{dy}{dx}(1,2)$
$= \frac{3(1)}{2} = \frac{3}{2}$

Slope of the normal at (1, 2)
$= \frac{-1}{m} = \frac{-2}{3}$
∴Equation of the normal at (1, 2) is
$(y-2) = \frac{-2}{3}(x - 1)$
$3y-6 = -2x+2$
$2x+3y-8 = 0$

50. (1)
$x^3y = 3x^4+ 1$
$y = 3x+\frac{1}{x^3}$

$$\frac{dy}{dx} = 3 - 3x^{-4}$$

$$\frac{dy}{dx} = 0$$

$\Rightarrow 3(1 - x^{-4}) = 0$
$\Rightarrow x^4 = 1$
$\Rightarrow x = \pm 1$
∴ $x = \pm 1$

$$\frac{d^2y}{dx^2} = 12x^{-5}$$

$= \frac{12}{x^5}$

$$\frac{d^2y}{dx^2}(x = -1) = \frac{12}{(-1)^5} = -12 < 0$$

∴ x = -1gives maximum.
when x = -1
y = -4
∴ (-1, -4) is the local maximum

51. (3)
Let $I = \int_{-x}^{x} \frac{\sin^6 x}{\cos^6 x+\sin^6 x}dx$
$= 2\int_{-x}^{x} \frac{\sin^6 x}{\cos^6 x+\sin^6 x}dx$
$\Rightarrow \frac{1}{2} = \int_{0}^{x} \frac{\sin^6 x}{\sin^6 x+\cos^6 x}dx$ ... (1)
Applying the formula
$\int_0^{2a} f(x)dx = 2\int_0^{a} f(x)dx$when f(x) is even
(1) $\Rightarrow \frac{1}{2}\int_0^{\frac{\pi}{2}} \frac{\sin^6 x}{\sin^6 x+\cos^6 x}dx$
$\Rightarrow \frac{1}{4} = \int_0^{\frac{\pi}{2}} \frac{\sin^6 x}{\sin^6 x+\cos^6 x}dx$

Using the formula $\int_0^a f(x)dx = \int_0^a f(a-x)dx$

(2) ⇒Using the

$$\Rightarrow \frac{1}{4} = \int_0^{\frac{\pi}{2}} \frac{\sin^6\left(\frac{\pi}{2}-x\right)dx}{\sin^6\left(\frac{\pi}{2}-x\right)+\cos^6\left(\frac{\pi}{2}-x\right)} dx$$

$$= \int_0^{\frac{\pi}{2}} \frac{\cos^6 x\, dx}{\cos^6 x+\sin^6 x} \quad \ldots (3)$$

Adding (2) and (3)

$$\Rightarrow \frac{1}{4}+\frac{1}{4} = \int_0^{\frac{\pi}{2}} \frac{\sin^6 x+\cos^6 x}{\sin^6 x+\cos^6 x} dx$$

$$\Rightarrow \frac{1}{2} = \int_0^{\frac{\pi}{2}} dx$$

$$= (x)_0^{\pi/2} = \frac{\pi}{2}$$

∴ I = π

52. (1)

$$(xe^{xy} + 2y)dy + (ye^{xy}) = 0$$

$$ye^{xy}dx + (xe^{xy} + 2y)dy = 0$$

= Mdx+Ndy

where $M = ye^{xy}$

$N = ye^{xy}+2y$

Now, $\frac{\partial M}{\partial y} = xye^{xy} + e^{xy}$

$$\frac{\partial N}{\partial x} = xye^{xy} + e^{xy}$$

$\therefore \frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

So given equation is exact.

$$\int Mdx = \frac{ye^{xy}}{y} = e^{xy}$$

(keep y as constant)

$$\int Ndy = \frac{xe^{xy}}{x} + \frac{2y^2}{2}$$

(keep x as constant)

$= e^{xy} + y^2$

53. (3)

Given differential equation is $(D^2+4D+3)y=0$

Auxiliary equation is

$m^2+4m+3 = 0$

⇒ (m+3)(m +1) = 0 .

⇒ m = -1,-3

∴Complementary function is $Ae^{-3x} + Be^{-x}$

54. (4)

Given velocity α $(\text{distance})^2$

$$\frac{dx}{dt} \propto x^2$$

$$\frac{dx}{dt} = kx^2$$

$$\frac{d^2x}{dt^2} = 2kx.\frac{dx}{dt}$$

$$= 2kx.kx^2$$

$= 2k^2x^3$

i.e., Acceleration = $2k^2(\text{distance})^3$

⇒ Acceleration ∝ $(\text{distance})^3$

∴ n = 3

55. (2)

Let $f(x, y) = 2(x^2-y^2)-x^4+y^4$

$$\frac{\partial f}{\partial x} = 4x - 4x^3$$

$$\frac{\partial^2 f}{\partial x^2} = 4 - 12x^2$$

$$\frac{\partial f}{\partial y} = -4y + 4y^3$$

$$\frac{\partial^2 f}{\partial y^2} = -4 + 12y^2$$

$$\frac{\partial^2 f}{\partial x \partial y} = 0$$

At (0, 1)

$$\frac{\partial^2 f}{\partial x^2}.\frac{\partial^2 f}{\partial y^2} - \left(\frac{\partial^2 f}{\partial x \partial y}\right)^2 = (4)(-4+12)$$

= 32 >0

Also $\frac{\partial^2 f}{\partial x^2} = 4 - 12x^2$

$$\frac{\partial^2 f}{\partial x^2}(0,1) = 4 > 0$$

∴The point (0, 1) is a minimum point.

56. (1)

Given equation is

$(x^2-4xy-2y^2)dx+(y^2-4xy-2x^2)dy = 0$

Let $M = x^2-4xy-2y^2$

N = y^2-4xy-2x^4

$$\frac{\partial M}{\partial y} = -4x + 4y$$

$$\frac{\partial N}{\partial x} = -4y - 4x$$

since$\frac{\partial M}{\partial y} = \frac{\partial N}{\partial x}$

Given equation is exact.

57. (3)

Auxiliary equation is m^2-2m+1 = 0

⇒ $(m-1)^2$= 0

⇒ m = 1,1

C.F = (A+Bx)e^x

P.I. = $\frac{e^x \cos x}{D^2-2D+1}$

$= e^x \left[\frac{\cos x}{(D+1)^2-2(D+1)+1}\right]$

$= e^x \left[\frac{\cos x}{D^2+2D+1-2D-2+1}\right]$

$= e^x \left[\frac{\cos x}{D^2}\right]$

$= -e^x \cos x$

General solution is

y = C.F+P.I.

$= (A + Bx)e^x - e^x \cos x$

$= e^x[A + Bx - \cos x]$

58. (2)

Formula:

$$\frac{\sin \alpha x}{D^2 + \alpha^2} = \frac{-x \cos \alpha x}{\alpha^2}$$

∴ P.I. = $\frac{\sin 2x}{D^2+4} = \frac{-x \cos 2x}{4}$

59. (2)

$$x^y = e^{x-y}$$

Taking log on both sides

log x^y = log e^{x-y}

⇒ y log x = (x-y)log e

= (x-y) [∵loge=1]

⇒ y log x+y = x

⇒y[1+logx] = x

⇒ y = $\frac{x}{1+\log x}$

$\therefore \frac{dy}{dx} = \frac{(1+\log x)-x.\frac{1}{x}}{(1+\log x)^2} = \frac{\log x}{(1+\log x)}$

60. (1)

Let I = $\int_0^\pi \log(1 - \cos x)dx$ ... (1)

Using the formula $\int_0^a f(x)dx = \int_0^a f(a - x)dx$

I = $\int_0^\pi \log(1 - \cos(\pi - x))dx$

= $\int_0^\pi \log(1 - \cos x)dx$ .... (2)

Adding (1) and (2)

2 I =$\int_0^\pi [\log(1 - \cos x) + \log(1 - \cos x)]dx$

= $\int_0^\pi \log(1 - \cos x)(1 + \cos x)dx$

= $\int_0^\pi \log(1 - \cos^2 x)dx$

= $\int_0^\pi \log \sin^2 x dx$

2 I = 2$\int_0^\pi \log \sin x\, dx$

I =$\int_0^\pi \log \sin x\, dx$

= 2$\int_0^{\frac{\pi}{2}} \log \sin x\, dx$

$\frac{I}{2} = \int_0^{\frac{\pi}{2}} \log \sin x\, dx$ ... (3)

Result:

$$\int_0^{\frac{\pi}{2}} \log \sin x\, dx = \frac{\pi}{2}\log 2$$

∴ (3) ⇒$\frac{1}{2} = \frac{-\pi}{2}\log 2$

⇒ I = $-\pi$ log 2

61. (4)

$$\frac{dy}{dx} = e^{2x+4y+5}$$

$$= e^{2x+5}e^{4y}$$

$\Rightarrow e^{-4y}dy = e^{2x+5}dx$

Integrate$\frac{e^{-4y}}{4} = \frac{e^{2x+5}}{2} + c$

62. (3)

Given D.E. is

$$p^2x^4 - xp - y = 0$$

P's discriminant is

$$x^2 + 4yx^4 = 0$$

$\Rightarrow 1 + 4yx^2 = 0$

$4yx^2 = -1$

$y + \frac{1}{4x^2} = 0$

63. $(1)(D^2 + 4)y = 4\tan 2x$

A.E = $m^2+4=0$

$m = \pm 2i$

C.F. = $c_1\cos 2x + c_2\sin 2x$

P.I. = $Pf_1 + Qf_2$

$f_1 = \cos 2x$

$f_2 = \sin 2x$

$f_1' = -2\sin 2x$

$f_2' = 2\cos 2x$

$f_1f_2' - f_2f_1' = 2\cos 2x\cos 2x - \sin 2x(-2\sin 2x)$

$= 2[\cos^2 2x + \sin^2 2x]$

$= 2$

$p = \int \frac{f_2 X\, dx}{f_1f_2' - f_1'f_2}$

where $x = 4\tan 2x$

$= \frac{-\int \sin 2x.4\tan 2x}{2}dx$

$= -2\int \frac{\sin^2 2x}{\cos 2x}dx$

$= -2\int \frac{1-\cos^2 2x}{\cos 2x}dx$

$= -2[\int \sec 2x\, dx - \int \cos 2x\, dx]$

$= -2\left[\frac{1}{2}\log(\sec 2x + \tan 2x) - \frac{\sin 2x}{2}\right]$

$= \sin 2x - \log(\sec 2x + \tan 2x)$

$Q = \int \frac{f_1 X}{f_1f_2' - f_1'f_2}$

$= \int \frac{\cos 2x.4\tan 2x}{2}dx$

$= 2\int \sin 2x\, dx$

$= -\cos 2x$

P.I. = $Pf_1 + Qf_2$

$= \cos 2x[\sin 2x - \log(\sec 2x + \tan 2x)] - \cos 2x\sin 2x$

$= -\cos 2x\log(\sec 2x + \tan 2x)$