17 — MATHEMATICS

(Answer ALL questions)

56. Least number of elements in a group is
1. 2
2. 3
3. 0
4. 1

57. Let $a \in G$ and $k \in I$ then which statement is correct if $O(a) = n$
1. $O(a^k) = n$
2. $O(a^k) < n$
3. $O(a^k) \leq n$
4. $O(a^k) \geq n$

58. The generator of the cyclic group of n^{th} roots of unity is
1. $e^{2\pi i/n}$
2. $e^{4\pi i/n}$
3. $e^{5\pi i/n}$
4. $e^{7\pi i/n}$

59. Which algebraic structure is not a ring?
1. $(2I, +, \cdot)$
2. $(N, +, \cdot)$
3. $(R, +, \cdot)$
4. $(C, +, \cdot)$

60. The cardinality of the centre of Z_{12} is
1. 1
2. 2
3. 3
4. 12

61. If F is a homomorphism of a group G in to a group G' with Kernal K then
1. K is a complex of G
2. K is a subgroup of G but not a normal subgroup of G
3. K is a normal subgroup of G
4. None of these

62. All the eigen values of the matrix $\begin{bmatrix} 1 & 2 & 0 \\ 2 & 1 & 0 \\ 0 & 0 & -1 \end{bmatrix}$ lie in the disc
1. $|\lambda + 1| \leq 1$
2. $|\lambda - 1| \leq 1$
3. $|\lambda + 1| \leq 0$
4. $|\lambda - 1| \leq 2$

63. All norms on a normed space vector space X are equivalent provided
1. X is reflexive
2. X is complete
3. X is finite dimensional
4. X is an inner product space

64. If V be a vector space over the field K then a linear function on V is a linear mapping from
1. V into K
2. K into V
3. V into itself
4. K into itself

65. The vectors $u = (6, 2, 3, 4), v = (0, 5, -3, 1), w = (0, 0, 7, -2)$ are
1. Dependent
2. Independent
3. Data are insufficient
4. None of the above

66. Let T be an arbitrary linear transformation from R^n to R^n which is not one-one then
1. $Rank\ T > 0$
2. $Rank\ T = n$
3. $Rank\ T < n$
4. $Rank\ T = n - 1$

67. The empty set is
1. Unbounded
2. Bounded
3. Neither bounded nor unbounded
4. Both bounded and unbounded

68. The Fourier expansion in the interval $[-4, 4]$ of the function $f(x) = \begin{cases} -x, & -4 \le x \le 0 \\ x, & 0 \le x \le 4 \end{cases}$ has
1. no cosine term
2. no sine term
3. both sine and cosine terms
4. none of the above

69. The radius of convergence of the power series of the function $f(z) = \dfrac{1}{1-z}$ about $z = \dfrac{1}{4}$ is
1. 1
2. $\dfrac{1}{4}$
3. $\dfrac{3}{4}$
4. 0

70. Diameter of a set S in a metric space with metric d is defined by $diam\ (S) = l.u.b\ \{d(x, y)\ /\ x,\ y \text{ in } S\}$. Thus, diameter of the cylinder $C = \{(x,y,z) \text{ in } R^3 \ /\ x^2 + y^2 = 1, -1 < z < 1\}$ in R^3 with standard metric, is
1. 2
2. $2\sqrt{2}$
3. $\sqrt{2}$
4. $\pi + 2$

71. Let $B = \{(x,y,z)\ /\ x,y,z \in R \text{ and } x^2 + y^2 + z^2 \le 4\}$. Let $\vec{v}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$ be a vector valued function defined on B. If $r^2 = x^2 + y^2 + z^2$, the value of the integral $\iiint_B \nabla \cdot \left(r^2 \vec{v}\,(x, y, z)\right) dv$ is
1. 16π
2. 32π
3. 64π
4. 128π

72. If $f(z) = z^3$, then it
1. has an essential singularity at $z = \infty$
2. has a pole of order 3 at $z = \infty$
3. has a pole of order 3 at $z = 0$
4. is analytic at $z = \infty$

73. Two families of curves $u(x, y) = c_1$ and $v(x, y) = c_2$ are orthogonal system
1. $u_x v_x = -u_y v_y$
2. $u_x v_x - u_y v_y = 0$
3. $u_x v_y - u_y v_x = 0$
4. $(u_x - v_x) + (u_y - v_y) = 0$

74. The harmonic conjugate of $u(x, y) = y^3 - 3x^2 y$ is
1. $-y^3 + 3x^2 y + c$
2. $y^3 - 3xy^2 + c$
3. $x^3 - 3x^2 y + c$
4. $x^3 - 3xy^2 + c$

75. The value of $\displaystyle\int_{|z|=2} \frac{e^{2z}}{|z+1|^4}\, dz$ is
1. $2\pi i e^{-1}$
2. $\dfrac{8\pi i}{2} e^{-2}$
3. $\dfrac{2\pi i}{3} e^{-2}$
4. 0

76. The cross ratio of four points (z_1, z_2, z_3, z_4) is real iff the four points lie on a
1. Circle
2. Straight line
3. Circle and on a straight line
4. Circle or on a straight line

77. In the Laurent series expansion of $f(z) = \dfrac{1}{z-1} - \dfrac{1}{z-2}$ valued in the region $|z| > 2$ the coefficient of $\dfrac{1}{z^2}$ is
1. -1
2. 0
3. 1
4. 2

78. The product of two Hausdorff space is a
1. Hausdorff space
2. Discrete space
3. Closed set
4. None of the above

79. The union of a collection of connected sets that have a point in common is
1. Connected
2. Disconnected
3. Separable
4. None of the above

80. Every compact subset of a Hausdorff space is
1. Open set
2. Closed set
3. Null set
4. None of the above

81. The product of completely regular space is
1. Normal
2. Regular
3. Completely Regular
4. None of the above

82. Separable space is
1. a topological space having a countable dense subset
2. a topological space having a countable non dense subset
3. a topological space having a uncountable dense subset
4. none of the above

83. A linear transformation T is bounded if
1. T is continuous
2. T is discontinuous
3. T is discrete
4. None of the above

84. A normed vector space is finite dimensional if
1. the open bounded sets are compact
2. the open bounded sets are not compact
3. the closed bounded sets are not compact
4. the closed bounded sets are compact

85. Orthonormal set is S such that
1. $(x, y) = \begin{cases} 1, & \text{if } x \neq y \\ 0, & \text{if } x = y \end{cases}, \quad x, y \in S$
2. $(x, y) = 0, \quad x, y \in S$
3. $(x, y) = \begin{cases} 0, & \text{if } x \neq y \\ 1, & \text{if } x = y \end{cases}, \quad x, y \in S$
4. None of these

86. If X is a normed vector space, $Y \subset X$ is total if
1. its closed linear span is X
2. its closed linear span is not X
3. its open linear span is X
4. none of the above

87. Improper subspace of vector space X are
1. X only
2. X and null vector space $\{0\}$ of X
3. null vector space $\{0\}$ of X only
4. none of the above

88. The n^{th} order ordinary linear homogenous differential equation have
1. n-singular solutions
2. One singular solution
3. No-singular solution
4. None of these

89. Solving by variation of parameter $y''-2y'+y=e^x \log x$, the value of Wronskian W is
1. 2
2. e^{2x}
3. e^{-2x}
4. 4

90. The linearity principle for ordinary differential equation holds for
1. non-homogeneous equation
2. linear differential equation
3. non-linear equation
4. none of the above

91. General solution of $\frac{d^3y}{dx^3} - 6\frac{d^2y}{dx^2} + 11\frac{dy}{dx} - 6y = 0$ is
1. $y = Ae^x + Be^{2x} + Ce^{3x}$
2. $y = A + Be^{2x}$
3. $y = 3e^x$
4. $y = Ae^{-x} + Be^{-2x} + Ce^{-3x}$

92. The solution of $(y-z)p + (z-x)q = x - y$ is
1. $f(x^2 + y^2 + z^2) = xyz$
2. $f(x+y+z) = x^2 + y^2 + z^2$
3. $f(x+y+z) = xyz$
4. $f(x^2 + y^2 + z^2, xyz) = 0$

93. A partial differential equation is a Quasi linear

1. if it is linear in the lowest order derivative of known function
2. if it is linear in the any derivative of the unknown function
3. if it is linear in the highest order derivative of the unknown function
4. if it is non-linear in the highest order derivative of the unknown function

94. The integral surface satisfying the PDE $\frac{\partial z}{\partial x}+z^2\frac{\partial z}{\partial y}=0$ and passing through the straight line $x=1,\ y=z$ is

1. $(x-1)\,z+z^2=y^2$
2. $(y-z)x+x^2=1$
3. $x^2+y^2-z^2=1$
4. $(x-1)z^2+z=y$

95. Solution of one dimensional heat equation $\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}$, $0<x<1, t>0,\ u(x,0)=2\cos^2\pi x$, $u_t(0,t)=0=u_x(1,\ t)$ is

1. $u(x,t)=1-e^{-4\pi^2 t}\cos 2\pi x$
2. $u(x,t)=1-e^{-4\pi^2 t}\sin 2\pi x$
3. $u(x,t)=1+e^{-4\pi^2 t}\cos 2\pi x$
4. $u(x,t)=1+e^{-4\pi^2 t}\sin 2\pi x$

96. It is required to solve the Laplace equation $\frac{\partial^2 u}{\partial x^2}+\frac{\partial^2 u}{\partial y^2}=0$, $0<x<a$, $0<y<b$ with $u(x,0)=0$, $u(x,3)=0$, $u(0,y)=0$ and $u(a,y)=f(y)$. If c_n's are constants, then the equation and the homogeneous boundary conditions determine the fundamental set of solutions of the form

1. $u(x,y)=\sum_{n=1}^{\infty}c_n\sinh\frac{n\pi x}{b}\sin\frac{n\pi y}{b}$
2. $u(x,y)=\sum_{n=1}^{\infty}c_n\sin\frac{n\pi x}{b}\sinh\frac{n\pi y}{b}$
3. $u(x,y)=\sum_{n=1}^{\infty}c_n\sin\frac{n\pi x}{b}\sin\frac{n\pi y}{b}$
4. $u(x,y)=\sum_{n=1}^{\infty}c_n\sinh\frac{n\pi x}{b}\sinh\frac{n\pi y}{b}$

97. A function $u(x,t)$ satisfies the wave equation $\frac{\partial^2 u}{\partial t^2}=\frac{\partial^2 u}{\partial x^2}$, $0<x<1$, $t>0$. If $u(1/2,0)=1/4$, $u(1,1/2)=1$ and $u(0,1/2)=1/2$, then $u(1/2,1)$ is

1. 7/4
2. 5/4
3. 4/5
4. 4/7

98. The integral surface of the PDE $x\frac{\partial u}{\partial x}+y\frac{\partial u}{\partial x}=0$ satisfying the condition $u(1,y)=y$ is given by

1. $u(x,y)=y/x$
2. $u(x,y)=y/(2-x)$
3. $u(x,y)=2y/(x+1)$
4. $u(x,y)=y+x-1$

99. The integral $\in\int_a^b\left(t\frac{\partial F}{\partial s}-\frac{dt}{dx}\frac{\partial F}{\partial s'}\right)dx$, where $y=s(x)$, $t=t(x)$ and $F=F(x,y,y')$ is referred as

1. First variation
2. Third variation
3. Second variation
4. Fourth variation

100. Given the function $F=F(x,y,y')$, the differential equation $\frac{\partial F}{\partial y}-\frac{d}{dx}\left(\frac{\partial F}{\partial y'}\right)=0$ is referred as

1. Lagrange's equation
2. Hamilton Equation
3. Euler's characteristic equation
4. De 'Alembert Equation

101. The integral I has strong minimum if

1. the arc AB of the arc of integration Γ_e, contains point conjugate to either A or B
2. the arc AB of the arc of integration Γ_e, contains point conjugate to both A and B
3. the arc AB of the arc of integration Γ_e, contains no point conjugate to either A or B
4. None of these

102. Let $k(x, y)$ be a given real function defined for $0 \le x \le 1$, $0 \le y \le 1$ and $f(x)$ a real valued function defined for $0 \le x \le 1$ and λ an arbitrary complex number. Then $\int_0^1 k(x, y)\varphi(y)\, dy = f(x)$, $(0 \le x \le 1)$ is the

1. Linear Fredholm integral equation of first kind
2. Linear Fredholm integral equation of second kind
3. Volterra integral equation of first kind
4. Volterra integral equation of second kind

103. A Volterra integral equation has

1. one eigen values
2. three eigen values
3. two eigen values
4. no eigen values

104. The singular points of the resolvent kernel H corresponding to a symmetric L_2 kernel $k(x, y)$ are

1. simple poles
2. not a simple pole
3. double poles
4. pole of order greater than 2

105. In a hypothesis testing problem, which of the following is NOT required in order to compute the p-value?

1. Value of the test statistic
2. Distribution of the test statistic under the null hypothesis
3. The level of significance
4. Whether the test is one tailed (or) two tailed

106. Suppose person A and person B draw random samples of sizes 15 and 20 respectively from $N(\mu, \sigma^2)$, $\sigma^2 > 0$ for testing $H_0 : \mu = 2$ against $H_1 : \mu > 2$. In both cases, the observed sample means and sample S.Ds are same with $\bar{x}_1 = \bar{x}_2 = 1.8$, $s_1 = s_2 = s$. Both of them use the usual t-test and state the p-values p_A and p_B respectively. Then which of the following is correct?

1. $p_A > p_B$
2. $p_A < p_B$
3. $p_A = p_B$
4. relation between p_A and p_B depends on the value of s

107. In a clinical trail 'n' randomly chosen persons were enrolled to examine whether two different skin creams A and B have different effects on the human body. Cream A was applied to one of the randomly chosen arms of each person, cream B to the other arm. Which statistical test is to be used to examine the difference? Assume that the response measured is a continuous variable.

1. Two sample t-test if normality can be assumed
2. Paired t-test if normality can be assumed
3. Two sample Kolmogorov-Smirnov test
4. Test for randomness

108. Let T be the matrix (occurring in a typical transportation problem) given by $\begin{pmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix}$. Then

1. Rank $T = 4$ and T is unimodular
2. Rank $T = 3$ and T is unimodular
3. Rank $T = 4$ and T is not unimodular
4. Rank $T = 3$ and T is not unimodular

109. Consider the linear programming formulation ($P2$) of optimally assigning 'n' men to 'n' jobs with respect to some costs $\{c_{ij}\}_{i,\, j=1, \ldots, n}$. Let A denote the coefficient matrix of the constraint set. Then

1. rank of A is $2n$–1 and every basic feasible solution of $P2$ is integer valued
2. rank of A is $2n$–1 and every basic feasible solution of $P2$ is non integer valued
3. rank of A is $2n$ and every basic feasible solution of $P2$ is integer valued
4. rank of A is $2n$ and every basic feasible solution of $P2$ is non integer valued

110. The daily wages (in Rupees) of workers in two cities are as follows

	Size of the sample	S.D. of wages
City A	16	25
City B	13	32

Test at 5% level, the equality of variances of the wage distribution in the two cities. [Tabulated value of F for (12, 15) degree of freedom at 5% LOS is 2.48]

1. Variances of the wage distribution in the two cities may be equal
2. Variances of the wage distribution in the two cities may not be equal
3. Data in sufficient
4. None of the above

111. A simple random sample of size 'n' is to be drawn from a large population to estimate the population proportion θ. Let p be the sample proportion. Using the normal approximation, determine which of the following sample size value will ensure $|p-\theta| \leq 0.02$ with probability at least 0.95, irrespective of the true value of θ? [You may assume, $\varphi(1.96) = 0.975$, $\varphi(1.64) = 0.95$, where φ denotes the cumulative distribution function of the standard normal distribution]

1. $n = 1000$
2. $n = 1200$
3. $n = 1500$
4. $n = 2500$

112. The random variable X has a t-distribution with υ degree of freedom. Then the probability distribution of X^2 is

1. Chi-square distribution with 1 degree of freedom
2. Chi-square distribution with υ degree of freedom
3. F-distribution with $(1, \upsilon)$ degree of freedom
4. F-distribution with $(\upsilon, 1)$ degree of freedom

113. The objective function of the dual problem for the following primal LPP :

$Max\, f = 2x_1 + x_2$ subject to

$x_1 - 2x_2 \geq 2$

$x_1 + 2x_2 = 8$

$x_1 - x_2 \leq 11$ with $x_1 \geq 0$ and x_2 unrestricted in sign, is given by

1. $\min z = 2y_1 - 8y_2 + 11y_3$
2. $\min z = 2y_1 - 8y_2 - 11y_3$
3. $\min z = 2y_1 + 8y_2 + 11y_3$
4. $\min z = 2y_1 + 8y_2 - 11y_3$

114. Let 'x' be a non-optimal feasible solution of a linear programming maximization problem and 'y' a dual feasible solution. Then

1. The primal objective value at x is greater than the dual objective value at y
2. The primal objective value at x could equal the dual objective value at y
3. The primal objective value at x is less than the dual objective value at y
4. The dual could be unbounded

115. Consider the following LPP

$Min\, z = 2x_1 + 3x_2 + x_3$ subject to constraints

$x_1 + 2x_2 + 2x_3 - x_4 + x_5 = 3$

$2x_1 + 3x_2 + 4x_3 + x_6 = 6,\ x_i \geq 0,\ i = 1, 2, \ldots, 6$.

A non degenerate basic feasible solution $(x_1, x_2, \ldots, x_6)$ is

1. (1, 0, 1, 0, 0, 0)
2. (0, 0, 0, 0, 3, 6)
3. (1, 0, 0, 0, 0, 7)
4. (3, 0, 0, 0, 0, 0)