Resonance - Study Materials

RESONANCE

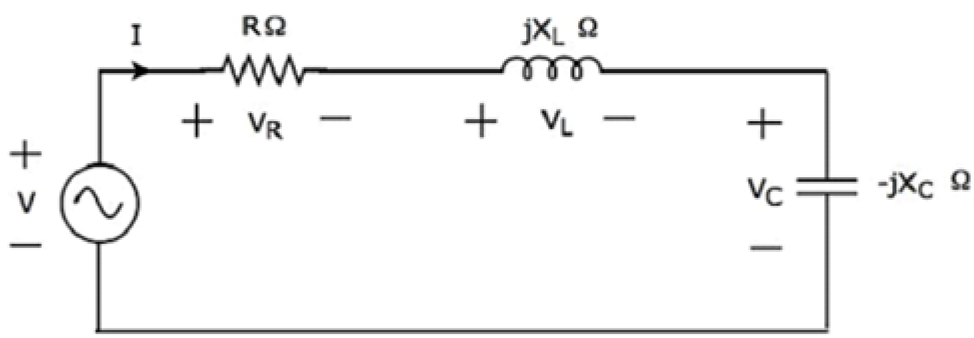
Resonance occurs in electric circuits due to the presence of energy storing elements like inductor and capacitor. It is the fundamental concept based on which, the radio and TV receivers are designed in such a way that they should be able to select only the desired station frequency.

There are **two types** of resonances, namely series resonance and parallel resonance. These are classified based on the network elements that are connected in series or parallel. In this chapter, let us discuss about series resonance.

Series Resonance Circuit Diagram

If the resonance occurs in series RLC circuit, then it is called as **Series Resonance**. Consider the following **series RLC circuit**, which is represented in phasor domain.

Here, the passive elements such as resistor, inductor and capacitor are connected in series. This entire combination is in **series** with the input sinusoidal voltage source.



Apply **KVL** around the loop.

$V-VR-VL-VC=0V-VR-VL-VC=0$

$\Rightarrow V-IR-I(jXL)-I(-jXC)=0\Rightarrow V-IR-I(jXL)-I(-jXC)=0$

$\Rightarrow V=IR+I(jXL)+I(-jXC)\Rightarrow V=IR+I(jXL)+I(-jXC)$

⇒V=I[R+j(XL−XC)]⇒V=I[R+j(XL−XC)]**Equation 1**

The above equation is in the form of ***V* = *IZ***.

Therefore, the **impedance Z** of series RLC circuit will be

Z=R+j(XL−XC)Z=R+j(XL−XC)

Parameters & Electrical Quantities at Resonance

Now, let us derive the values of parameters and electrical quantities at resonance of series RLC circuit one by one.

Resonant Frequency

The frequency at which resonance occurs is called as **resonant frequency** f_r. In series RLC circuit resonance occurs, when the imaginary term of impedance *Z* is zero, i.e., the value of XL−XCXL−XC should be equal to zero.

⇒XL=XC⇒XL=XC

Substitute XL=2πfLXL=2πfL and XC=12πfCXC=12πfC in the above equation.

2πfL=12πfC2πfL=12πfC

⇒f2=1(2π)2LC⇒f2=1(2π)2LC

⇒f=1(2π)LC−−−√⇒f=1(2π)LC

Therefore, the **resonant frequency** f_r of series RLC circuit is

fr=1(2π)LC−−−√fr=1(2π)LC

Where, ***L*** is the inductance of an inductor and ***C*** is the capacitance of a capacitor.

The **resonant frequency** f_r of series RLC circuit depends only on the inductance ***L*** and capacitance ***C***. But, it is independent of resistance ***R***.

Impedance

We got the **impedance Z** of series RLC circuit as

$Z=R+j(XL-XC)Z=R+j(XL-XC)$

Substitute $XL=XCXL=XC$ in the above equation.

$Z=R+j(XC-XC)Z=R+j(XC-XC)$

$\Rightarrow Z=R+j(0)\Rightarrow Z=R+j(0)$

$\Rightarrow Z=R\Rightarrow Z=R$

At resonance, the **impedance *Z*** of series RLC circuit is equal to the value of resistance ***R***, i.e., ***Z* = *R***.

Current flowing through the Circuit

Substitute $XL-XC=0XL-XC=0$ in Equation 1.

$V=I[R+j(0)]V=I[R+j(0)]$

$\Rightarrow V=IR\Rightarrow V=IR$

$\Rightarrow I=VR\Rightarrow I=VR$

Therefore, **current** flowing through series RLC circuit at resonance is $I=VRI=VR$.

At resonance, the impedance of series RLC circuit reaches to minimum value. Hence, the **maximum current** flows through this circuit at resonance.

Voltage across Resistor

The voltage across resistor is

$VR=IRVR=IR$

Substitute the value of ***I*** in the above equation.

VR=□ VR□ RVR=□ VR□ R

⇒VR=V⇒VR=V

Therefore, the **voltage across resistor** at resonance is $\boldsymbol{V_R = V}$.

Voltage across Inductor

The voltage across inductor is

VL=I(jXL)VL=I(jXL)

Substitute the value of ***I*** in the above equation.

VL=□ VR□ (jXL)VL=□ VR□ (jXL)

⇒VL=j□ XLR□ V⇒VL=j□ XLR□ V

⇒VL=jQV⇒VL=jQV

Therefore, the **voltage across inductor** at resonance is VL=jQVVL=jQV.

So, the **magnitude** of voltage across inductor at resonance will be

|VL|=QV|VL|=QV

Where ***Q*** is the **Quality factor** and its value is equal to XLRXLR

Voltage across Capacitor

The voltage across capacitor is

VC=I(−jXC)VC=I(−jXC)

Substitute the value of *I* in the above equation.

VC=□ VR□ (−jXC)VC=□ VR□ (−jXC)

⇒VC=−j□ XCR□ V⇒VC=−j□ XCR□ V

$\Rightarrow V_C = -jQV$⇒VC=−jQV

Therefore, the **voltage across capacitor** at resonance is $V_C = -jQV$VC=−jQV.

So, the **magnitude** of voltage across capacitor at resonance will be

$|V_C| = QV$|VC|=QV

Where ***Q*** is the **Quality factor** and its value is equal to $\frac{X_C}{R}$XCR

Note − Series resonance RLC circuit is called as **voltage magnification** circuit, because the magnitude of voltage across the inductor and the capacitor is equal to *Q* times the input sinusoidal voltage *V*.